Non-rigid supports

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Non-rigid supported hollow core floor
Introduction

Need for flexible, adaptable space

- Reduce the number of columns
- Long spans >> hollow core floor
- With integrated steel, concrete or composite beams
- Minimize the structural depth >> possibilities for an additional storey
- Clear route for services
Non-rigid supported hollow core floor

- Reduce the number of columns
- Long spans >> hollow core floor
- With integrated steel, concrete or composite beams
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- Clear route for services
Introduction

Integrated beams – Shallow beams

Non-rigid supported hollow core floor
Preliminary tests showed that the **resisting shear force was influenced** by the behaviour of the integrated beam.
Mechanical behaviour

Bending in two directions

Non-rigid supported hollow core floor
**Mechanical behaviour**

**Bending in two directions**

Due to loading the floor and the beam deflect.

Cracks at the interface of beam and in situ concrete or in situ concrete and end of the hollow core unit.

Longitudinal cracks along strands (A)

Shear deformation of the hollow core slab (B) or sliding of hollow core slab along the beam (B)

=> *non-intended composite action*
Full scale tests

Tests in the 1990’s and 2000’s

Non-rigid supported hollow core floor
## Full scale tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Hollow core slab</th>
<th>length of slab</th>
<th>Beam</th>
<th>Length of beam</th>
<th>$V_{pim}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DE265</td>
<td>h=265</td>
<td>6000</td>
<td>Delta beam</td>
<td>5000</td>
</tr>
<tr>
<td>2</td>
<td>WQ265</td>
<td>h=265</td>
<td>6000</td>
<td>Steel beam</td>
<td>5000</td>
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<tr>
<td>3</td>
<td>PC265</td>
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<td>6000</td>
<td>Prestressed concrete</td>
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<tr>
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<td>7200</td>
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<tr>
<td>5</td>
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<td>Prestressed concrete with filled cores</td>
<td>5000</td>
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<td>7</td>
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<td>h=265</td>
<td>6000</td>
<td>Prestressed concrete with topping</td>
<td>5000</td>
</tr>
<tr>
<td>8</td>
<td>PC265N</td>
<td>h=265</td>
<td>6000</td>
<td>Prestressed concrete</td>
<td>5000</td>
</tr>
<tr>
<td>9</td>
<td>PC265C</td>
<td>h=265</td>
<td>6000</td>
<td>Prestressed concrete continues beam</td>
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<td>11</td>
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<td>h=265</td>
<td>6000</td>
<td>Prestressed concrete</td>
<td>7200</td>
</tr>
<tr>
<td>12</td>
<td>LBL320</td>
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<td>7200</td>
<td>LBL-beam</td>
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<td>13</td>
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<td>h=400</td>
<td>8500</td>
<td>Delta beam</td>
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</tr>
<tr>
<td>14</td>
<td>SUP320</td>
<td>h=320</td>
<td>10000</td>
<td>Superbeam</td>
<td>4800</td>
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<td>15</td>
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<td>LB-beam</td>
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</tr>
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<td>9000</td>
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<td>20</td>
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<td>A-beam</td>
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<tr>
<td>21</td>
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<td>6000</td>
</tr>
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<td>5000</td>
<td>IFB-beam</td>
<td>6000</td>
</tr>
<tr>
<td>23</td>
<td>IFB250</td>
<td>h=250</td>
<td>5000</td>
<td>IFB-beam</td>
<td>6000</td>
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<tr>
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<td>IFB-beam</td>
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<td>25</td>
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<td>IFB-beam</td>
<td>6000</td>
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<tr>
<td>26</td>
<td>IFB265RD</td>
<td>h=265</td>
<td>5000</td>
<td>IFB-beam</td>
<td>6000</td>
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<tr>
<td>27</td>
<td>IFB250M</td>
<td>h=250</td>
<td>5000</td>
<td>IFB-beam</td>
<td>6000</td>
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<tr>
<td>28</td>
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<td>h=250</td>
<td>5000</td>
<td>IFB-beam</td>
<td>6000</td>
</tr>
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<td>29</td>
<td>PC270Bi</td>
<td>h=270</td>
<td>4300</td>
<td>Prestressed concrete</td>
<td>6000</td>
</tr>
<tr>
<td>30</td>
<td>IFB150</td>
<td>h=150</td>
<td>4200</td>
<td>IFB-beam</td>
<td>3600</td>
</tr>
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<td>31</td>
<td>IFB150T</td>
<td>h=150</td>
<td>4200</td>
<td>IFB-beam</td>
<td>3600</td>
</tr>
</tbody>
</table>
Integrated beams – Shallow beams

Non-rigid supported hollow core floor
Non-rigid supported hollow core floor

Full scale tests

Shear-tension failure
Non-rigid supported hollow core floor
Design model

*fib* Bulletin 6
Codecard 18
CUR/BmS Aanbeveling 104

Roggendorf:

Shows the shear forces in a composite structure when the cracks in b) cannot transfer any stresses.
The failure is controlled by the tensile principal stress of the concrete, which equals to the design strength value when the failure is assumed to follow.

The principal tensile stress is calculated according to the transformed plane stress condition:

$$\sigma_i = f_{ct} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau_1^2 + \tau_2^2}$$
Design model

\[ \sigma_I = f_{ct} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau_1^2 + \tau_2^2} \]

Where \( \tau_2 \) is shear stress due to the non-intended composite action in the direction of the beam.

Challenge: how to describe/derive \( \tau_2 \)?
Design model

Beam model

The horizontal shear flow due to the non-intended composite action:

\[ \tau = V \cdot \frac{S}{b \cdot I} \]

\[ \nu_{yd} = V_{beam} \cdot \frac{ES_{topflanges}}{EI_{beam}} \]
The shear stress in transverse direction due to the non-intended composite action:

\[ \tau_2(y) = \frac{3 \cdot vyd \cdot b_{slab}}{4 \cdot b_{cr}(y) \cdot b_w(y)} \]
Design model

Beam model

\[ b_{cr} \]

*fib* Bulletin 6:

Current:

\[ b_{cr}(y) = 2y \geq h \]
Design model

Beam model

The principal stress included the transverse shear stress:

\[ \sigma_I = fct = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau_1^2 + \tau_2^2} \]

\(\tau_2\) is a function of \(ES_{\text{topflangens}}\); respectively a function of \(b_{\text{eff}}\).

How to derive \(b_{\text{eff}}\)?
Calibration of the design model to the tests

**Coefficient $k_{cd}$**

<table>
<thead>
<tr>
<th>Depth of hc slab</th>
<th>Concrete beam</th>
<th>With concrete filled steel beam</th>
<th>Steel beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 mm</td>
<td>0.026</td>
<td>0.021</td>
<td>0.010</td>
</tr>
<tr>
<td>200 mm</td>
<td>0.026</td>
<td>0.021</td>
<td>0.010</td>
</tr>
<tr>
<td>260 mm</td>
<td>0.029</td>
<td>0.023</td>
<td>0.011</td>
</tr>
<tr>
<td>320 mm</td>
<td>0.031</td>
<td>0.022</td>
<td>0.013</td>
</tr>
<tr>
<td>400 mm</td>
<td>0.035</td>
<td>0.022</td>
<td>0.014</td>
</tr>
<tr>
<td>500 mm</td>
<td>0.040</td>
<td>0.028</td>
<td>0.020</td>
</tr>
</tbody>
</table>

The design width:  

$$b_{eff} = L \cdot k_{cd}$$

where $L$ is the effective span length of the beam (distance between moment zero points)
Design model

Line of failure

Zone affected by support reaction

Centroidal axis

Critical point

β = 35°

Considered points

Considered sections

Non-rigid supported hollow core floor
Influence of filled cores

Filled cores prevent the deformation of the hollow core cross section, in care of filled cores the shear stress $\tau_2$ can be reduced with a factor $\beta$:

<table>
<thead>
<tr>
<th>Slab thickness [mm]</th>
<th>200</th>
<th>265</th>
<th>320</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filling length &lt; 50 mm</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Filling length at least equal</td>
<td>0.70</td>
<td>0.70</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>to the depth of void</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All voids filled</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values for $\beta$. For filling lengths shorter than depth of the core, linear interpolation may be applied.
Debonding of strands

Check (ULS) the slab at midspan (highest curvature), when exceeding $f_{ctd}$ a number of debonded strands should be taken into account.

<table>
<thead>
<tr>
<th>Strands per web</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of debonded strands per slab unit</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of debonded strands per web</td>
<td></td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Length of debonding (mm)</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 4.8-3: Debonding of strands

Non-rigid supported hollow core floor
Serviceability limit state

Check allowable strain and/or crack width:

<table>
<thead>
<tr>
<th>$\varepsilon_{cr} \times 10^3$</th>
<th>$w_k$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>1.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

$$\varepsilon_2 = \frac{M_{Ed\text{add}} \cdot Z}{EI}$$
Transverse shear stress

Example calculation

Slab length: 9.0 meter
Self weight: 3.6 kN/m²
Dead load: 1.2 kN/m²
Live load: 5.0 kN/m²
Transverse shear stress

Example calculation

Beam: THQ 265.10-240.30-450.20

Span of the beam: \( L_{\text{beam}} = 5000 \text{ mm} \)

Depth of filled concrete in the cores: \( \text{FillingDepth} = 50 \text{ mm} \)
Transverse shear stress

Example calculation

The composite beam:

Effective width: \[ b_{\text{eff}} := L_{\text{beam}} \cdot k_{\text{cd}} \]

\[ k_{\text{cd}} := 0.011 \]

\[ b_{\text{eff}} = 55 \text{ mm} \]

Influence of filled cores:

\[ \beta_f := \beta_{\text{filledcores}}(\text{FillingDepth}) = 1.0 \]
Transverse shear stress

Example calculation

The composite cross section for the calculation of the transverse shear stresses:

\[
\begin{align*}
T_{xt} &= \begin{pmatrix}
"Beam" \\ "Joint" \\ "Top flange" \\ "Under flange" \\ "Topping flange" \\ "Topping beam"
\end{pmatrix} \\
A &= \begin{pmatrix}
21500 \\ 7950 \\ 6800 \\ 0 \\ 0 \\ 0
\end{pmatrix} \\
Y &= \begin{pmatrix}
112.2 \\ 132.5 \\ 248.0 \\ 0.0 \\ 0.0 \\ 0.0
\end{pmatrix} \\
I &= \begin{pmatrix}
575804167 \\ 186096250 \\ 418882267 \\ 0 \\ 0 \\ 0
\end{pmatrix} \\
E &= \begin{pmatrix}
210000 \\ 29962 \\ 36283 \\ 36283 \\ 29962 \\ 29962
\end{pmatrix}
\end{align*}
\]
Transverse shear stress

Example calculation

\[
EA_{0.1} := \sum_{j = 0}^{\text{hollowcore}} \left( E_j \cdot A_j \right) = 4999923191
\]

\[
ES_{0.1} := \sum_{j = 0}^{\text{hollowcore}} \left( E_j \cdot A_j \cdot Y_j \right) = 599321638805
\]

\[
Y_{0.1} := \frac{ES_{0.1}}{EA_{0.1}} = 119.9 \text{ mm}
\]

\[
EI_{0.1} := \sum_{j = 0}^{\text{hollowcore}} \left( \frac{E_j \cdot I_j}{EA_{0.1}} \right) - \frac{ES_{0.1}^2}{EA_{0.1}} = 69854676938884
\]
Transverse shear stress

Example calculation

Moment of area of the top flange:

\[ E_{S_{f.1}} := E_{\text{topflange}} \cdot A_{\text{topflange}} \cdot \left( Y_{\text{topflange}} - Y_{0.1} \right) = 31613906539 \]
Transverse shear stress

Example calculation

Loads on the beam:
design load beam due to added load:

\[ q_{d.add.beam} := \left( \gamma_g \cdot p_{g3} + \gamma_q \cdot p_q \right) \left( SL + 2 \cdot \frac{\text{SlabOffset}}{1000} \right) = 83.1 \text{ kN/m} \]

The shear force in the beam inducing a transversal shear flow:

\[ V_{Ed} = \left( q_{d.add.beam} \right) \cdot \frac{L_{beam}}{2} = 207631 \text{ N} \]
Transverse shear stress

Example calculation

Horizontal shear flow $v_{yd}$ in the composite beam:

$$v_{yd} := V_{Ed} \cdot \frac{E\sigma_{f.1}}{E I_{0.1}}$$

$v_{yd} = 94.0$ N/mm
Transverse shear stress

Example calculation

Shear stress in the hollow core webs in transverse direction:

$$\tau_2(y) := \frac{3 \cdot v_{yd} \cdot b_{sl}}{4 \cdot b_{cr}(y) \cdot b_1(y)}$$

$$\tau_2(90) = 1.36 \text{ MPa}$$

Shear stress due to the vertical load:

$$V_{Ed, hc} = 70633 \text{ N}$$

$$\tau_1(y) := \frac{V_{Ed, hc} \cdot S_{c1}(y)}{b_1(y) \cdot I_{i1}}$$

$$\tau_1(90) = 0.98 \text{ MPa}$$
Transverse shear stress

Example calculation

Principal stress:

\[
\sigma_1(\text{limit, } x, y) := -\frac{\sigma_{cp}(\text{limit, } x, y)}{2} + \sqrt{\left(\frac{\sigma_{cp}(\text{limit, } x, y)}{2}\right)^2 + \left(\tau_1(y) + \tau_{cp}(\text{limit, } x, y)\right)^2 + \left(\beta_2 \tau_2(y)\right)^2}
\]
Transverse shear stress

Example calculation

Critical point found:

\[ y := \text{Position}(\sigma_1, \text{upper}) = 90 \text{ mm} \]

\[ x := X_{\text{fail}}(y) = 0.21 \text{ m} \]

\[ \sigma_1(\text{upper}, x, y) = 1.37 \text{ MPa} \]

\[ \sigma_1(\text{lower}, x, y) = 1.25 \text{ MPa} \]

\[ \sigma_{\text{l,max}} = 1.37 \text{ MPa} \]

\[ f_{\text{ctd1}} = 1.77 \text{ MPa} \]

Requirement \( (\sigma_{\text{l,max}} \leq f_{\text{ctd1}}) \) = "is fulfilled"
Transverse shear stress

Example calculation

The stress components:

- Principle stress
- Normal stress: Sig_cp
- Shear stress: Tau_1
- Shear stress: Tau_cp
- Transverse shear stress: Tau_2
- Concrete tensile strength

Non-rigid supported hollow core floor
Non-rigid supported hollow core floor

Fire design

For shear resistance in fire:
Go down to the shear resistance in fire condition in cracked situation:

\[ V_{Rd,c,\bar{f}} = \left[ C_{\theta,1} + \alpha_k \cdot C_{\theta,2} \right] \cdot b_w \cdot d \]
Non-rigid supported hollow core floor

Summary

- The shear tension resistance is influenced by the properties of the beam.
- It is about non-intended composite action.
- For design: use the same approach as for shear tension capacity on rigid support; but add a shear stress due to this possible composite action
- For cracked sections the known shear flexure capacity applies
- In case of fire: cracked section => same design formula as rigid support
Thank you for your attention