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Restrained support and cantilever

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Restrained support and cantilever

design of restrained supports, preliminary design rules, design of cantilevered slabs, unintended support restraint

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Restrained support design

HC slabs are mainly designed as simply supported floor units.

HC floor slabs can also be constructed with restrained support conditions, especially in combination with cast in-situ beams and walls, or in composite structures.

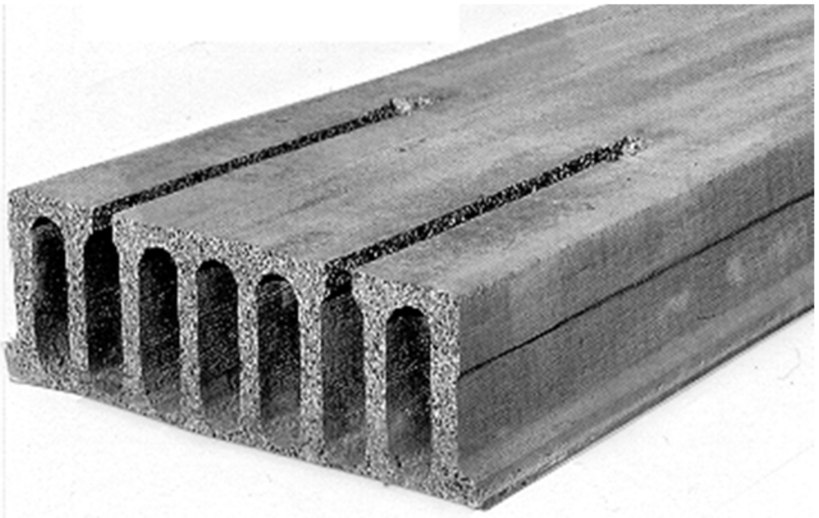
Restrained support design

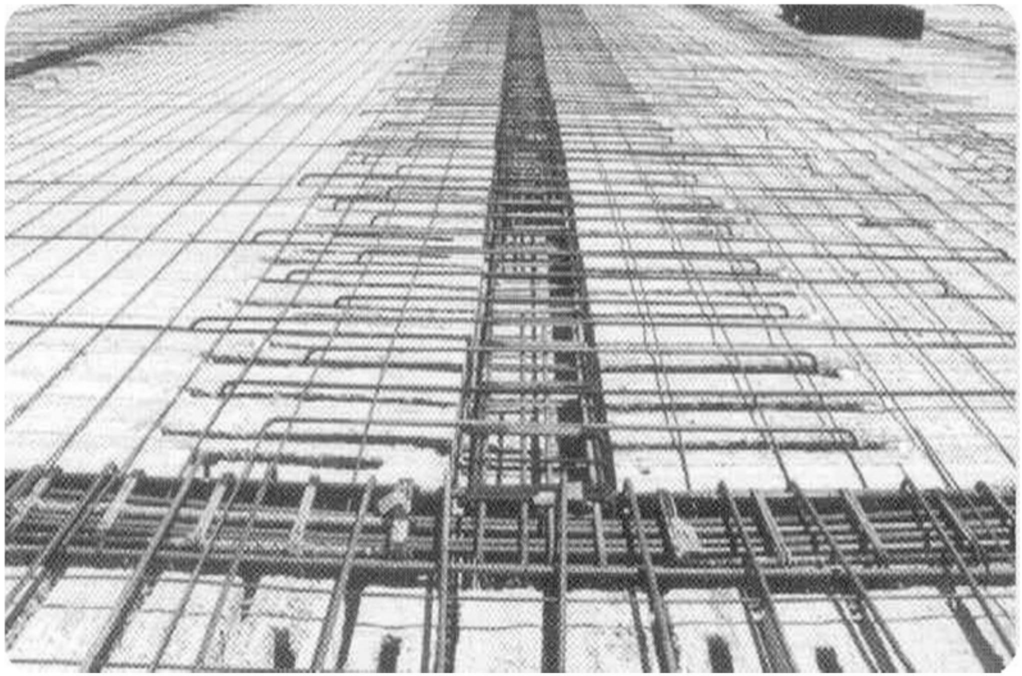
Advantages of the design with floor continuity and/or restrained support:

- **Reduce the magnitude of positive sagging moment** (lowering the shear capacity), leading to shallower floor depths and/or greater spans;
- **Reduce deflections and crack widths** at SLS ;
- **Increase structural integrity between floor unit and support** and allow composite support connections without direct support under the precast hollow core slabs;
- **Improve diaphragm action** in resisting horizontal loads (wind, earth pressure or seismic actions).

Restrained support design

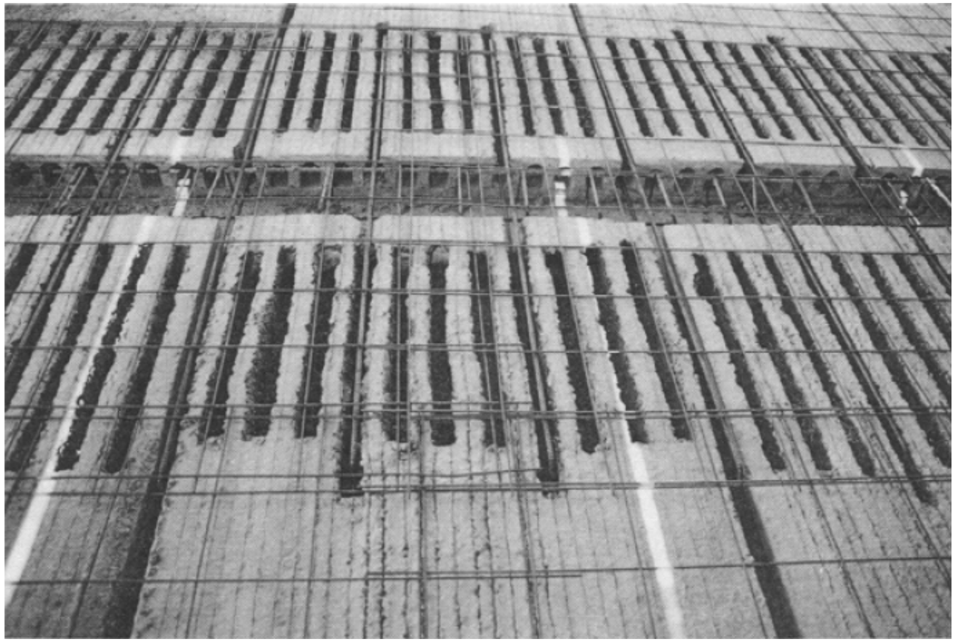
The additional in situ reinforcement and concrete is placed in open cores at the slab end.



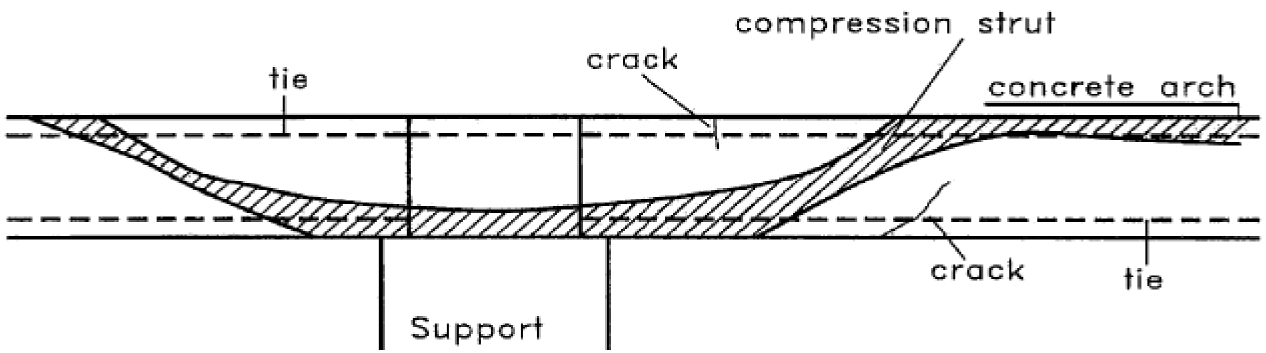


Restrained support design

The longitudinal continuity at the support zone of HC floors can be achieved by suitable reinforcement able to withstand negative moments.

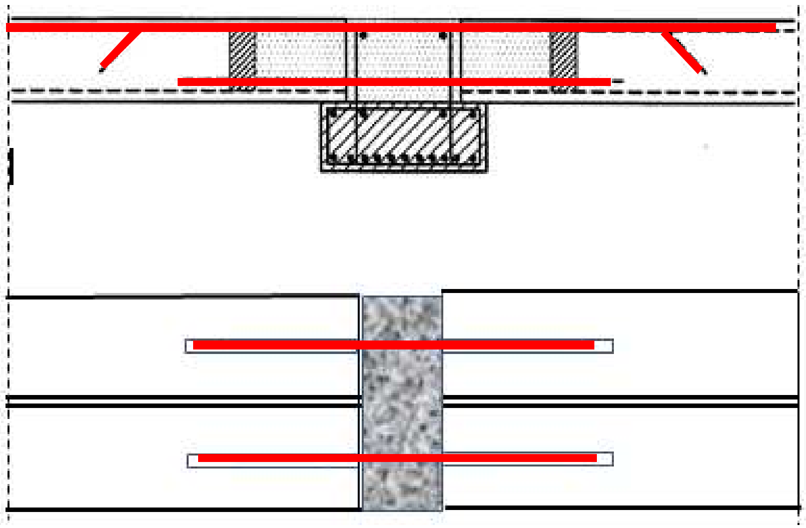


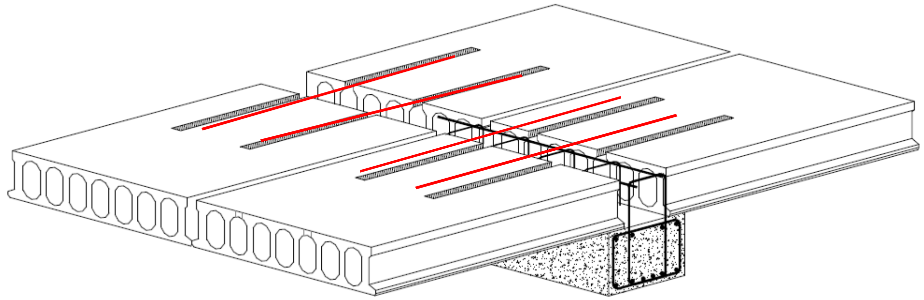
Arch and tie mechanism



Possible support conditions for restrained support systems

- ***Direct support*** *of the hollow core floor on a wall or a beam*



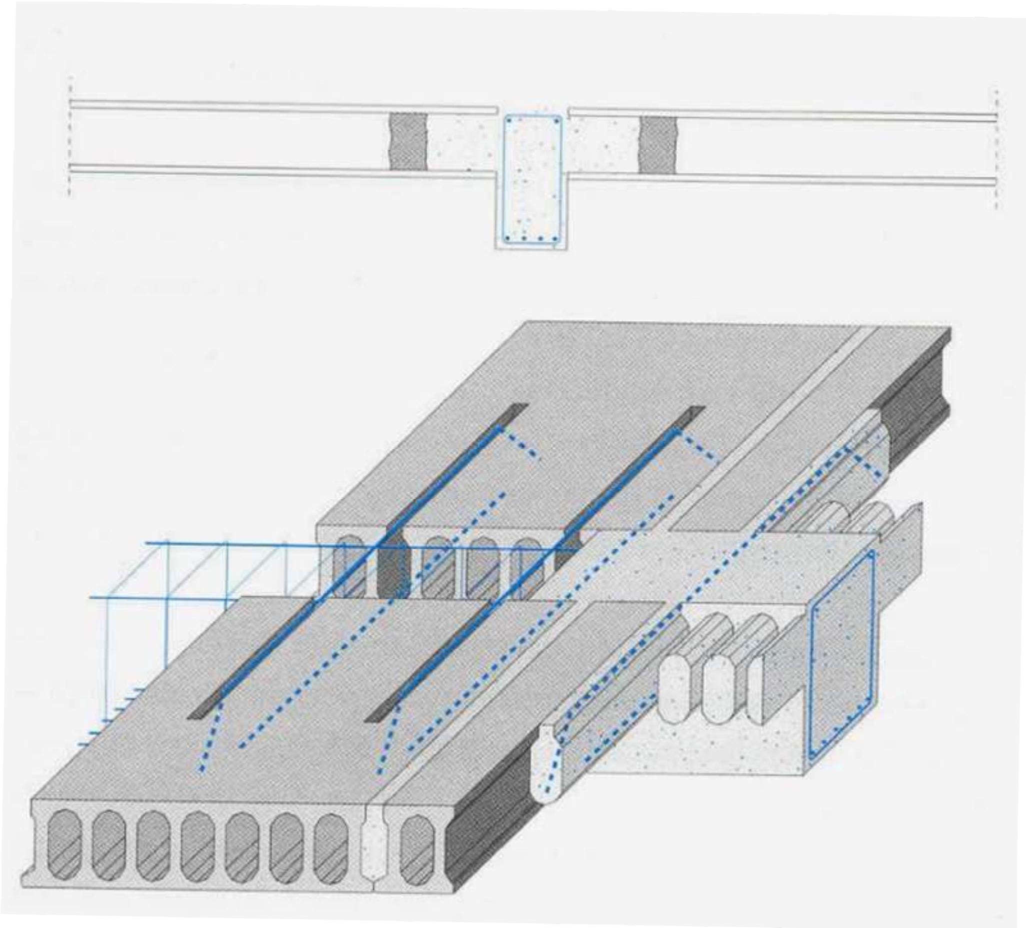


The connection can aim at:

- fully restrained both at SLS and ULS,
- partially restrained only at SLS for the frequent load combination. The ULS is designed as simply supported.

Possible support conditions for restrained support systems

- ***Indirect composite support***

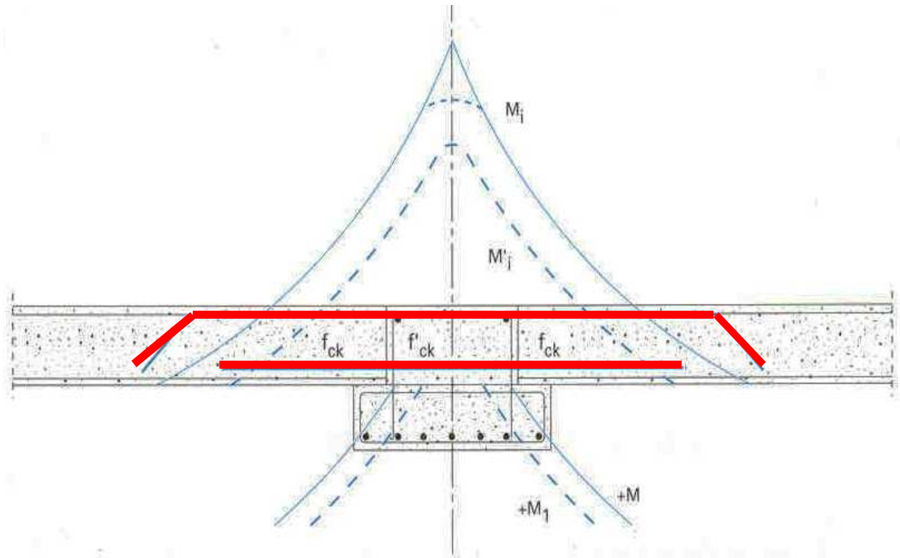


Note:
“Composite support“ may also be adopted in case of insufficient support length, as an alternative to “protruding strands”.
In this case the connection of the hollow core elements to the support is realised by adequate in-situ bottom reinforcement in open cores.
(min. 2-3 per slab unit –
- length 0.8-1.0m).

Negative bending moments

The negative moment shall be determined according to the floor continuity scheme, taking into account all variable and permanent loads acting on the floor after hardening of the composite connection and long term effects.

Top prestressing strands are generally required to limit:
- the length of additional rebars ;
- cracking due to negative bending moment.

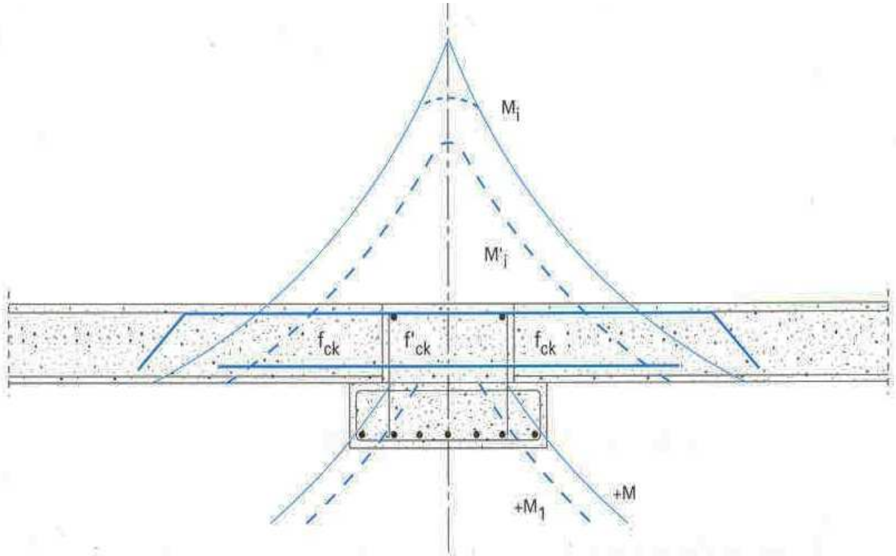


The design negative moment M^{-}_{sd} at ULS may be reduced by approx. 25 %, due to the ductility of the connection.

The design value of the negative moment may sometimes be increased by long term effects.

Negative bending moments

$$M^{-}_{sd} = (M^{-}_{sdq} + M'_{(P.G)d}) \times \delta$$



where:

M^{-}_{sd} is the design negative moment.

M^{-}_{sdq} is the max negative bending moment due to variable and permanent loads acting after continuity is achieved but before redistribution.

$M_{(P,G)d}$ is the additional negative bending moment due to long term effect (relaxation, creep, shrinkage), dead weight and in-situ concrete.
For a uniformly distributed floor load $q_{G.Q}$ between 4.0 and 8 kN/m^2 and floor span/depth ratio in the range 30 to 40, the value of $M'_{S(P.G)d}$ is very small and may be neglected.

δ is the redistribution ratio ($\delta \cong 0.75$)

Positive sagging moments

$$M^{+}_{sd} = M^{+}_{sdG} + M^{+}_{sdq}$$

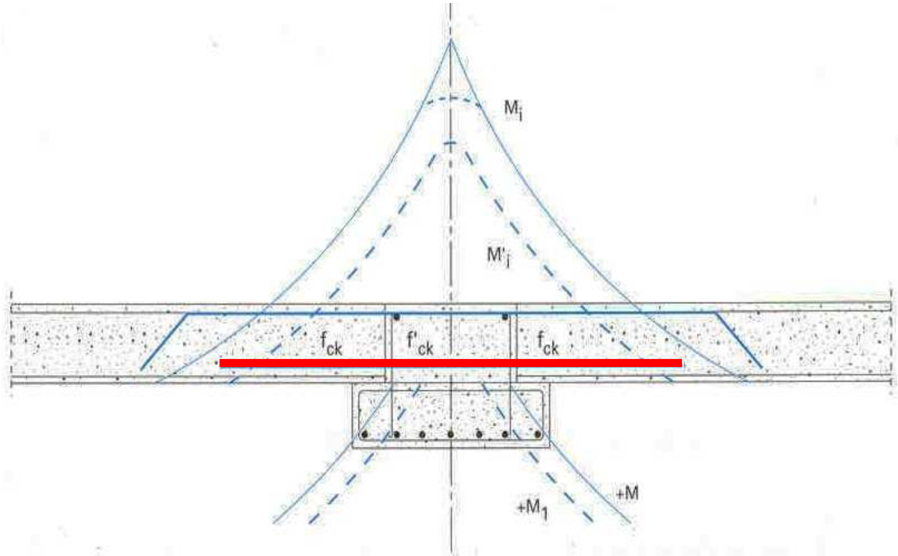
where:

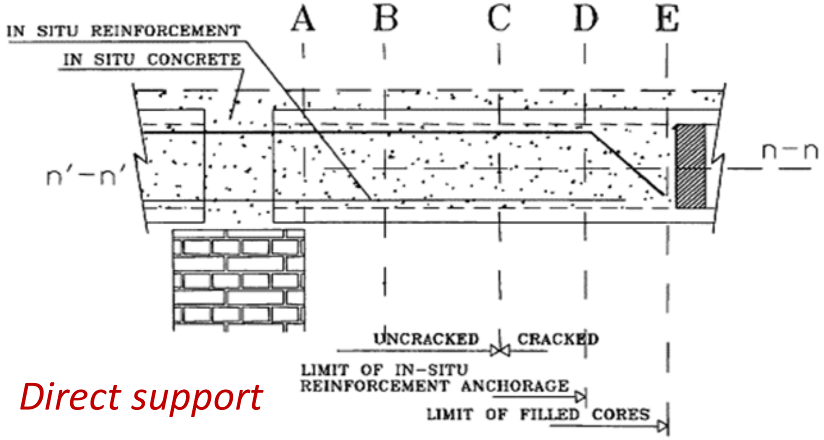
M^{+}_{sd} is the design positive moment in hollow core.

M^{+}_{sdG} is the positive moment in hollow core for simply supported condition.

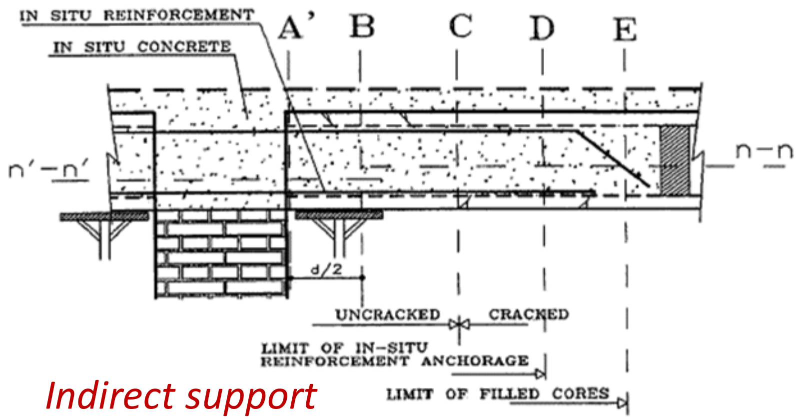
M^{+}_{sdq} is the positive moment due to variable and permanent loads, acting on the floor after hardening of the connection at support.

Positive continuity reinforcement may also be required at the support to resist rheological effects in prestressed structures and in order to ensure that the shifted positive moment diagram is covered.



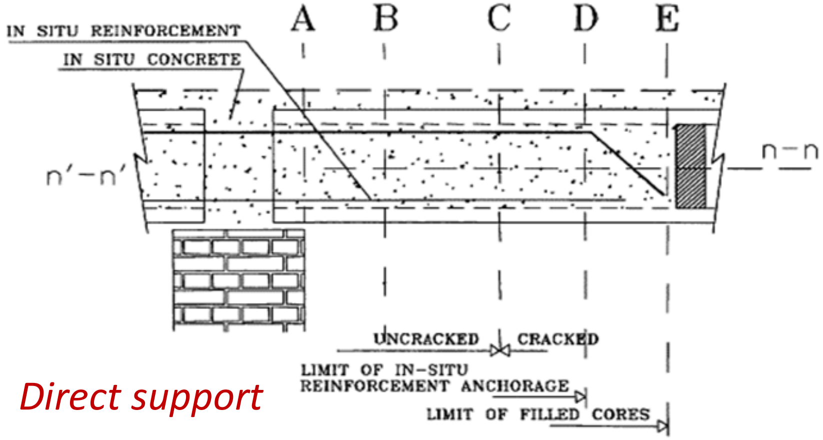


Direct support



Indirect support

Verification condition	A	A'	B	C	D	E
Bending capacity of the composite member without prestress contrubution (also long term effects)						
Anchorage capacity of the reinforcement in a filled core						
Interface shear capacity of the composite member						
Shear capacity of the composite member, under negative moment, **without contribution from prestressing**						
Flexural/shear tension capacity, under bending moment and contribution from prestressing						
Anchorage resistance of the prestressing steel						
Negative bending capacity of the hollow-core slab						
Shear and spalling stresses						



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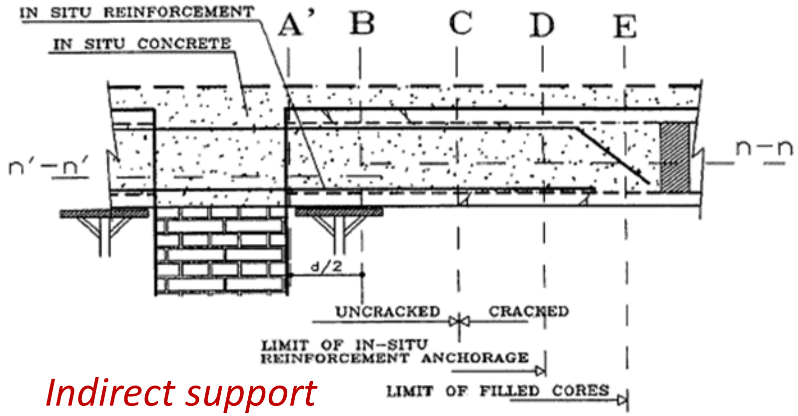

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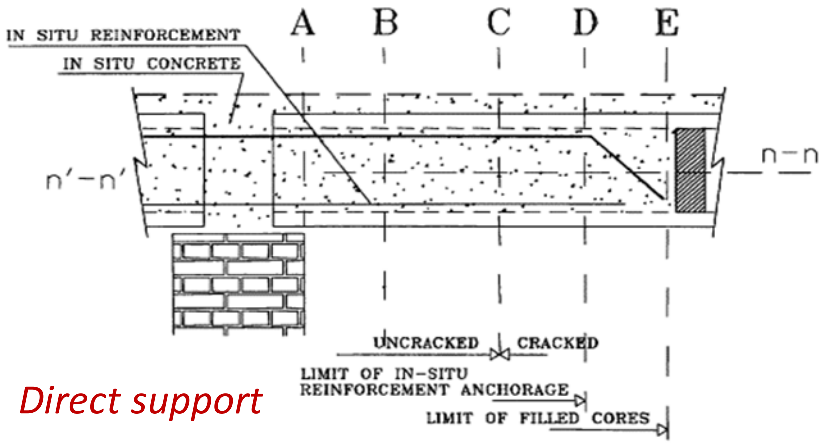

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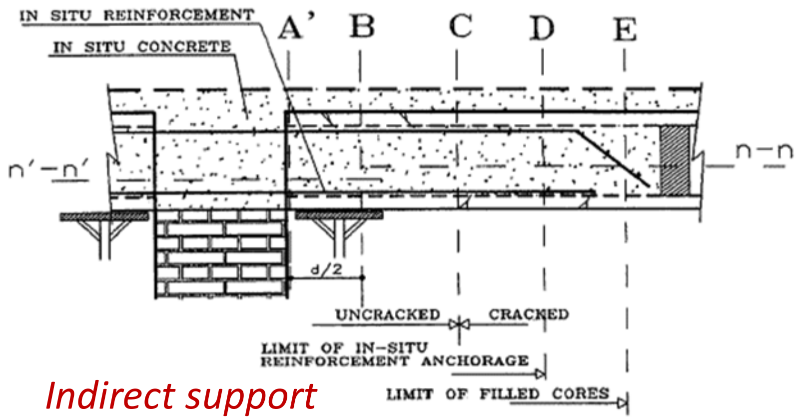

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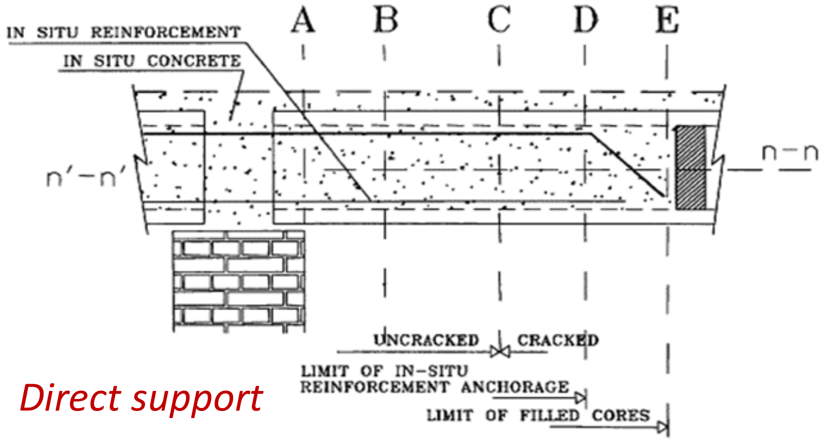
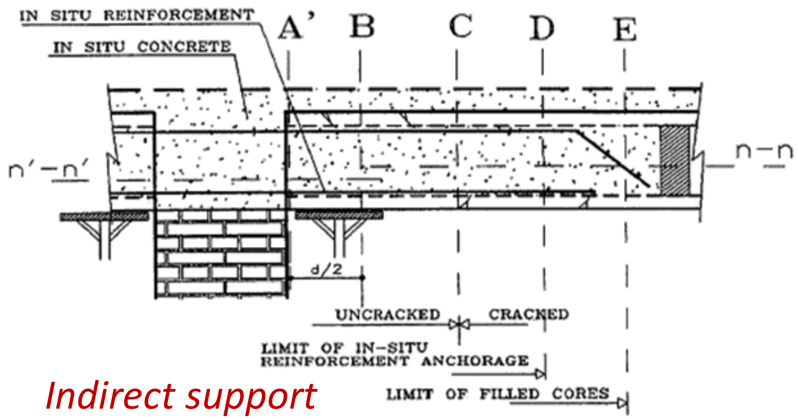


Direct support



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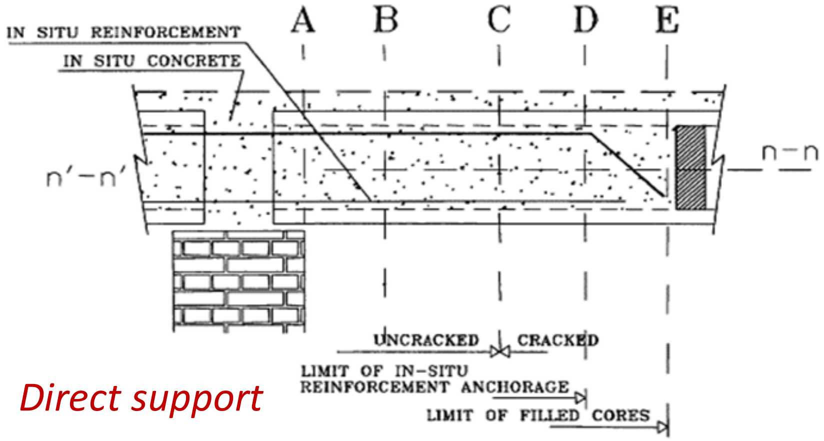

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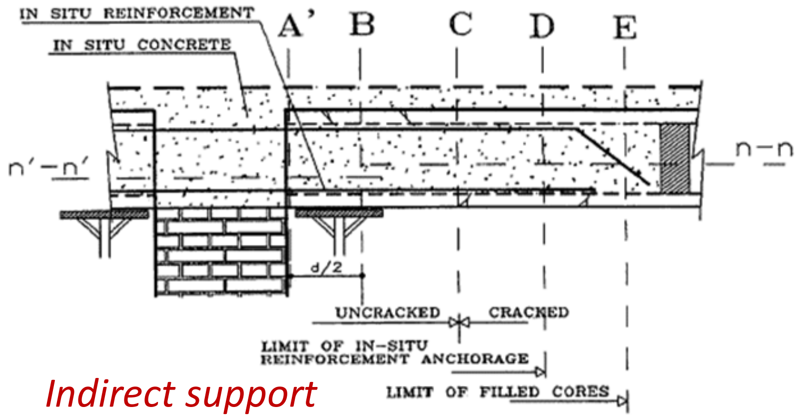

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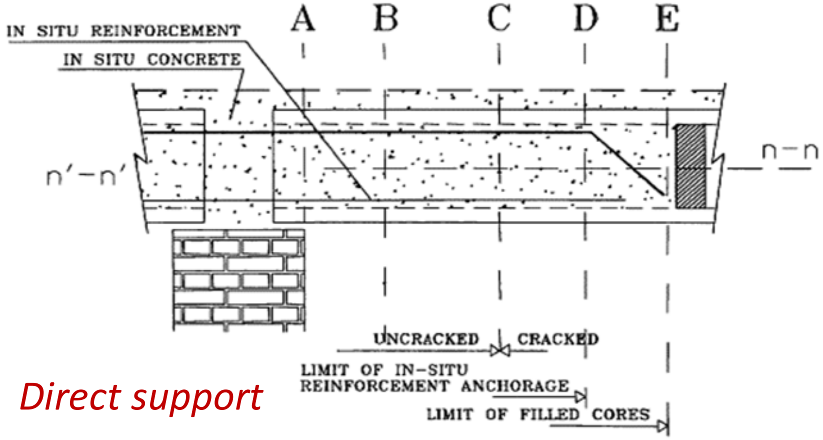

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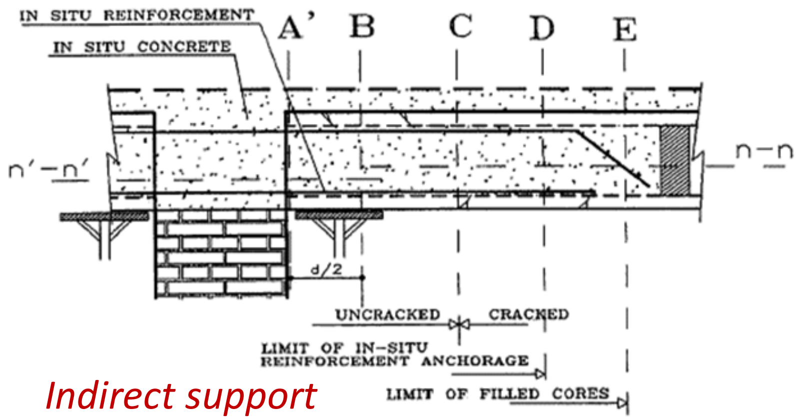

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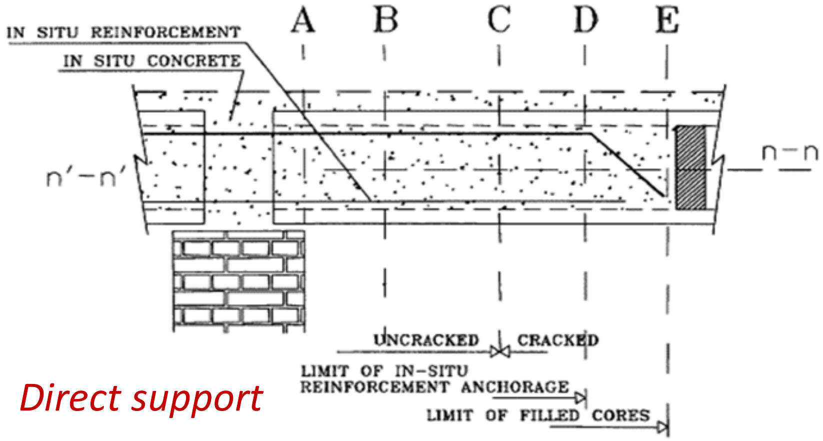

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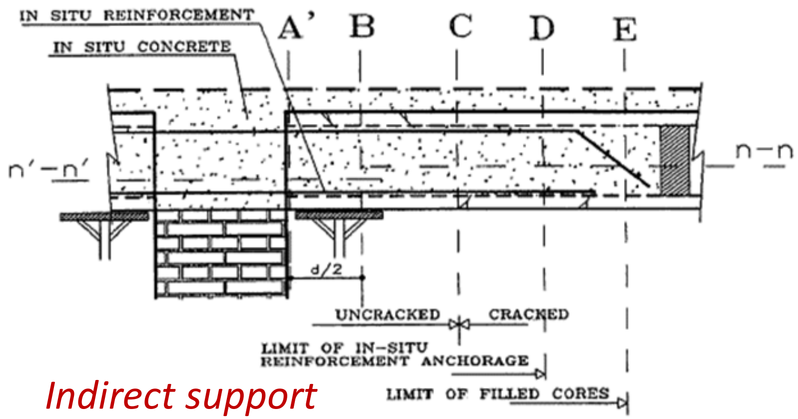

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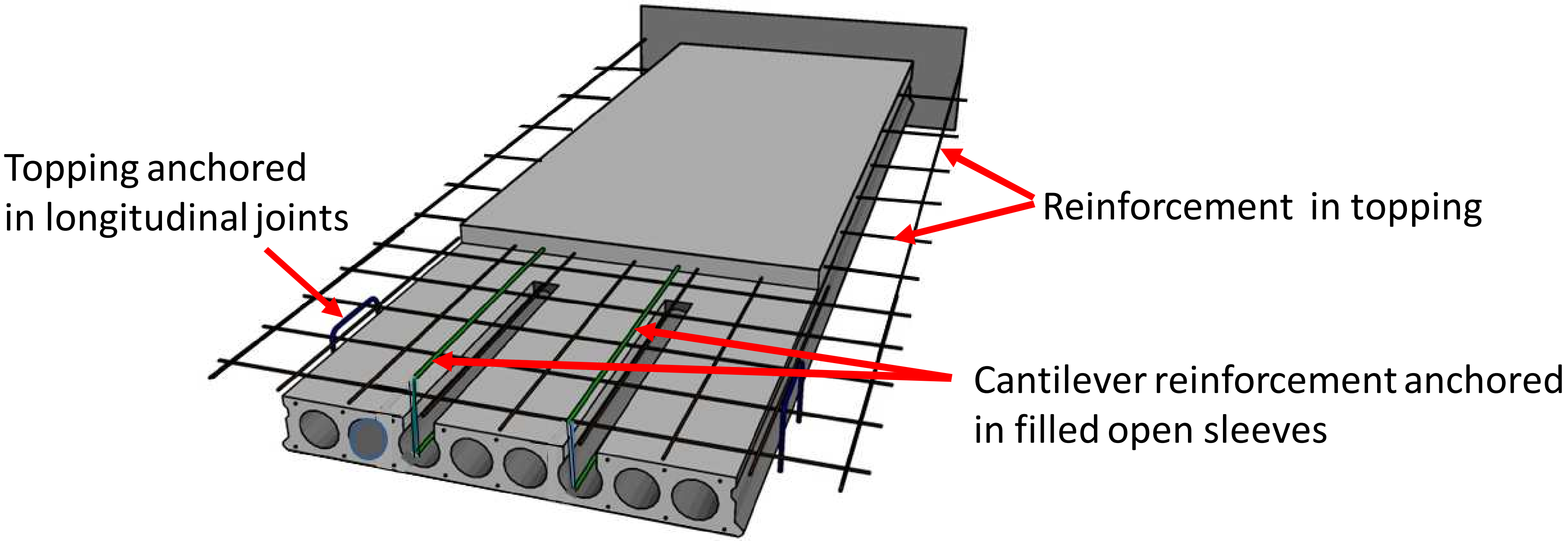
Design of cantilevered slabs

Hollow core units may be used for small direct cantilever action.

The slabs could be designed with prestressing tendons at the upper and the lower part of the cross-section.

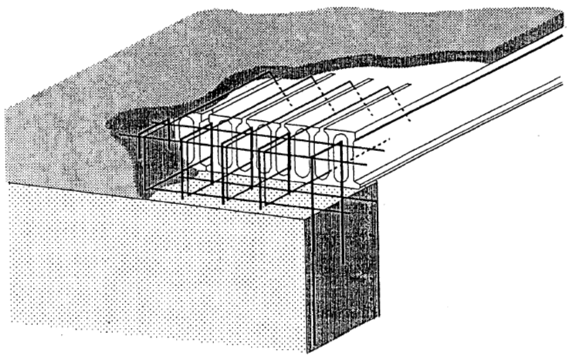
The cantilevering action could also be taken up by a reinforced structural topping anchored in concreted open cores.

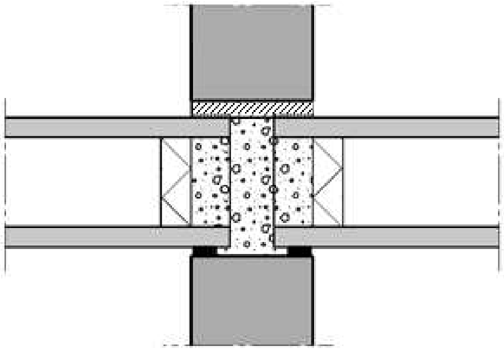


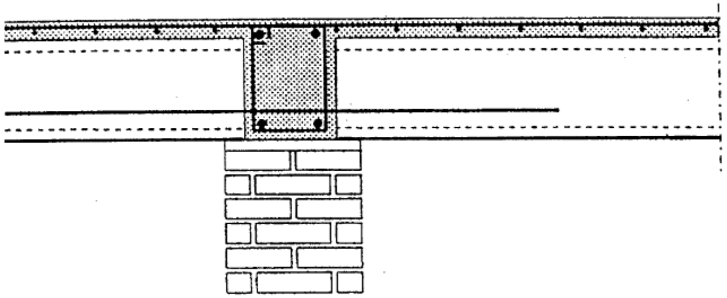
Unintended support restrainment

Restraining effects may appear in unintended way, for instance due to heavy wall loads on the ends of the floor units.

If the support rotation is prevented by the support connection, restraint stresses appear when the element is loaded.
Restraint stresses can also occur due to intrinsic effects, such as shrinkage, creep and thermal strains.



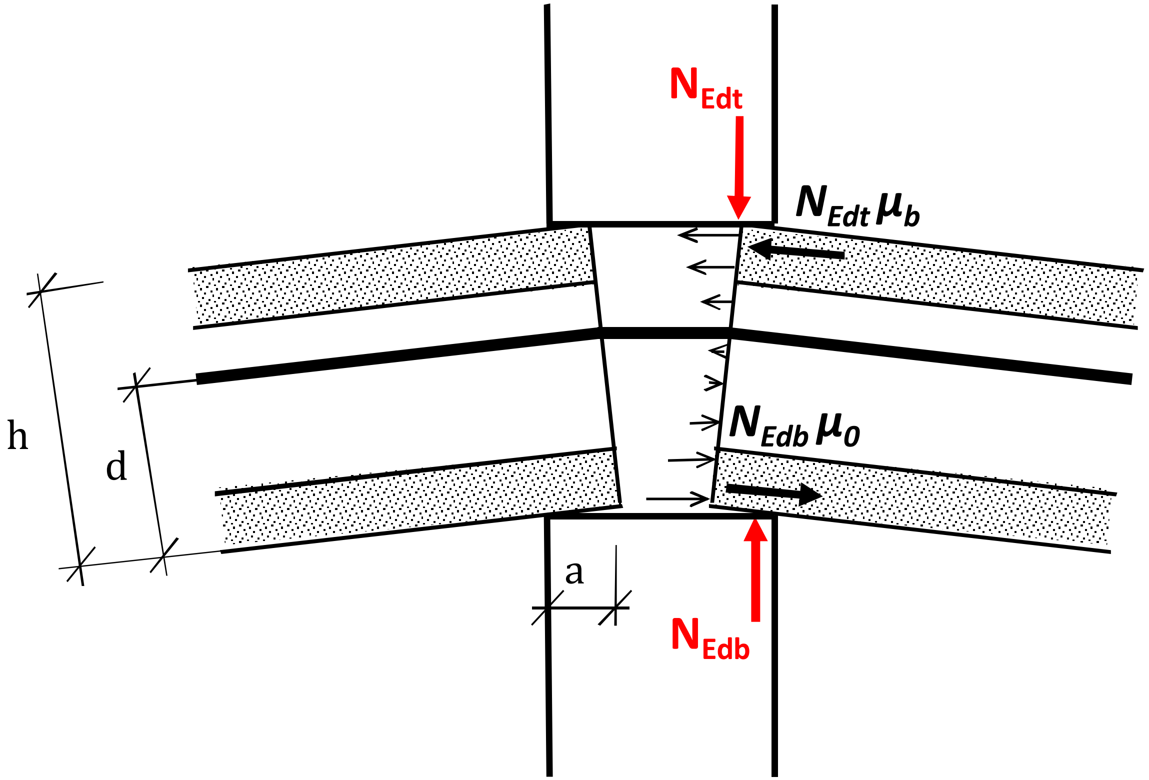




Possible consequences of unintended restraint must be evaluated and appropriate measures should be taken to avoid problems.

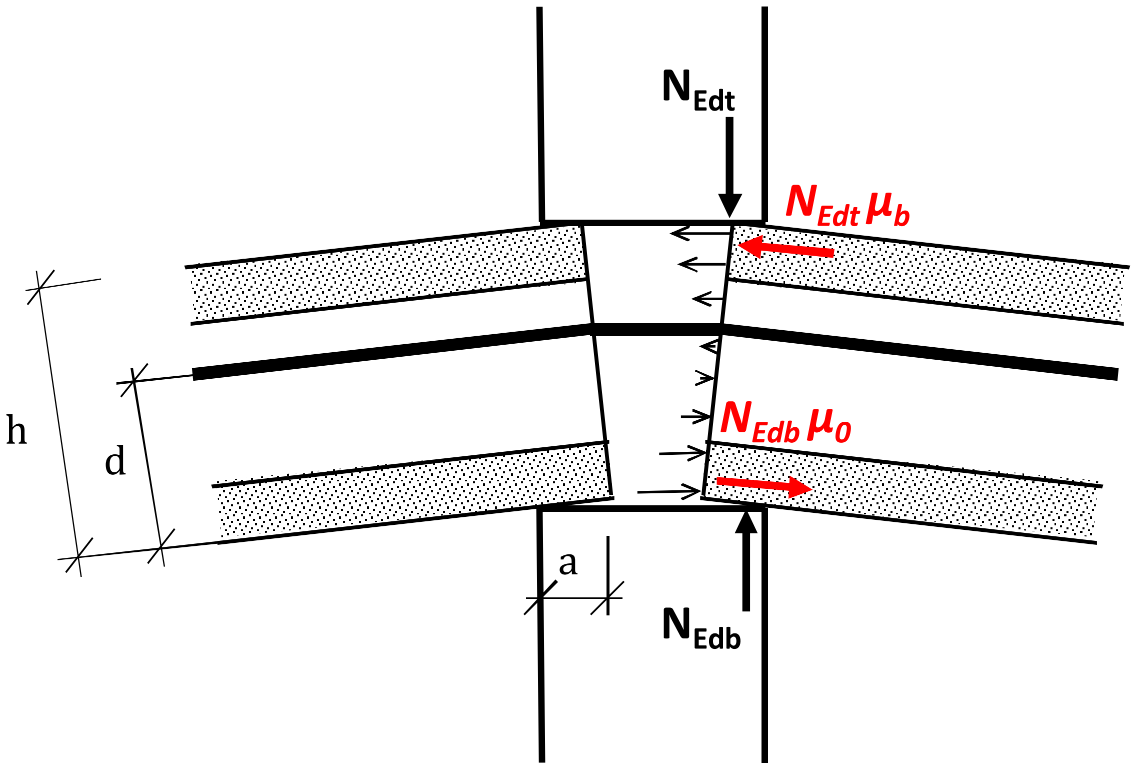
Unintended restraining effects

Clamping effect of wall loads introduced on the end of the hollow core unit at the support



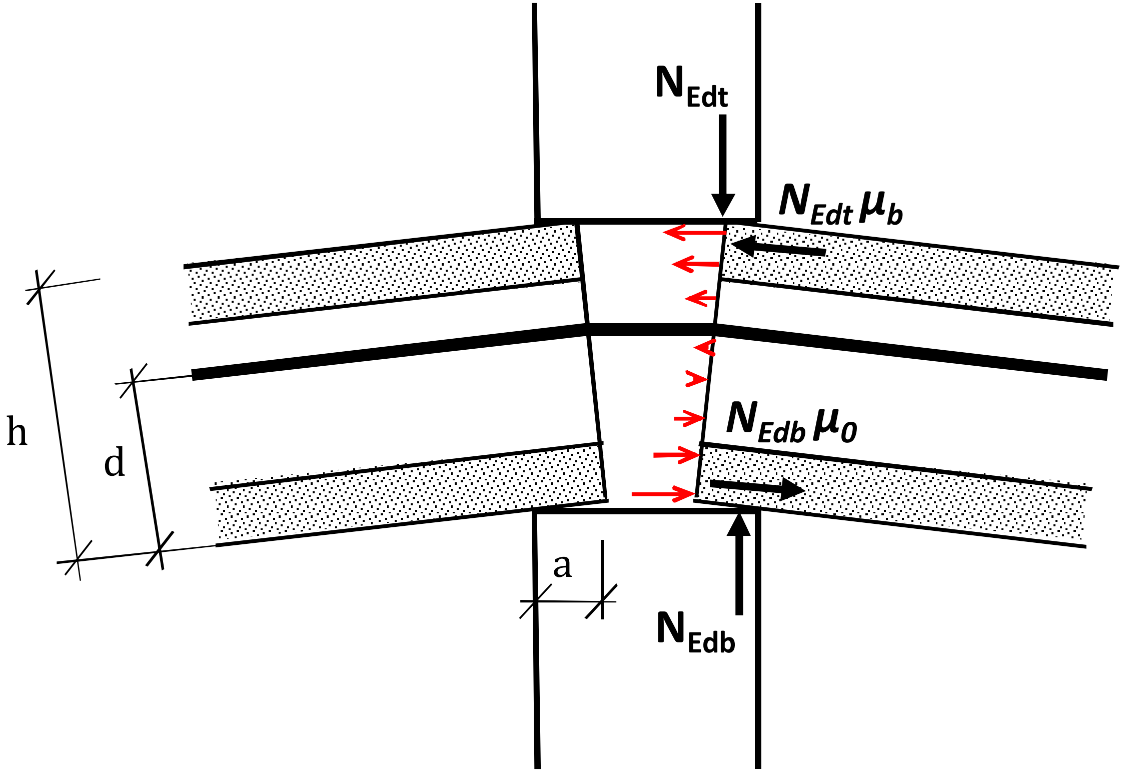
Unintended restraining effects

Friction developed at the interfaces between the hollow core unit and the upper and lower wall panels



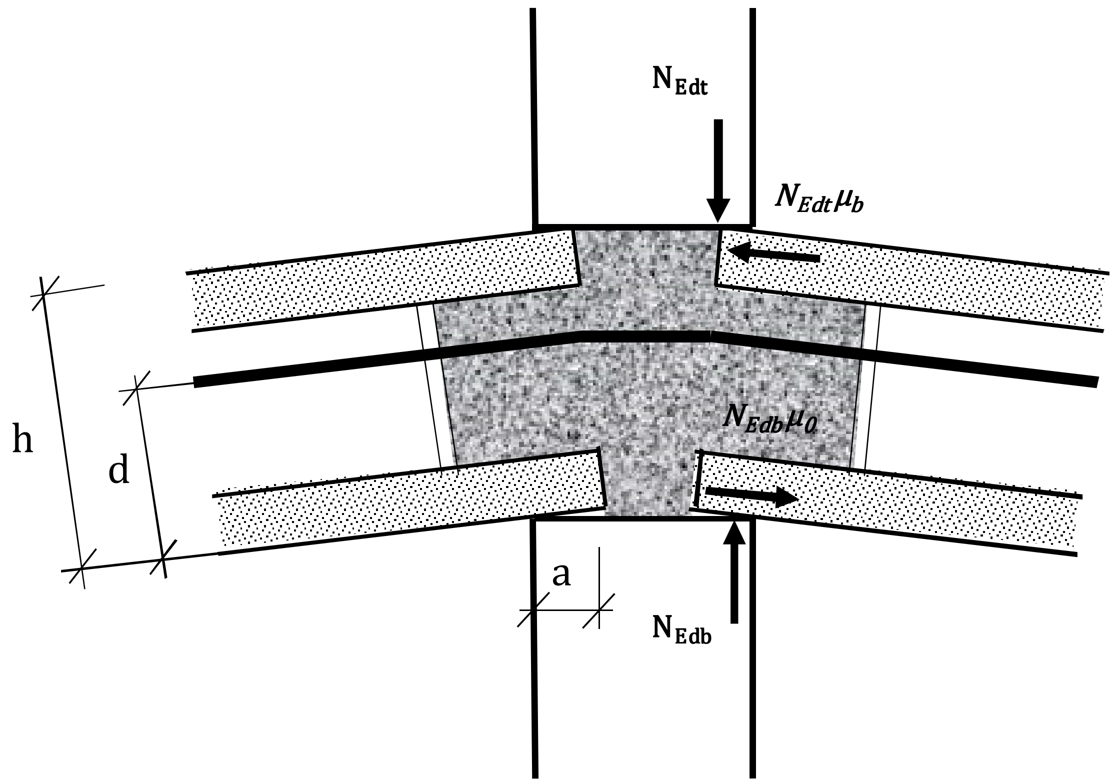
Unintended restraining effects

Adhesive bond between the end face of the hollow core unit and the in-situ joint concrete or grout in the transverse joint at the support



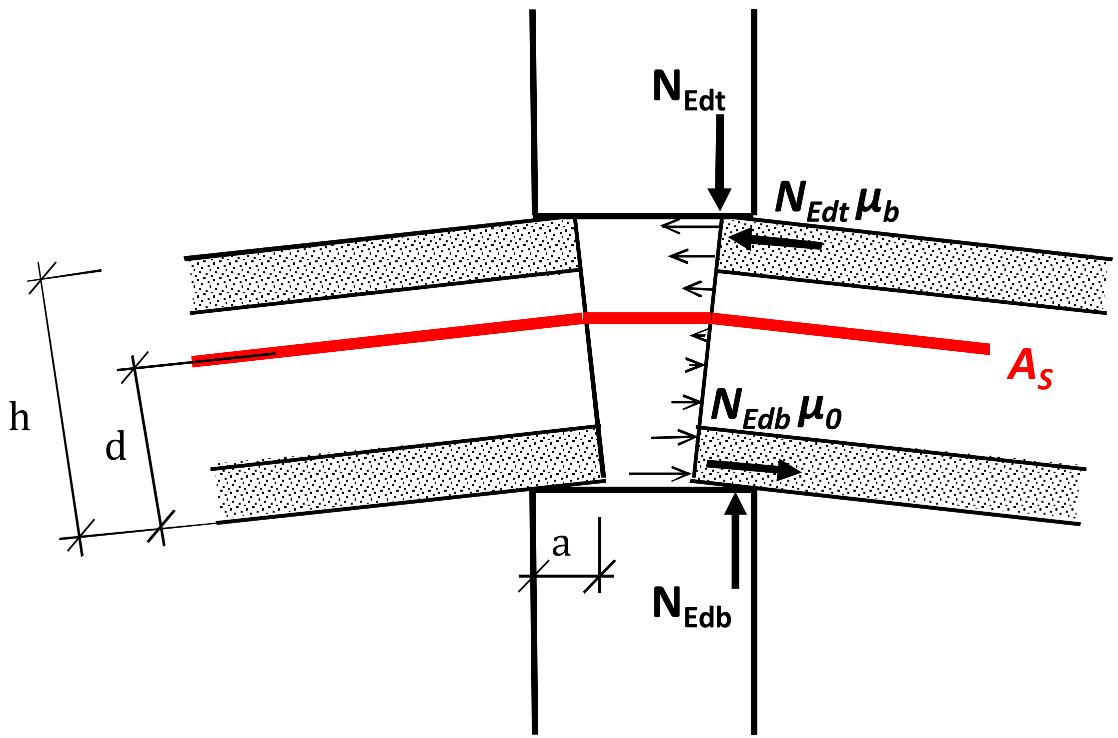
Unintended restraining effects

Dowel action of joint concrete which has filled the ends of the cores of the hollow core unit

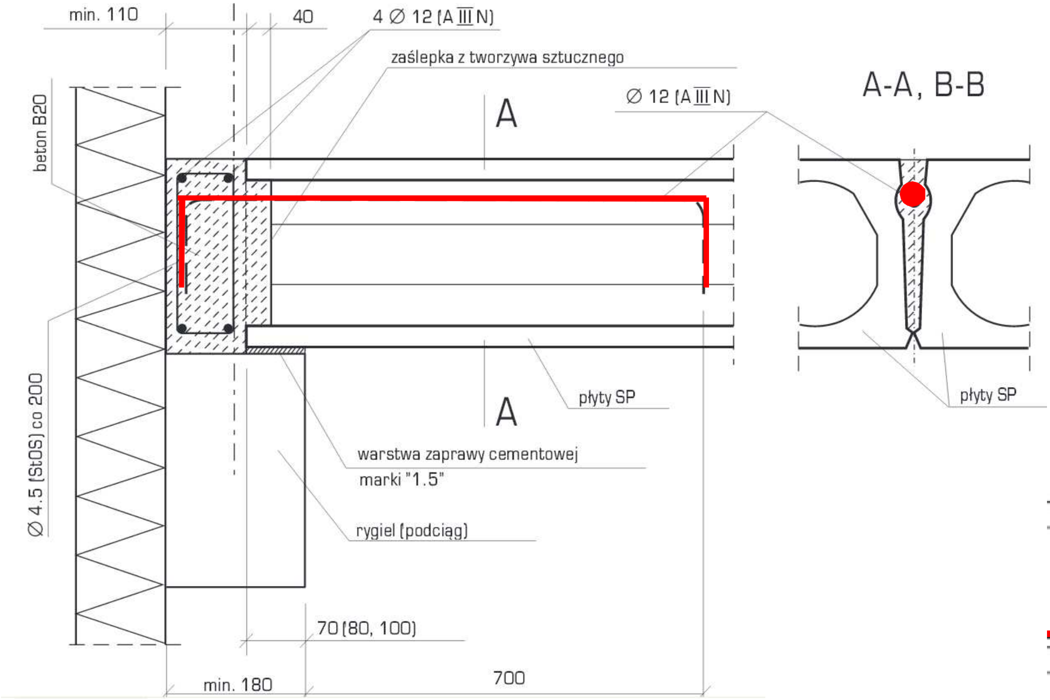


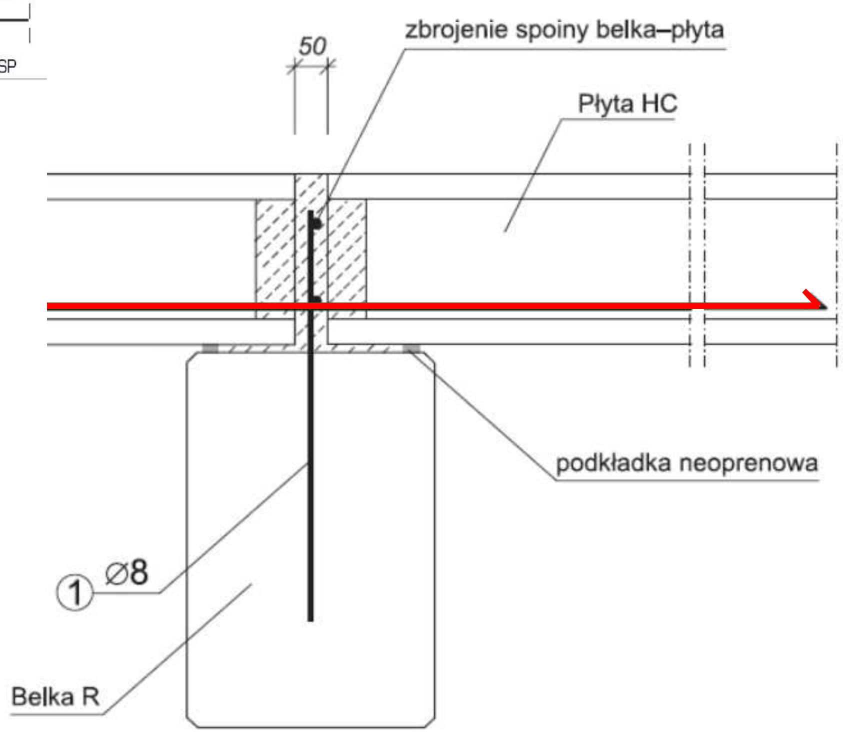
Unintended restraining effects

Tie arrangements between the hollow core unit and the support or between adjacent hollow core units across the support.



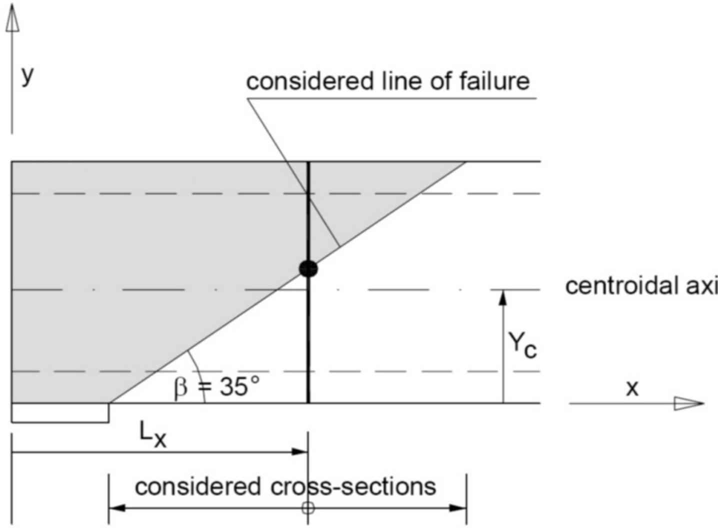
Location of tie reinforcement





Influence of bending moment on the shear resistance

Accurate procedure
(EN 1168, p. 4.3.3.2.2.2):



$$V_{Rdc} = \frac{Ib_w(y)}{S_c(y)}\left(\sqrt{(f_{ctd})^2 + \sigma_{cp}(y) f_{ctd}} - \tau_{cp}(y)\right)$$

$\sigma_{cp}(y)$ is the concrete compressive stress at the height y and distance l_x :

$$\sigma_{cp}(y) = \sum_{t=1}^{n}\left\{\left[\frac{1}{A} + \frac{(Y_c - y)(Y_c - Yp_t)}{I}\right] \cdot P_t(l_x)\right\} - \frac{M_{Ed}}{I} \cdot (Y_c - y)$$

$\tau_{cp}(y)$ is the concrete shear stress due to transmission of prestress at height y and distance l_x :

$$\tau_{cp}(y) = \frac{1}{b_w(y)} \cdot \sum_{t=1}^{n}\left\{\left[\frac{A_c(y)}{A} - \frac{S_c(y) \cdot (Y_c - Yp_t)}{I} + Cp_t(y)\right] \cdot \frac{dP_t(l_x)}{dx}\right\}$$

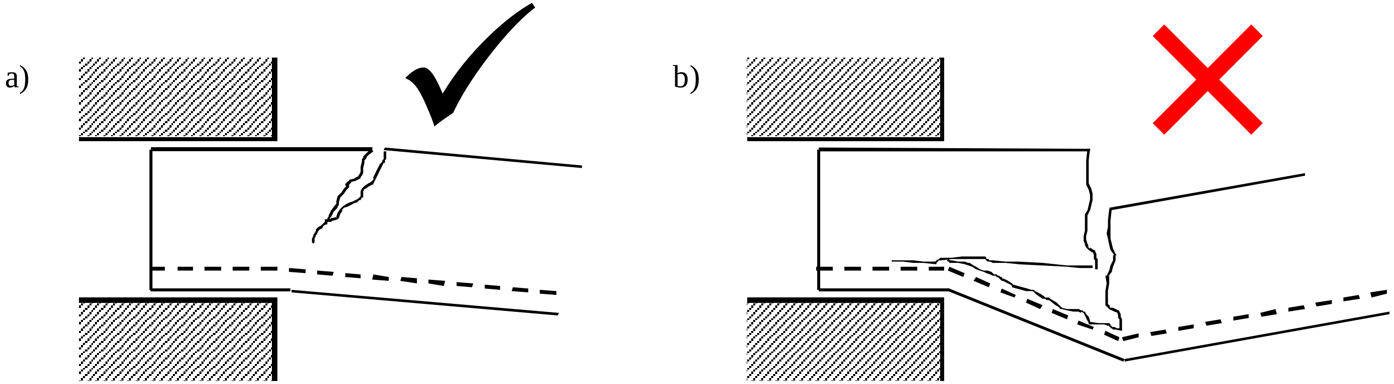
Cracking of the support zone

The end zone of the hollow core unit should be analysed with regard to the **risk of cracking**.

When the connection zone is not provided with reinforcement in the upper part of the section flexural cracks can start from the top surface.

- **the connection can be designed to be effectively simply supported;**
- **the restraint moment can be reduced or limited so that restraint cracks can be avoided;**
- **a restraint crack in the permited location, the connection zone is strengthened so that the shear capacity is sufficient in spite of this crack.**

Cracking of the support zone



Favorable location

Crack in the immediate vicinity of the support face

Shear transfer by dowel action of the bottom reinforcement can be balanced by the support pressure

Unfavorable location

Crack located in a certain distance from the support

Bottom concrete cover might spall off when the shear force is resisted mainly by dowel action of the bottom reinforcement

Ways to reduce the risk of cracking

- restraint moment can be reduced by limiting the load on the wall;
- the concrete fill in cores without tie bars should not extend outside the wall;
- tie bars anchored in cores or longitudinal joints should not be cut off within the critical region;
- slanted ends of the hollow core slabs are proposed

Restraint moment and axial restraint force

End face assumed to be uncracked:

$$M_{Edf} = \frac{2}{3} N_{Edt} \cdot a + f_{ctd} W$$

$$N_{Edf} = \left(\mu_b N_{Edt} + \mu_0 N_{Edb}\right) + f_{ct} A_{cj}$$

End face assumed to be cracked:

$$M_{Edf}^{cr} = \frac{2}{3} N_{Edt} \cdot a + f_{yd} A_y d + \mu_b N_{Edt} h$$

$$N_{Edf}^{cr} = \left(\mu_b N_{Edt} + \mu_0 N_{Edb}\right) + f_{yd} A_y$$

Values of μ:
0.80 for concrete-concrete;
0.60 for mortar-concrete;
0.25 for the rubber or neoprene pad

where:

a is the support length
μ_t is the frictional coefficient at top edge;
μ_b is the frictional coefficient at bottom edge;
f_{ct} is the tensile strength of joint concrete;
N_{edt} is the normal force in the wall (on one side of the connection);
W_{cj} is the sectional modulus of the joint interface (including the cores).

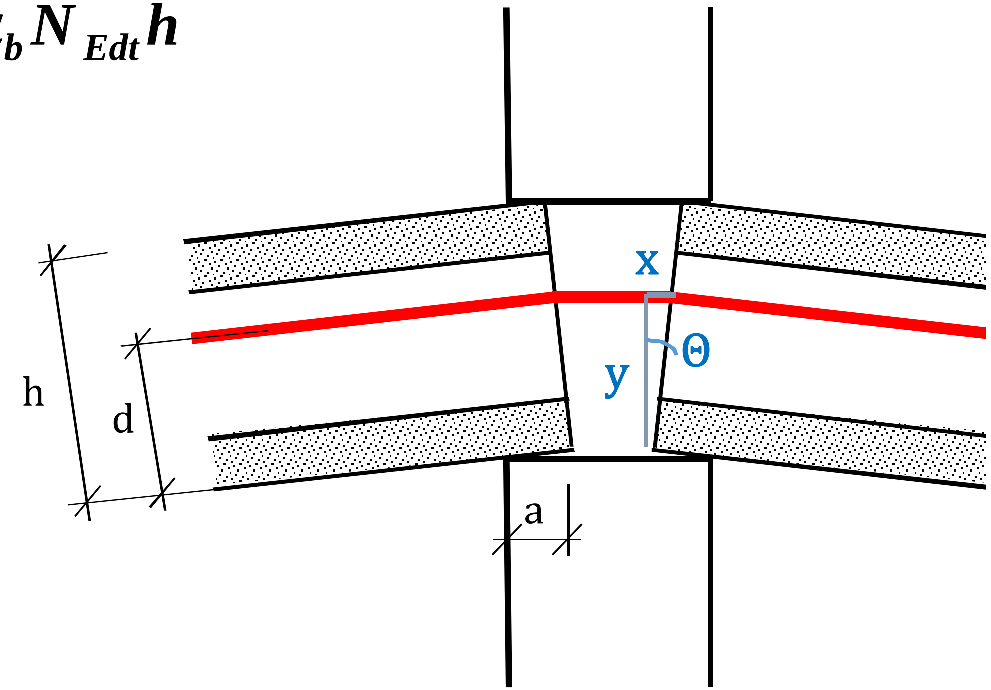
Unintended negative moment resulting from tying

The value of negative bending moment depends on the amount and location of reinforcing bars (longitudinal ties, reinforcement in the topping).

$$M_{restr} = \tfrac{2}{3} N_{Edt} l_s + f_{yd} A_s d + \mu_b N_{Edt} h$$

assumes yielding of reinforcement

In fact, the stress in the reinforcement depends on the rotation of the HC slab on the support



Proposal for determine of the value of the negative bending moment on the basis of the real rebars effort due to rotation of HC slab on the support (due to deflection).

Stiffness of the reinforcing bars: $K_t = \frac{A_s \cdot E_s}{l_{t.eq}}$ $\quad l_{t.eq} = \min\left(30\varphi_{12} \cdot \frac{\sigma_s}{f_{yd}}, 0.8b\right)$

The rotation angle on a support of the simply supported beam:

$$\Theta_1 = \frac{(\Delta g + q) \cdot b \cdot l_{eff}^3}{24 E_{cm} \cdot I_{cs}}$$

Real force in the rebars, due to rotation of HC slab:

$$x = y \cdot \tan(\Theta_1) \qquad F_1 := x_1 \cdot K_t$$

Unintended negative moment due to rotation:

$$M_{sup1} = y \cdot \tan(\Theta_1) \cdot K_t \cdot y$$

The assumption that the beam is simply supported was not correct.

So the rotation angle on a support can be calculated as:

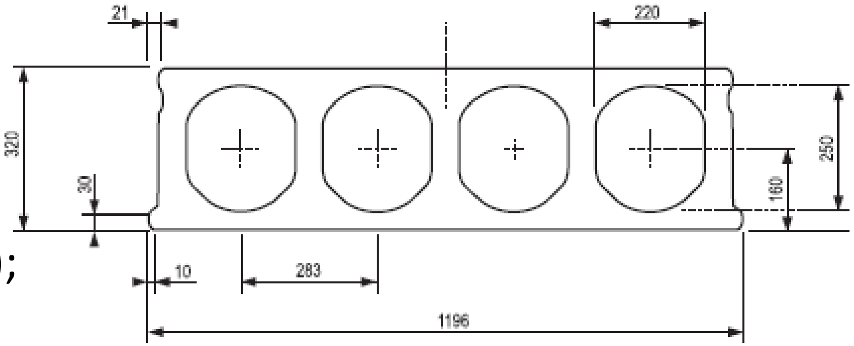
$$\Theta_2 = \frac{(\Delta g + q)\cdot b\cdot l_{eff}^{\,3}}{24E_{cm}\cdot I_{cs}} - \frac{M_{sup1}\cdot l_{eff}}{2E_{cm}\cdot I_{cs}}$$

The unintended negative moment due to rotation of the angle θ_2:

$$M_{sup2} = y\cdot \tan\left(\Theta_2\right)\cdot K_t\cdot y$$

In an iterative process one can determine the actual value of unintended negative bending moment at the support.

Exemplary calculation results



Assumptions:

- HC320 slab
- simply supported with a span L = 10,0 m;
- concrete grade C50/60 (HC) and C25/30 (in situ);
- 8 prestressing strands of diameter 12,5 mm;
- in each joint between the HC slabs is located one rebar with a diameter of 12 mm, at a height of 16 cm (h/2).

$$M_{restr} = \tfrac{2}{3} N_{Edt} l_s + f_{yd} A_s d + \mu_b N_{Edt} h$$

12,7 kNm

Proposed accurate method: 7,6 kNm

Result: revaluation of approx. 40%.

Thank you,

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