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Shear resistance of hollowcore slabs

Ronald Klein-Holte

VBI Ontwikkeling bv

in cooperation with

Contents

- Introduction
- Overview shear design according EC2
- Full scale testing
- Shear flexure resistance
- Shear tension resistance
- Conclusion

Introduction

- **As long as pre-stressed hollow core slabs exists as long as there are questions about their shear resistance**

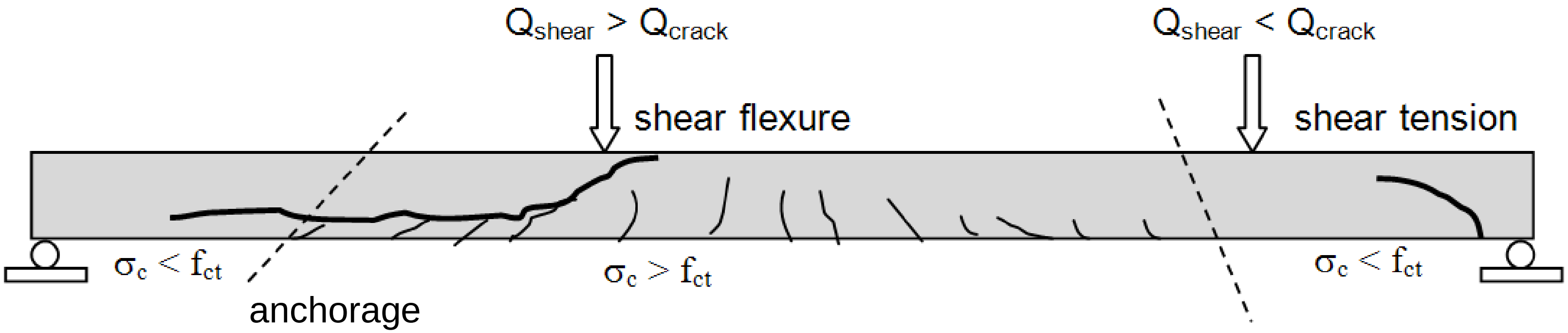
  The shear tension capacity of a slab element is well known now.

  But what about…

  - Shear flexure capacity?
  - Influence of structural topping?
  - Influence of filled cores?
  - Interaction shear and bending?

Introduction

- **Shear failure modes in cracked and uncracked regions**



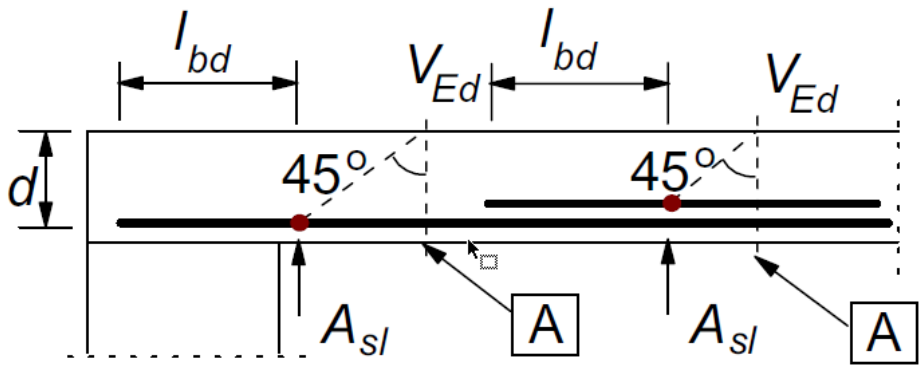
EC2 Design

- **Shear flexural resistance:**

$$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$$

with a minimum of

$$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\,b_w d$$



EC2 Design

- **Shear tension resistance:**

$$V_{\mathrm{Rd,c}} = \frac{I \cdot b_{\mathrm{w}}}{S} \sqrt{\left(f_{\mathrm{ctd}}\right)^2 + \alpha_l \sigma_{\mathrm{cp}} f_{\mathrm{ctd}}}$$

In regions uncracked in bending (where the flexural tensile stress is smaller than $f_{ctk,0,05}/\gamma_c$) the shear resistance should be limited by the tensile strength of the concrete.
For cross-sections where the width varies over the height, the maximum principal stress may occur on an axis other than the centroidal axis. In such a case the minimum value of the shear resistance should be found by calculating $V_{Rd,c}$ at various axes in the cross-section.

EC2 Design

- **Shear tension resistance extended formula in EN 1168**

$$V_{\mathrm{Rdc}} = \frac{I b_{\mathrm{w}}(y)}{S_{\mathrm{c}}(y)} \left(\sqrt{(f_{\mathrm{ctd}})^2 + \sigma_{\mathrm{cp}}(y) f_{\mathrm{ctd}}} - \tau_{\mathrm{cp}}(y) \right)$$

where

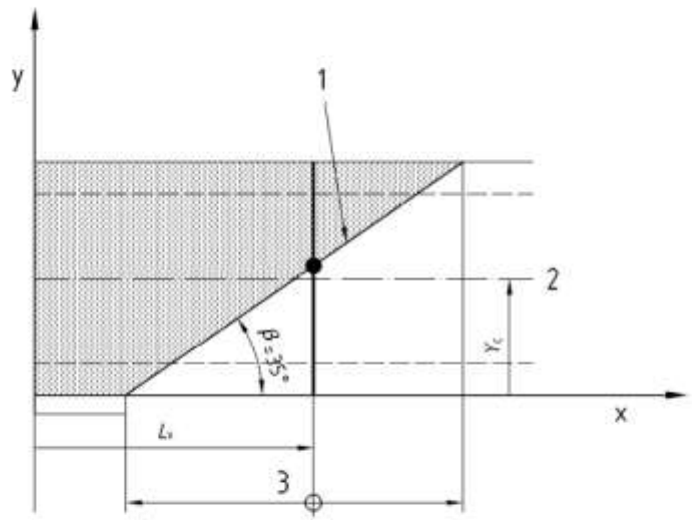
$$\sigma_{\mathrm{cp}}(y) = \sum_{t=1}^{\mathrm{n}} \left\{ \left[\frac{1}{A} + \frac{(Y_{\mathrm{c}} - y)(Y_{\mathrm{c}} - Yp_{\mathrm{t}})}{I} \right] \times P_{\mathrm{t}}(l_{\mathrm{x}}) \right\} - \frac{M_{\mathrm{Ed}}}{I} \times (Y_{\mathrm{c}} - y)$$ (positive if compressive)

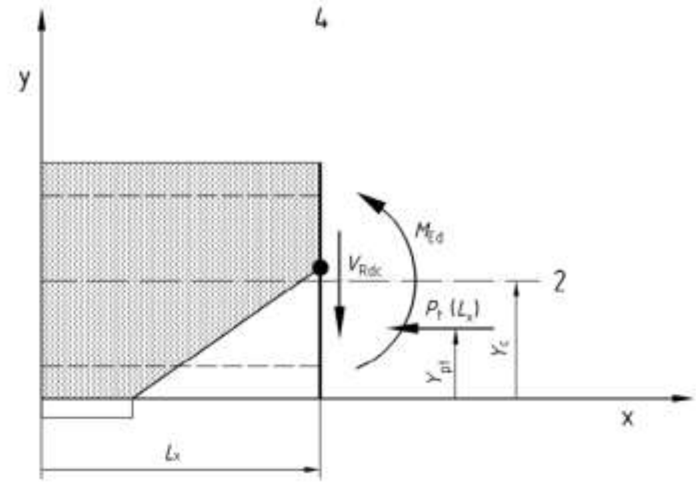
$$\tau_{\mathrm{cp}}(y) = \frac{1}{b_{\mathrm{w}}(y)} \times \sum_{t=1}^{\mathrm{n}} \left\{ \left[\frac{A_{\mathrm{c}}(y)}{A} - \frac{S_{\mathrm{c}}(y) \times (Y_{\mathrm{c}} - Yp_{\mathrm{t}})}{I} + Cp_{\mathrm{t}}(y) \right] \times \frac{dP_{\mathrm{t}}(l_{\mathrm{x}})}{dx} \right\}$$

EC2 Design

- **Shear tension resistance extended formula in EN 1168**

This equation shall be applied with reference to the critical points of a straight line of failure rising from the edge of the support with an angle $\beta = 35°$ with respect to the horizontal axis. The critical point is the point on the quoted line where the result of the expression of $V_{Rd,c}$ is the lowest.

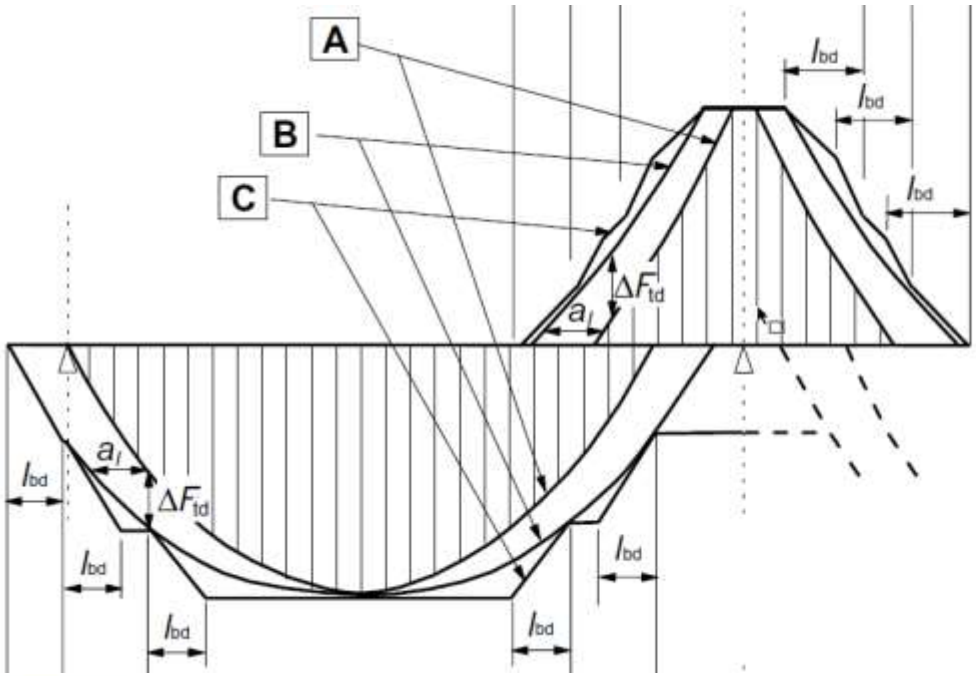




EC2 Design

- **Anchorage**

  For members with shear reinforcement the additional tensile force, ΔF_{td}, should be calculated according to 6.2.3 (7). For members without shear reinforcement ΔF_{td} may be estimated by shifting the moment curve a distance $a_l = d$ according to 6.2.2 (5). This "shift rule“ may also be used as an alternative for members with shear reinforcement.



EC2 Design

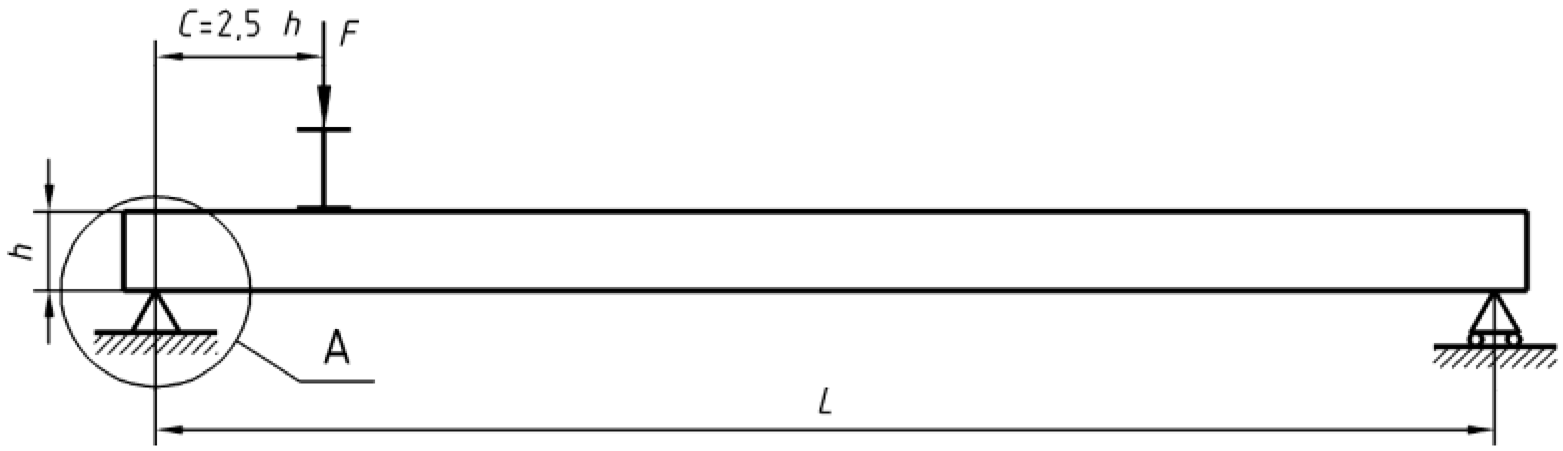
- **Anchorage**

Anchorage of tensile force for the ultimate limit state

The anchorage of tendons should be checked in sections where the concrete tensile stress exceeds $f_{ctk,0,05}$. The tendon force should be calculated for a cracked section, including the effect of shear according to 6.2.3 (6); see also 9.2.1.3.
Where the concrete tensile stress is less than $f_{ctk,0,05}$, no anchorage check is necessary.

Full scale test

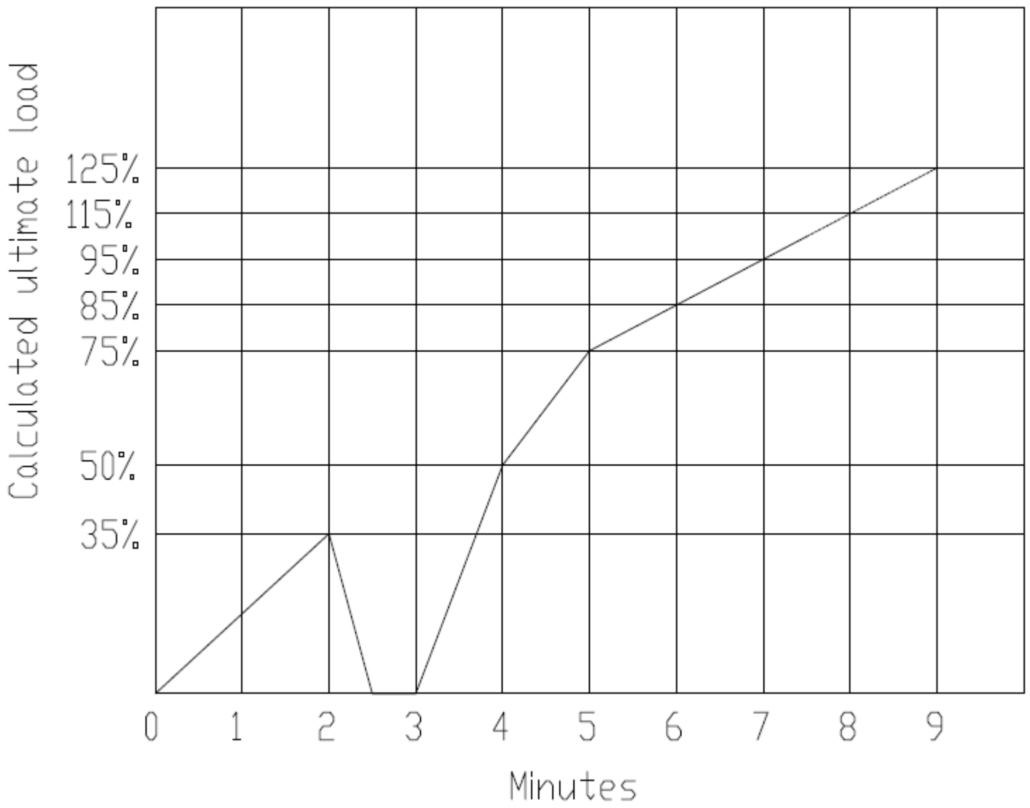
- **EN 1168 Annex J Test Setup**



General test setup for testing shear and anchorage strength

Full scale test

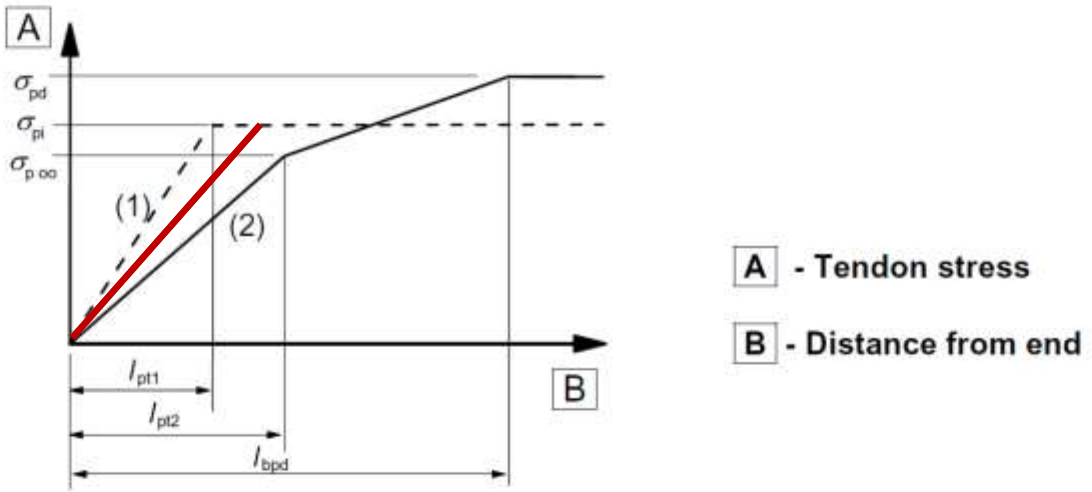
- **EN 1168 Annex J – Loading sequence**



Full scale test

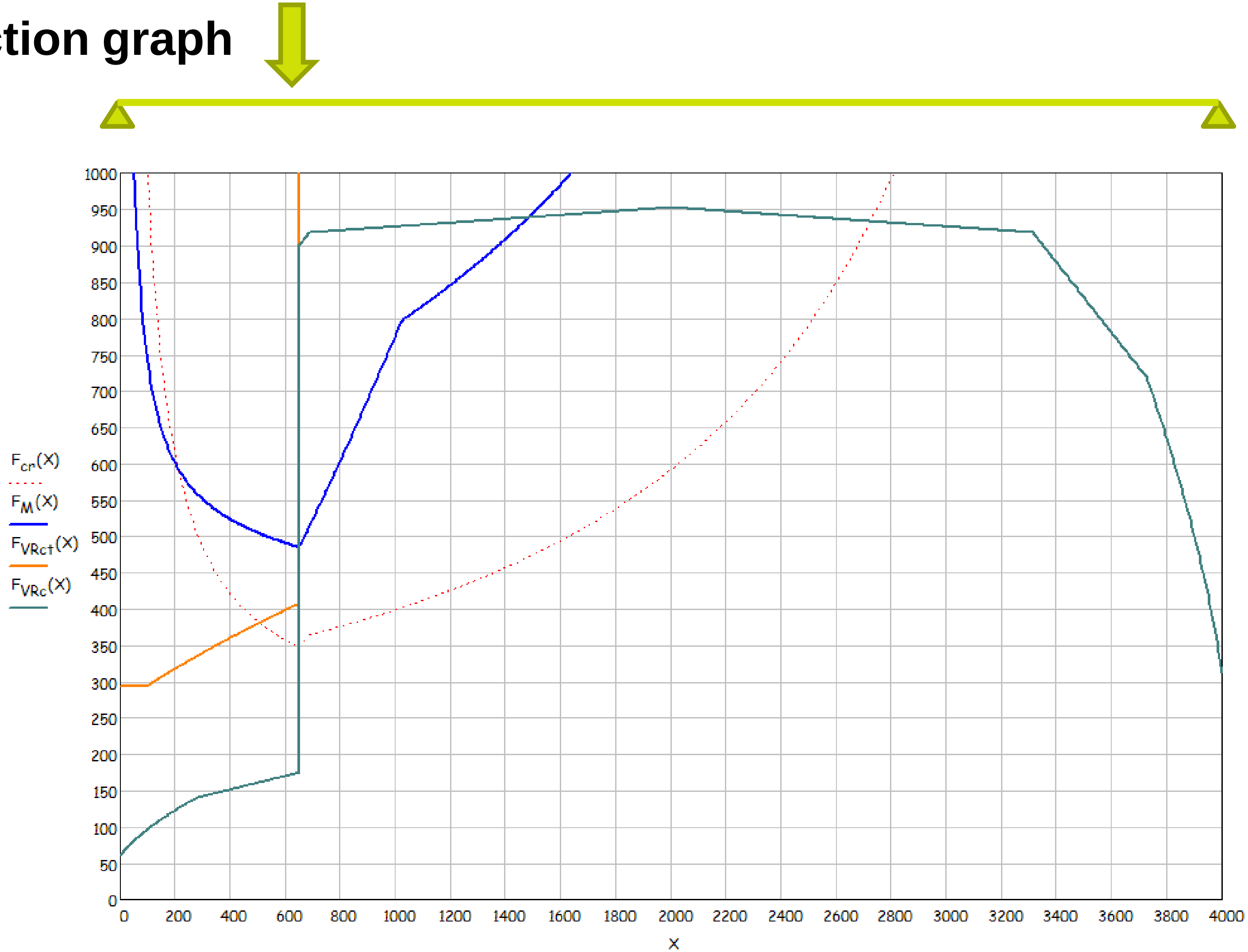
- **Calculating the expected failure load**
  - Mean material strength
  - Design principles based on this mean material strength

    e.g.:
    The bond strength based on mean tensile strength (anchorage)
    Transfer of pre-stress based on mean transmission length



Full scale test

- **Interaction graph**

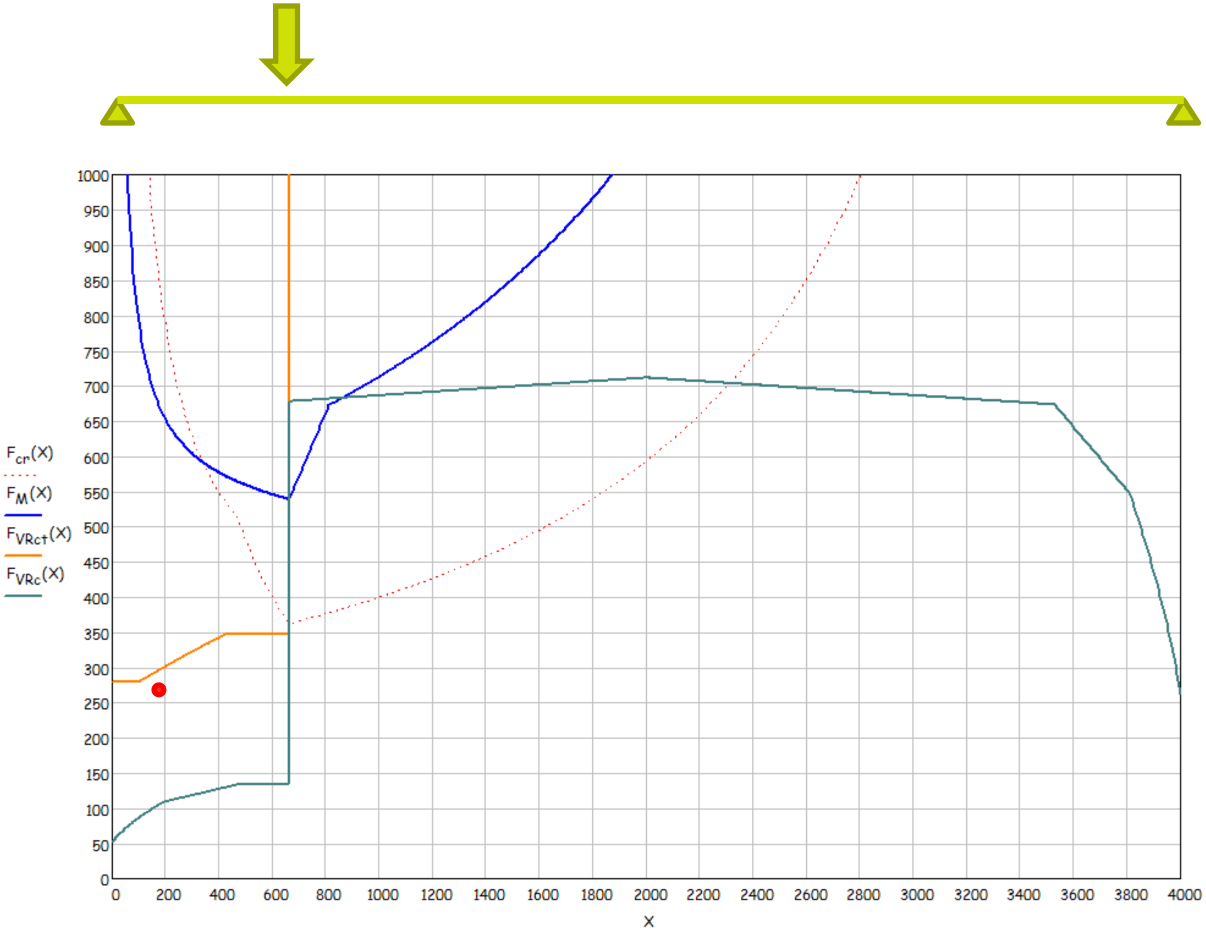


Shear tension resistance

VX265 10 Ø12,5

F_{exp} = 267 kN

F_{calc} = 290 kN



Shear flexure resistance

$$V_{Rd,c} = [C_{Rd,c} k (100\, \rho_l\, f_{ck})^{1/3} + k_1\, \sigma_{cp}]\, b_w d$$

$C_{Rd,c} = 0{,}18$
no material factor

$f_{ck} = f_{cm}$

Effective depth of total cross section
(structural topping included)

Reinforcement ratio:

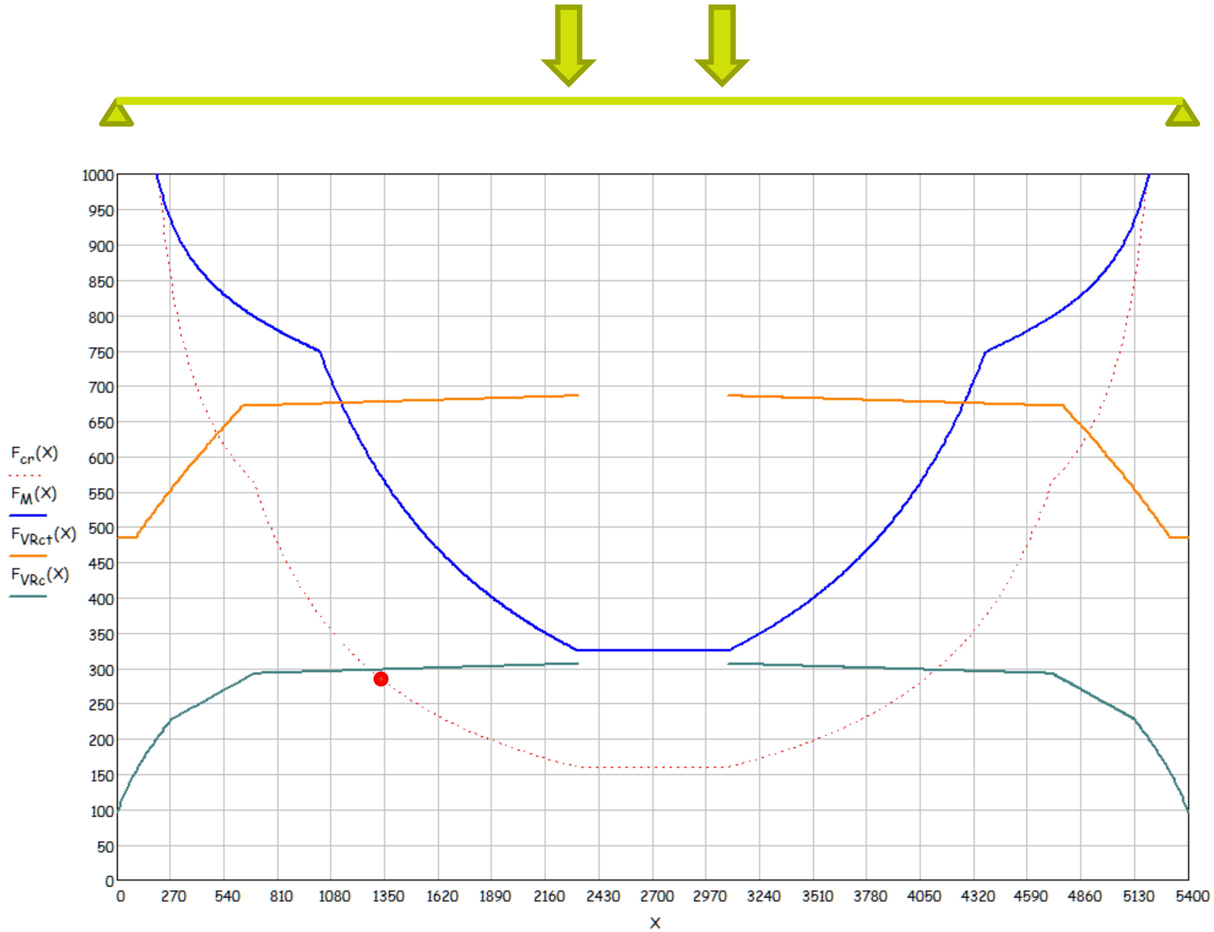
$$\rho_1(x) := \min\left(\frac{A_{sl}(x)}{b_w(x) \cdot d}, 0.020\right)$$

Equivalent tensile reinforcement:

$$A_{sl}(x) := \frac{F_{anchorage}(x)}{500}$$

Shear flexure resistance

$$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l\,f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$$

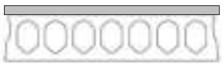


A260 12 Ø12,5
no topping

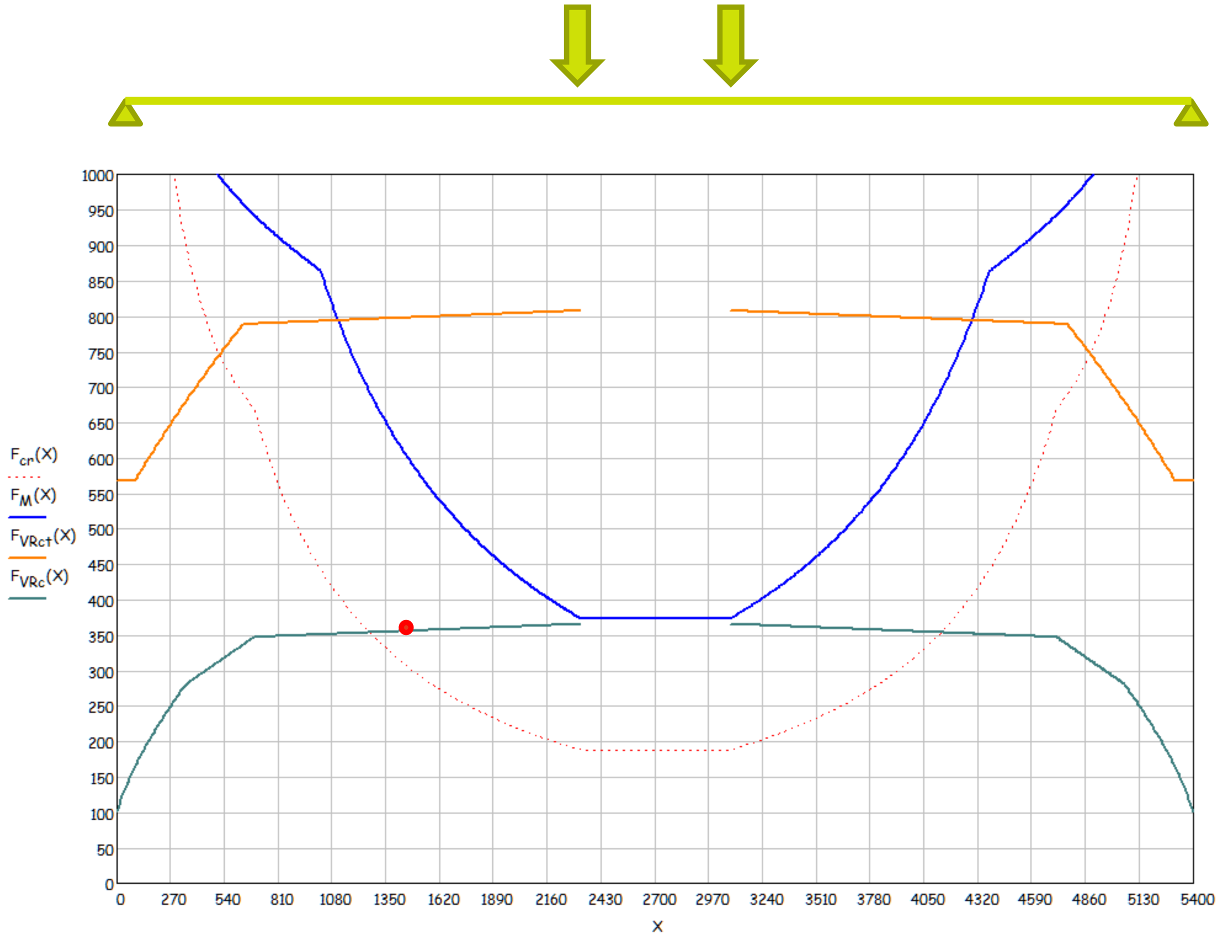
F_{exp} = 284 kN

F_{calc} = 300 kN

Shear flexure resistance



$$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l\,f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$$

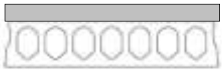


A260 12 Ø12,5
50 mm topping

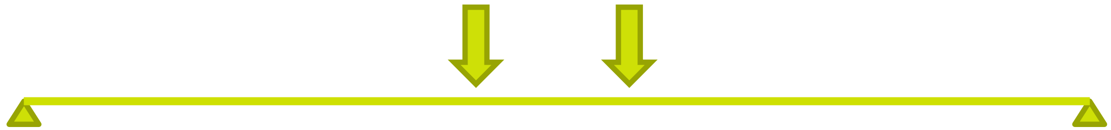
F_{exp} = 353 kN

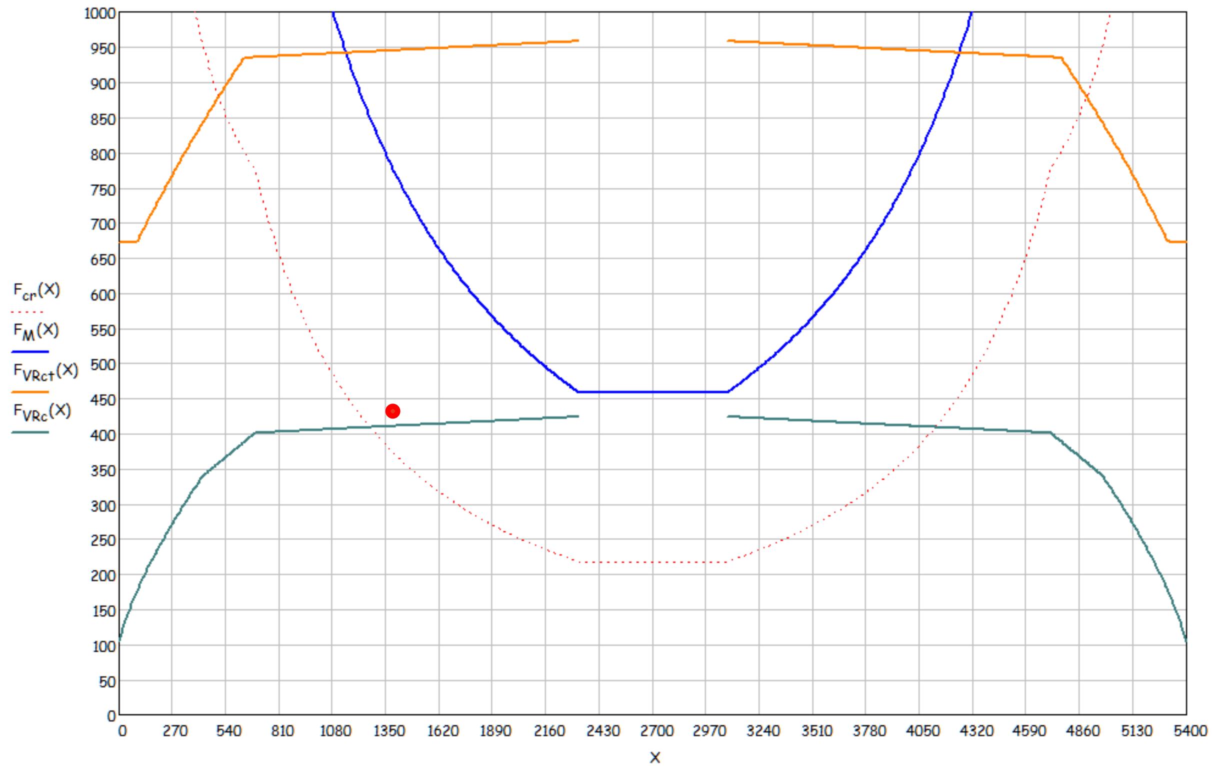
F_{calc} = 351 kN

Shear flexure resistance



$$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$$





A260 12 Ø12,5
100 mm topping

F_{exp} = 430 kN

F_{calc} = 411 kN

Shear tension resistance

- **General formula according to EC2**

$$V_{\mathrm{Rd,c}} = \frac{I \cdot b_{\mathrm{w}}}{S} \sqrt{\left(f_{\mathrm{ctd}}\right)^2 + \alpha_l \sigma_{\mathrm{cp}} f_{\mathrm{ctd}}}$$

f_{ctm}

where

I	is the second moment of area
b_{w}	is the width of the cross-section at the centroidal axis, allowing for the presence of ducts in accordance with Expressions (6.16) and (6.17)
S	is the first moment of area above and about the centroidal axis
α_l	$= l_x/l_{\mathrm{pt2}} \leq 1,0$ for pretensioned tendons $= 1,0$ for other types of prestressing
l_x	is the distance of section considered from the starting point of the transmission length
l_{pt2}	is the upper bound value of the transmission length of the prestressing element according to Expression (8.18).
σ_{cp}	is the concrete compressive stress at the centroidal axis due to axial loading and/or prestressing ($\sigma_{\mathrm{cp}} = N_{\mathrm{Ed}}/A_{\mathrm{c}}$ in MPa, $N_{\mathrm{Ed}} > 0$ in compression)

l_{pt}

Shear tension resistance

- **Transmission length**

8.10.2.2 Transfer of prestress

(1) At release of tendons, the prestress may be assumed to be transferred to the concrete by a constant bond stress f_{bpt}, where:

$$f_{bpt} = \eta_{p1}\, \eta_1\, f_{ctd}(t) \qquad (8.15)$$

where:

η_{p1} is a coefficient that takes into account the type of tendon and the bond situation at release
η_{p1} = 2,7 for indented wires
η_{p1} = 3,2 for 3 and 7-wire strands

η_1 = 1,0 for good bond conditions (see 8.4.2)
= 0,7 otherwise, unless a higher value can be justified with regard to special circumstances in execution

γ_c=1.5

$f_{ctd}(t)$ is the design tensile value of strength at time of release; $f_{ctd}(t) = \alpha_{ct} \cdot 0,7 \cdot f_{ctm}(t) / \gamma_c$ (see also 3.1.2 (8) and 3.1.6 (2)P)

Note: Values of η_{p1} for types of tendons other than those given above may be used subject to a European Technical Approval

Shear tension resistance

- **Transmission length**

(2) The basic value of the transmission length, l_{pt}, is given by:

$$l_{pt} = \alpha_1 \alpha_2 \phi \sigma_{pm0} / f_{bpt} \tag{8.16}$$

where:

α_1 = 1,0 for gradual release
= 1,25 for sudden release
α_2 = 0,25 for tendons with circular cross section
= 0,19 for 3 and 7-wire strands
ϕ is the nominal diameter of tendon
σ_{pm0} is the tendon stress just after release

(3) The design value of the transmission length should be taken as the less favourable of two

Shear tension resistance

Cross sectional properties

Composed section properties:

$$E_{cm}(fck) := 22000 \cdot \left(\frac{fck + 8}{10}\right)^{0.3} \qquad n_{top} := \frac{E_{cm}(f_{ck.topping})}{E_{cm}(f_{ck})} = 0.868 \qquad n_{fc} := \frac{0.7\, E_{cm}(f_{ck.core})}{E_{cm}(f_{ck})} = 0.607$$

Section properties depending on distance x:

$$A_{ci}(x) := A_c + n_{top} A_{topping} + n_{cores}(x) \cdot A_{core}$$

$$y_{ci}(x) := \frac{A_c \cdot y_c + n_{top} A_{topping} \cdot y_{topping} + n_{cores}(x) \cdot A_{core} \cdot y_{core}}{A_{ci}(x)}$$

$$I_{ci}(x) := I_c + A_c \cdot \left(y_c - y_{ci}(x)\right)^2 + n_{top} \cdot I_{topping} + n_{top} \cdot A_{topping} \cdot \left(y_{topping} - y_{ci}(x)\right)^2 + n_{cores}(x) \cdot I_{core} + n_{cores}(x) \cdot A_{core} \cdot \left(y_{core} - y_{ci}(x)\right)^2$$

$$S_{ci}(x) := S_c \frac{y_{Sc} + \left(y_{ci}(x) - y_c\right)}{y_{Sc}} + n_{cores}(x) \cdot A_{Sc.core} \cdot \left[y_{Sc.core} + \left(y_{ci}(x) - y_c\right)\right]$$

Shear tension resistance

Shear tension capacity: (simplified formula added with core filling)

$\varphi := 1.0 \qquad \beta := 1.0$

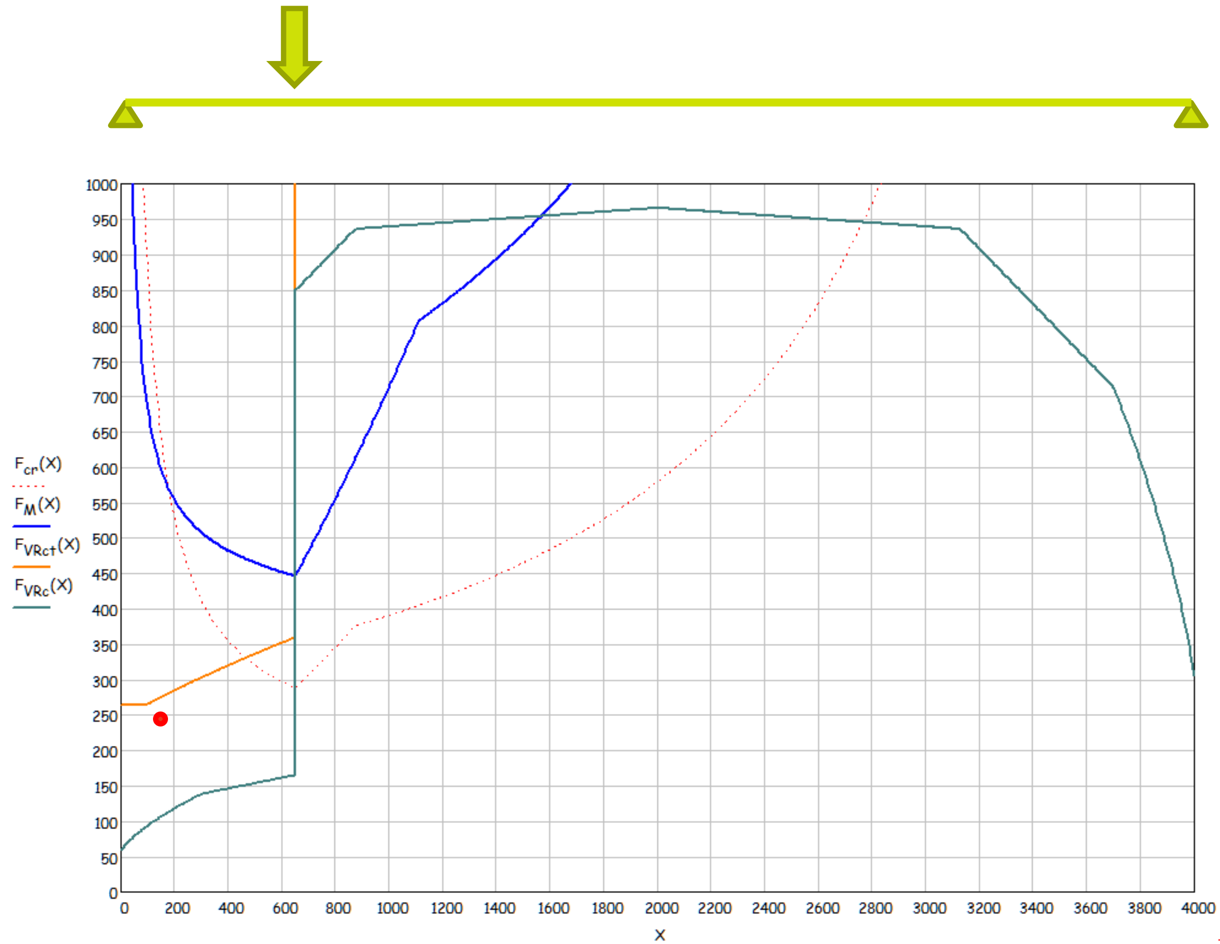
$$V_{Rct}(x) := \begin{array}{|l} Lx \leftarrow \max\left(0.5 \cdot a_1 + Y_c, \text{distance}(x)\right) \\ \sigma_c \leftarrow \dfrac{F_p(Lx)}{A_c} \\ \dfrac{I_{ci}(x)}{S_{ci}(x)}\left(\varphi \cdot b_{w.slab} \cdot \sqrt{f_{ctm}^{\,2} + \beta\,\sigma_c \cdot f_{ctm}} + b_{w.core}(x) \cdot f_{ctm.core}\right) \end{array}$$

Shear tension resistance

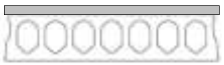
A260 12 Ø12,5
no topping

F_{exp} = 243 kN

F_{calc} = 274 kN



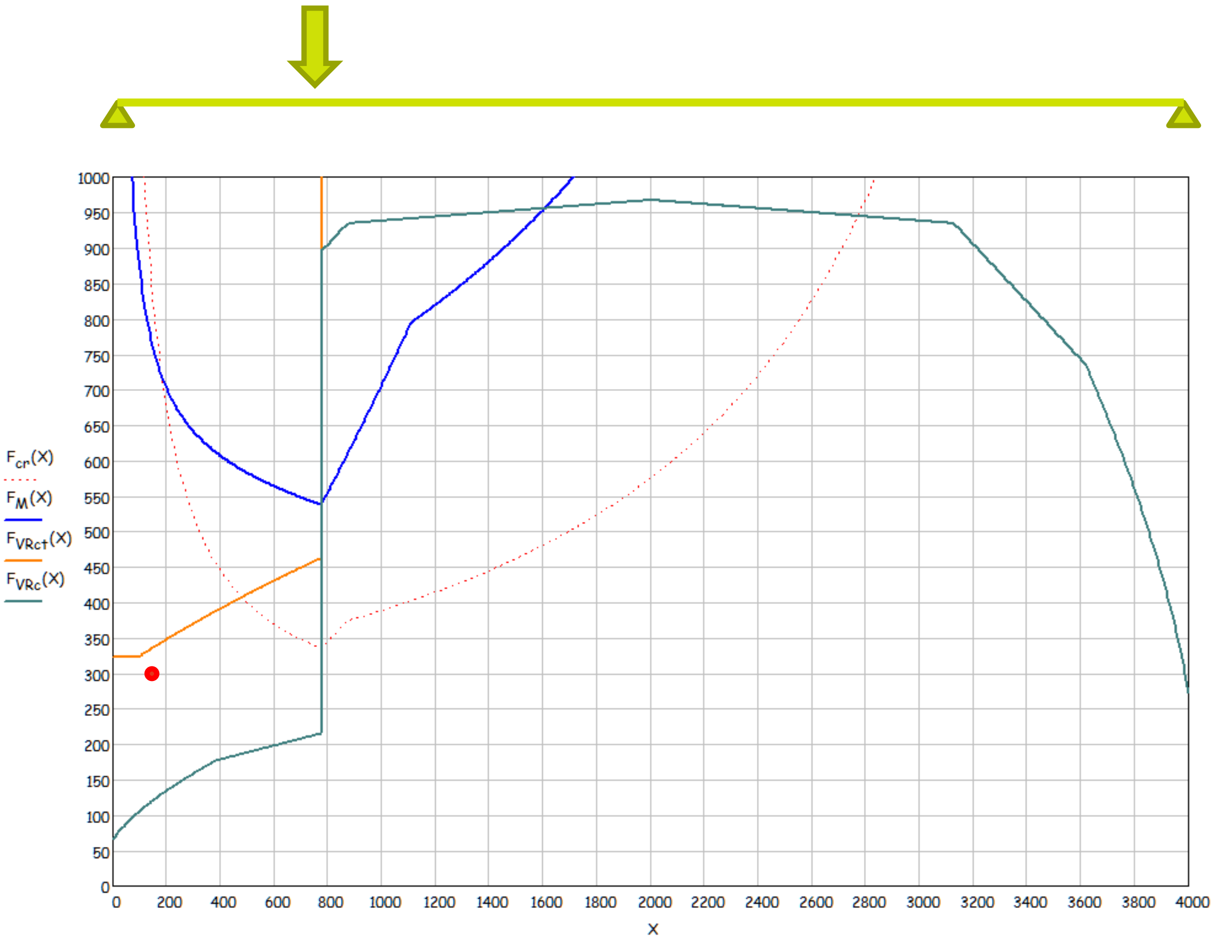
Shear tension resistance



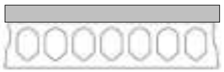
A260 12 Ø12,5
50 mm topping

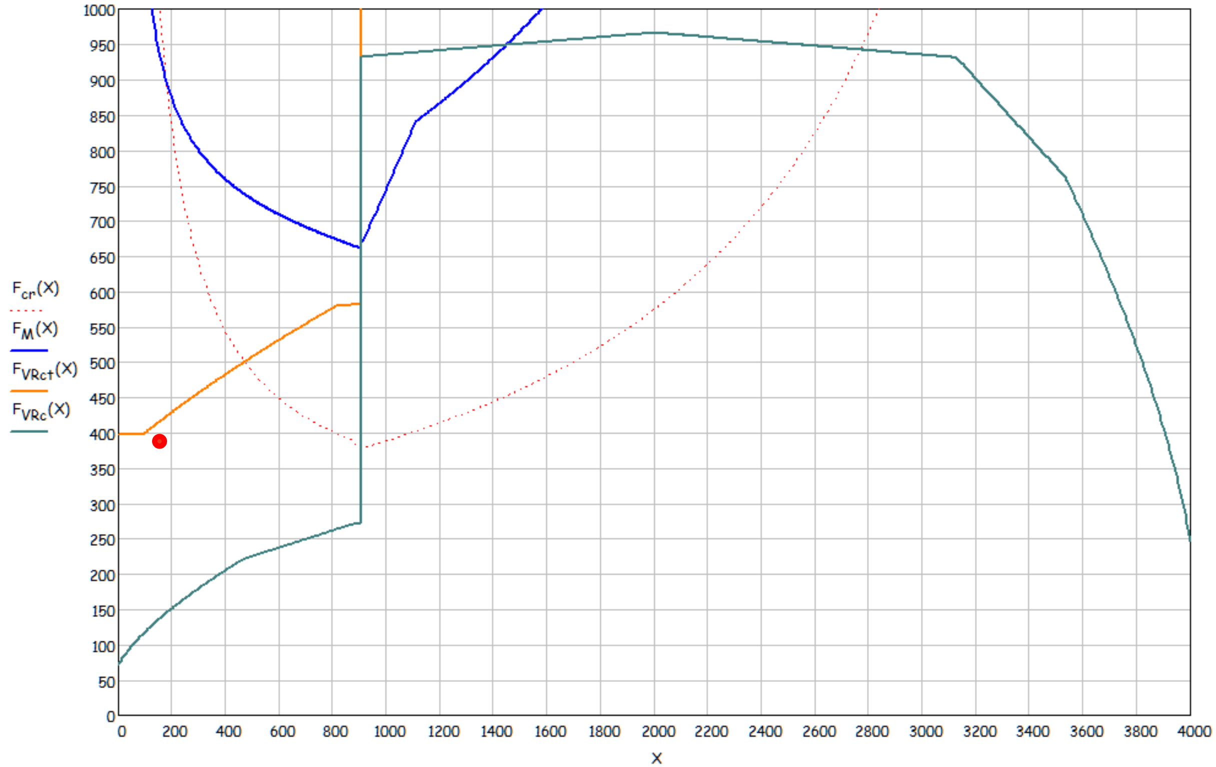
F_{exp} = 299 kN

F_{calc} = 336 kN



Shear tension resistance



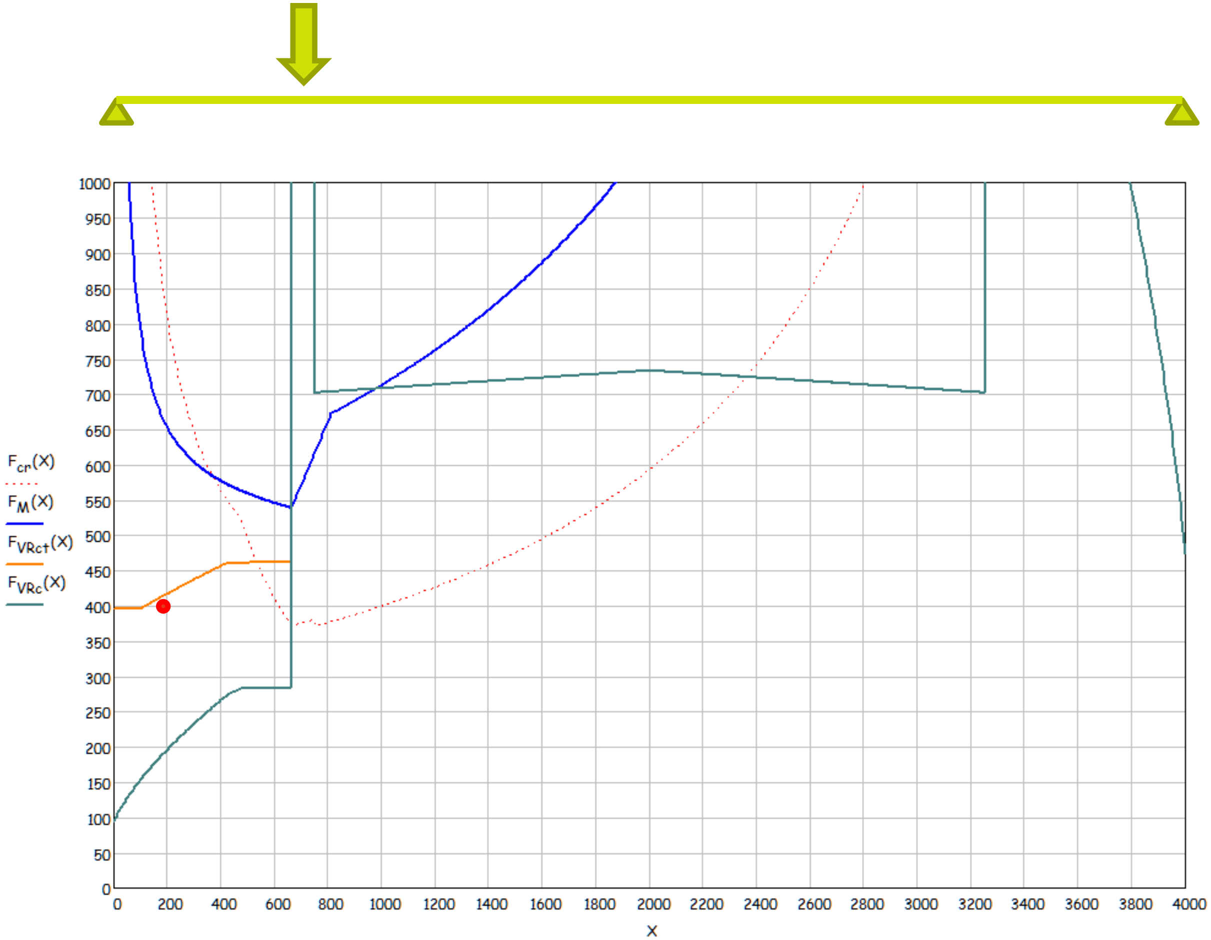


A260 12 Ø12,5
100 mm topping

F_{exp} = 384 kN

F_{calc} = 414 kN

Filled cores

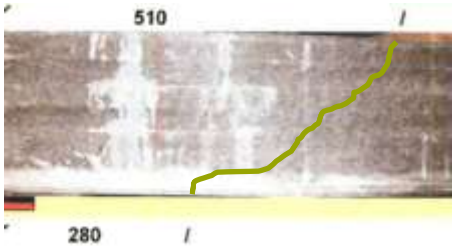


VX265 10 Ø12,5
no topping

F_{exp} = 398 kN

F_{calc} = 407 kN

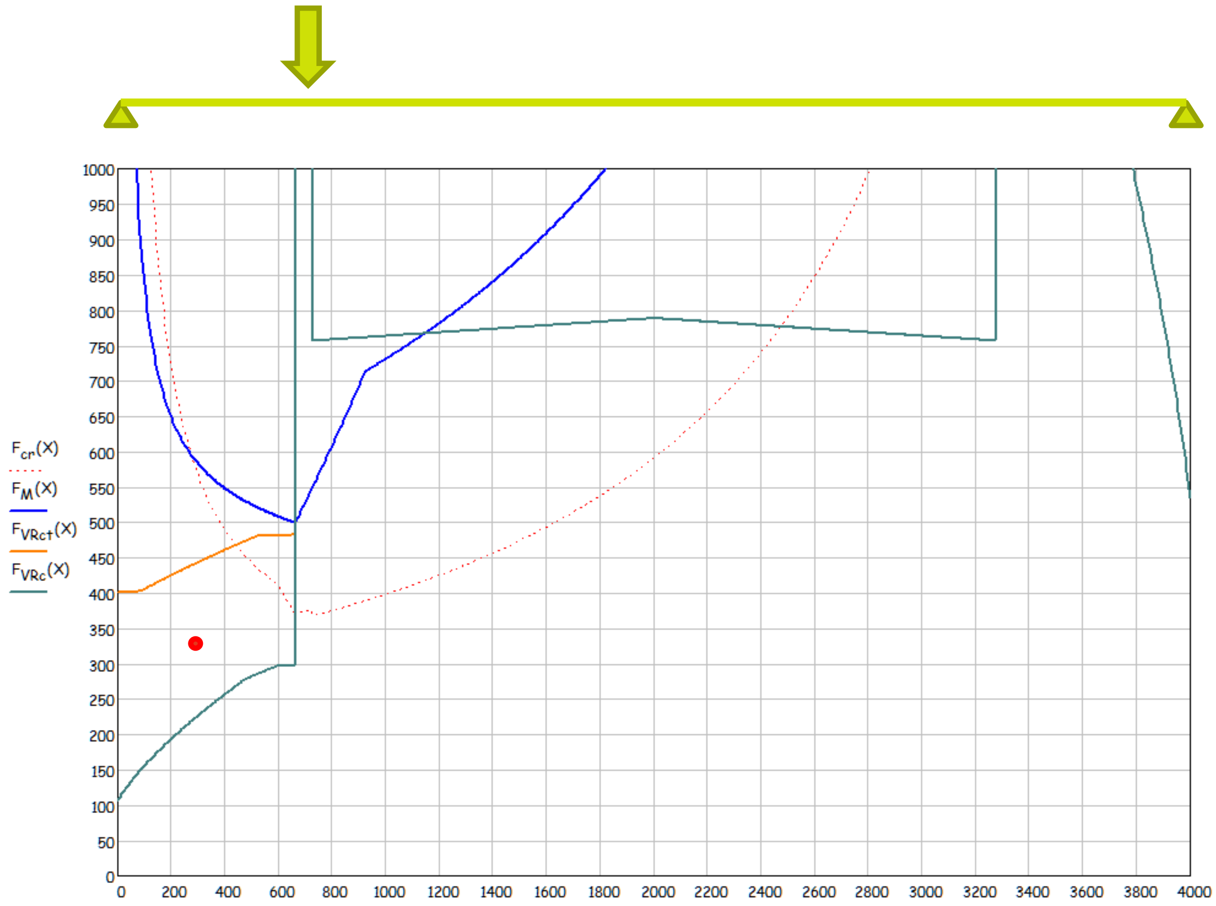
Filled cores



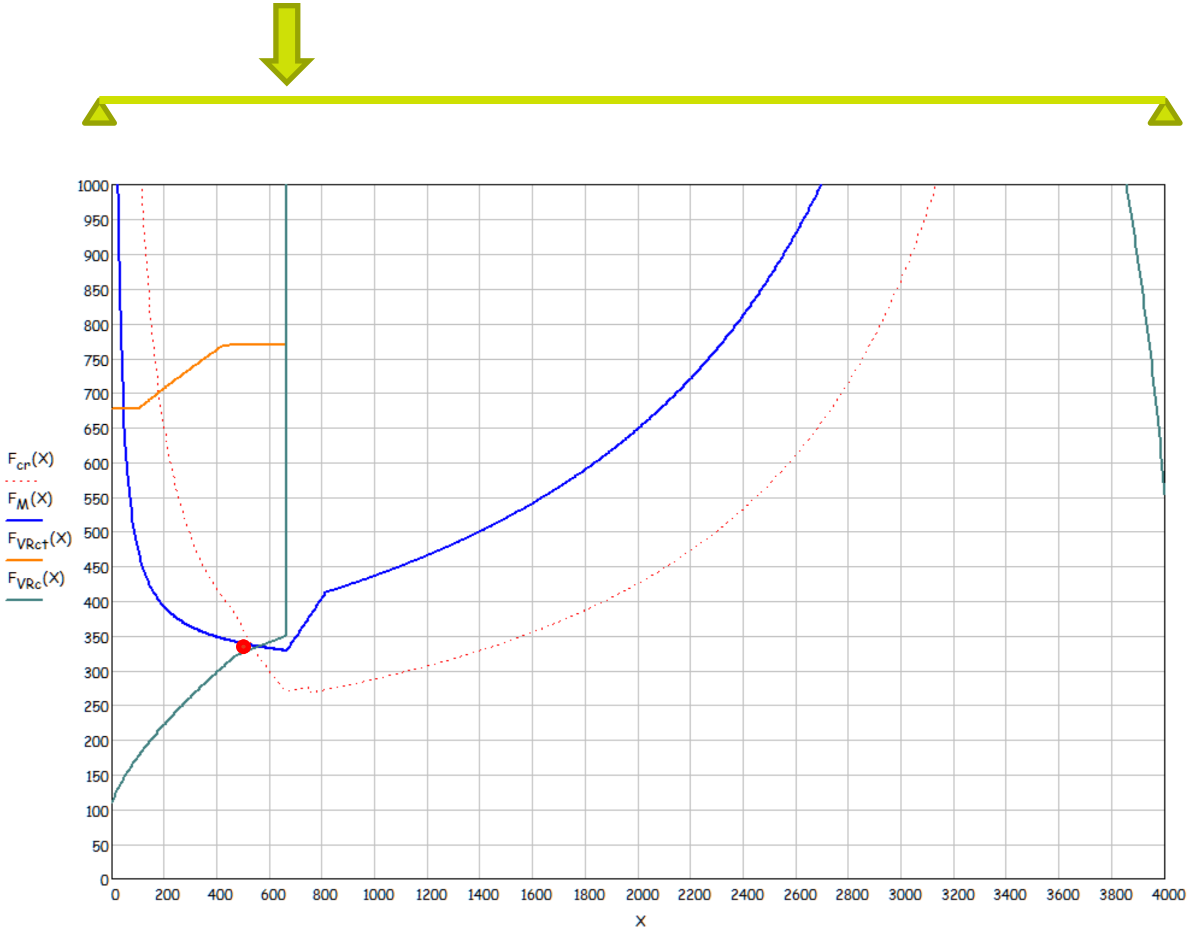
5-265 10 Ø12,5
no topping

F_{exp} = 330 kN

F_{calc} = 405 kN



Filled cores



260 5 Ø12,5
no topping

F_{exp} = 340 kN

F_{calc} = 350 kN

Conclusions

- **Structural topping increases the shear resistance as well for shear tension as for shear flexural capacity**
- **The design model according to EC2 predict the same failure mode as in the tests.**
- **Magnitude of the capacity of the tests corresponds with EC2 calculation model.**
- **The bond of the core filling is a critical parameter.**
- **Filled cores increases the shear capacity but also the failure can shift to another mode. (to anchorage failure)**