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Shear and torsion

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Organized by

INTERNATIONAL PRESTRESSED HOLLOWCORE ASSOCIATION

in cooperation with

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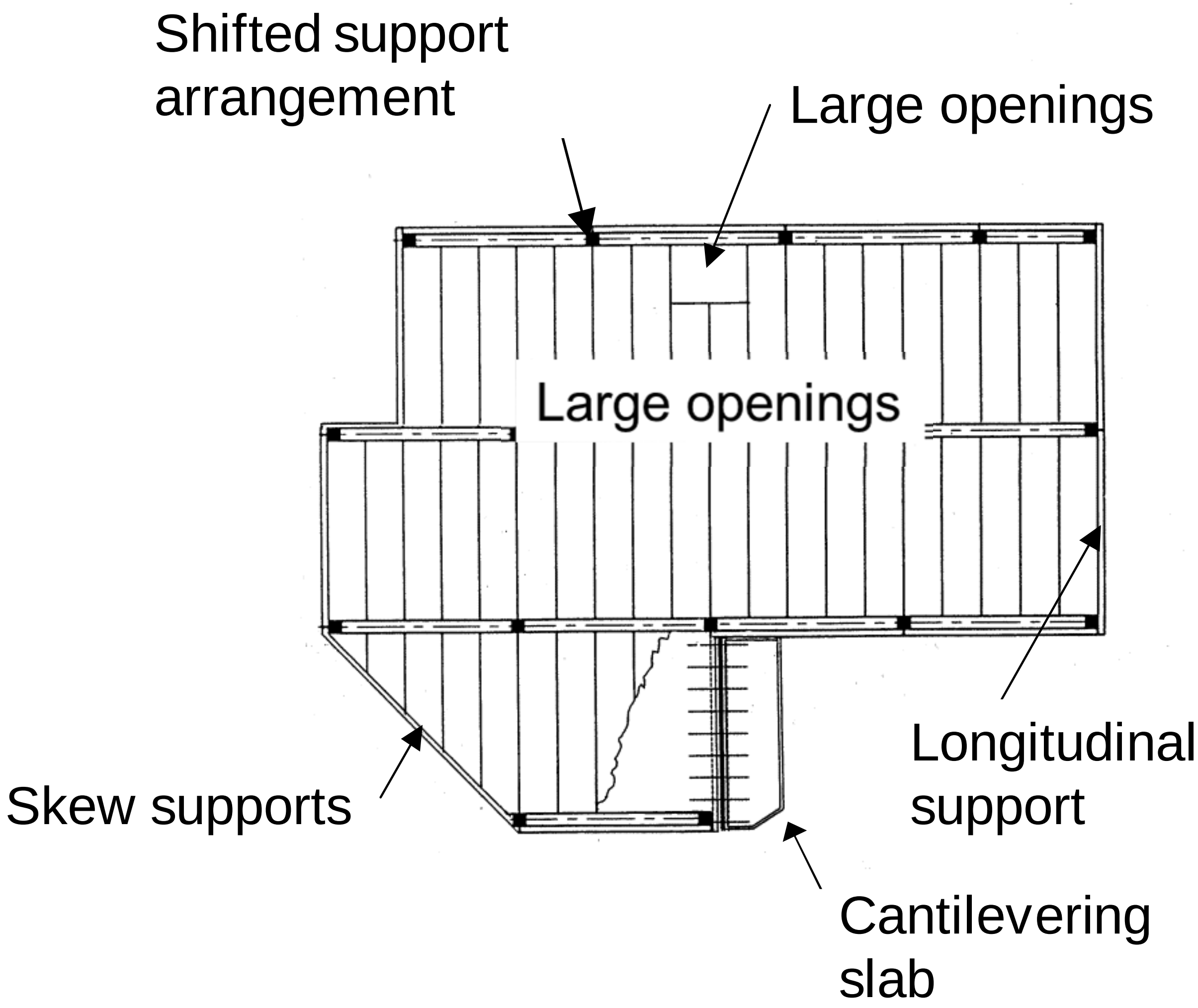
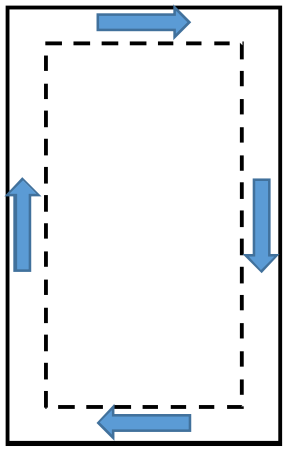
Reasons for torsion

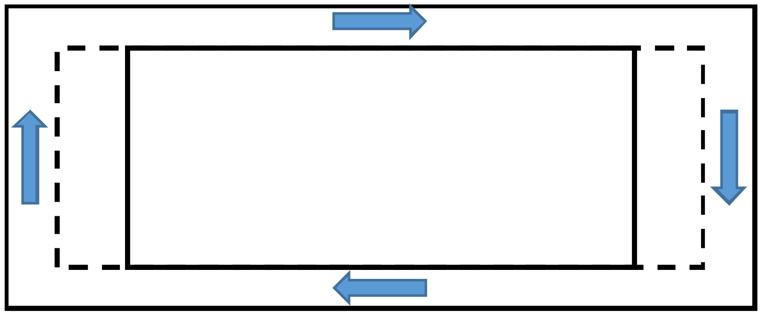


Trimmer beam at large opening

Response to torsion



Rectangular
solid section



Rectangular
hollow section

Torsion results in shear stresses in outer part of solid section or in actual thin walls of hollow section

Shear flow – torsional shear stress

Shear flow q (force per unit length) assumed to be constant

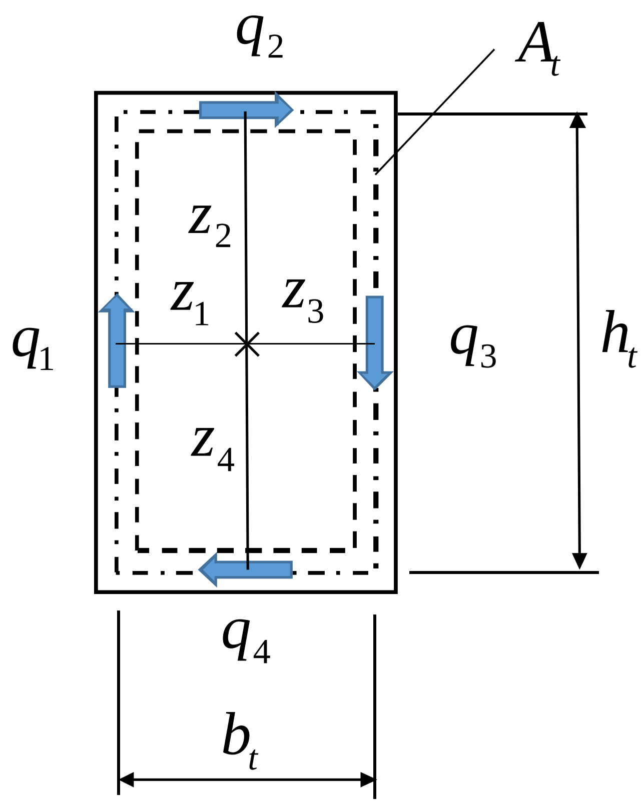
$$q = \tau \cdot t$$

Torsional shear stress

$$\tau = \frac{T}{2A_t \cdot t} = \frac{T}{W_t}$$

Torsion modulus of section

$$W_t = 2A_t \cdot t$$

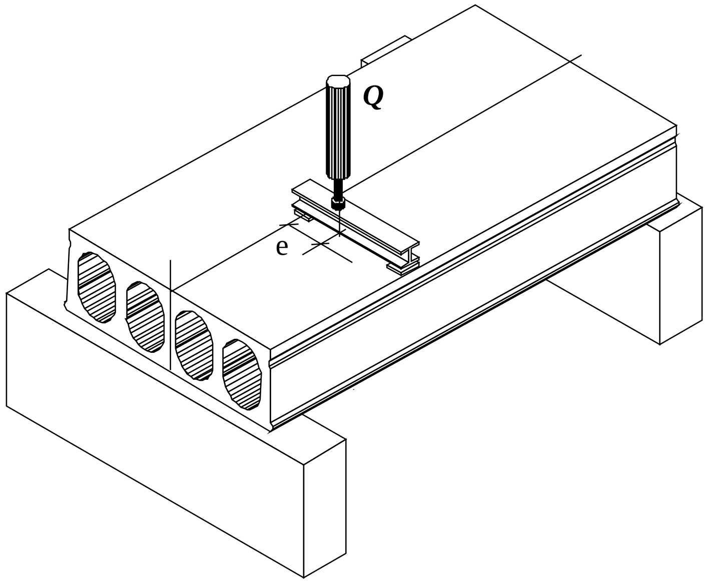


$$T = q_1 \cdot h_t \cdot \frac{b_t}{2} + q_2 \cdot b_t \cdot \frac{h_t}{2} + q_3 \cdot h_t \cdot \frac{b_t}{2} + q_4 \cdot b_t \cdot \frac{h_t}{2} == q[2b_t \cdot h_t] = q \cdot 2A_t$$

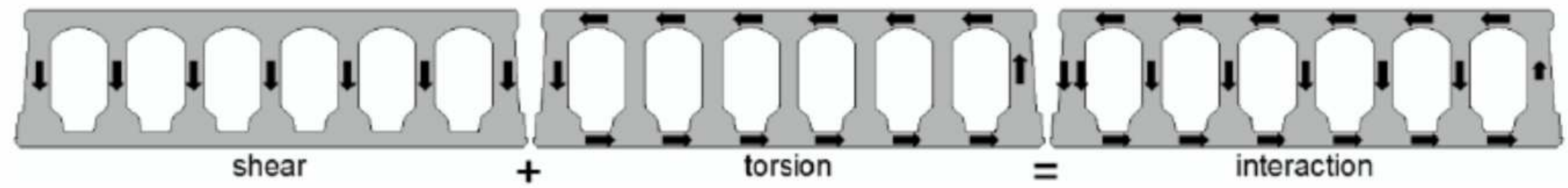
Torsional shear stress

- Rectangular solid section
  - The shear stress is largest at the outside of the section at the midpoint of the wider side
- Thin walled section
  - The shear flow is constant and the shear stress is largest in the thinner wall

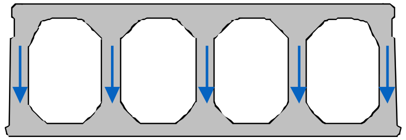
Interaction vertical shear and torsion



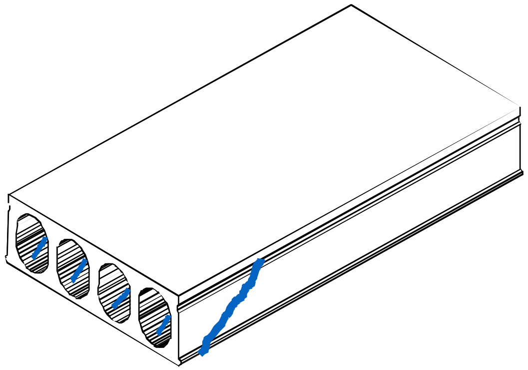
Eccentric load results in both vertical shear and torsion

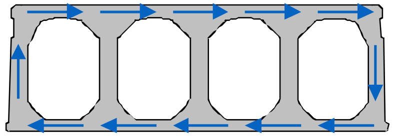


Cracking due to shear and torsion

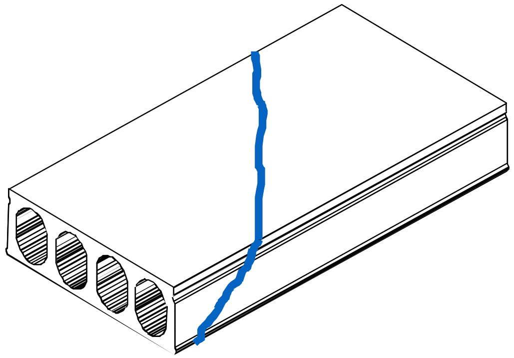


Web shear tension crack



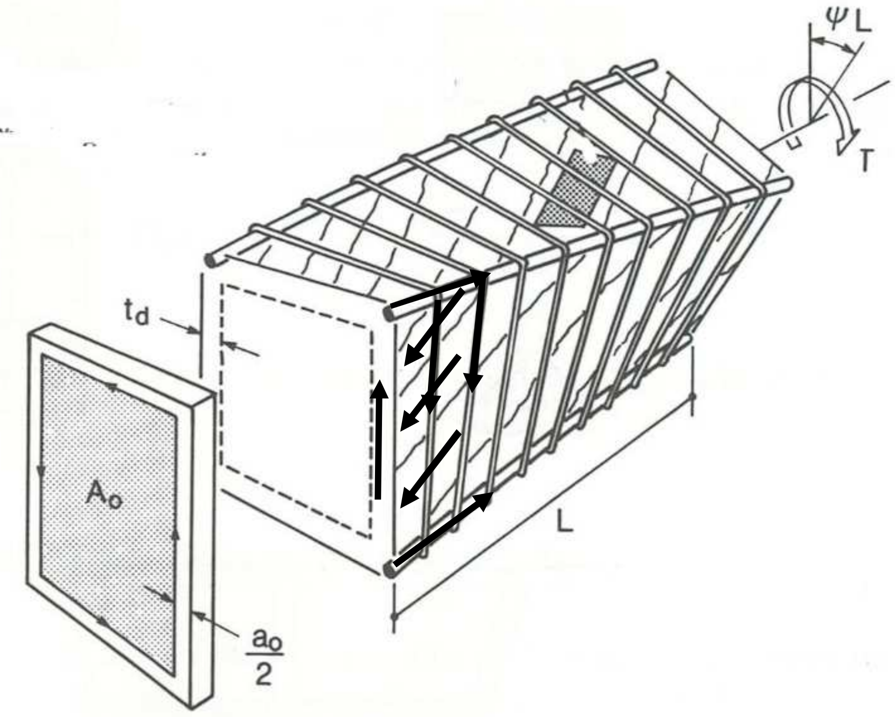


Torsional crack



Reinforced concrete beam

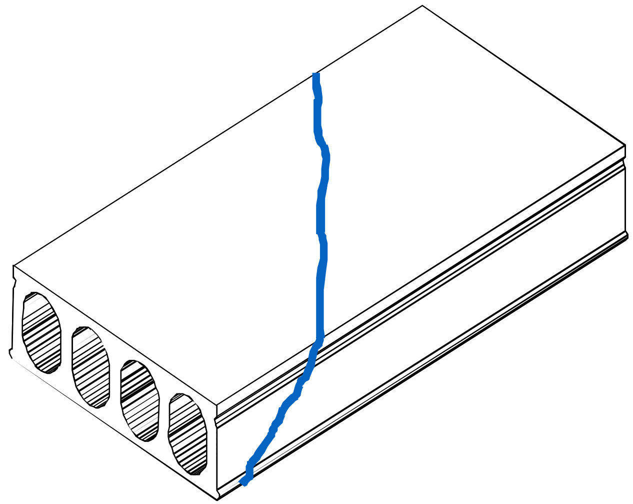
- Uncracked stage
  - Torsional stiffness in uncracked stage
- Torsional cracking
- Cracked stage
  - Torsional stiffness in cracked stage
- Torsional resistance



- After inclined cracking the torsional shear is carried by transverse components of inclined compression.
- The inclined compressive struts need to be balanced by transverse and longitudinal steel reinforcement.
- Torsional failure due to crushing of struts or yielding of all steel

Hollow core slab

- Uncracked stage
  - Torsional stiffness in uncracked stage
- Torsional cracking = torsional resistance

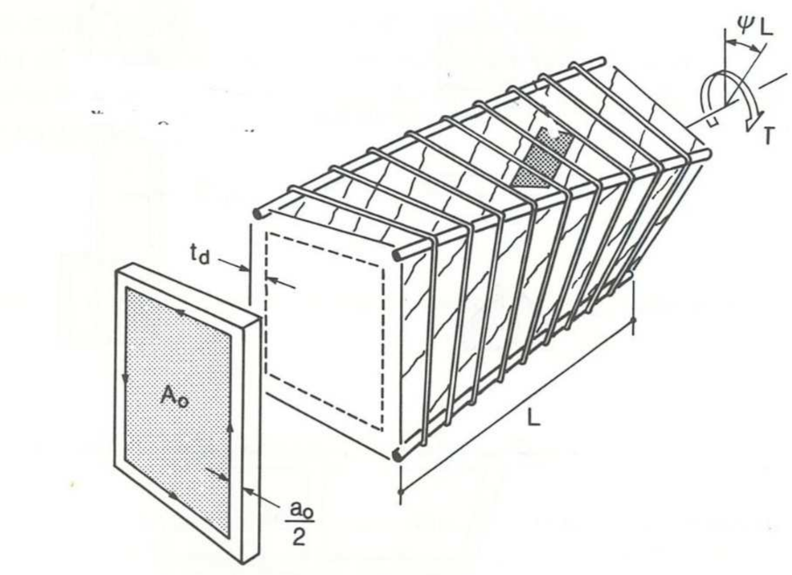


- Since transverse and some longitudinal reinforcement is missing, it is not possible to achieve a state of equilibrium in the cracked stage
- Skew cracking in top flange or web means that the torsional resistance is reached

Torsional deformation and stiffness

- Twist per unit length

$$\frac{d\varphi}{dx} = \frac{T}{C}$$



- Torsional rigidity (uncracked)

$$C = GK_T$$

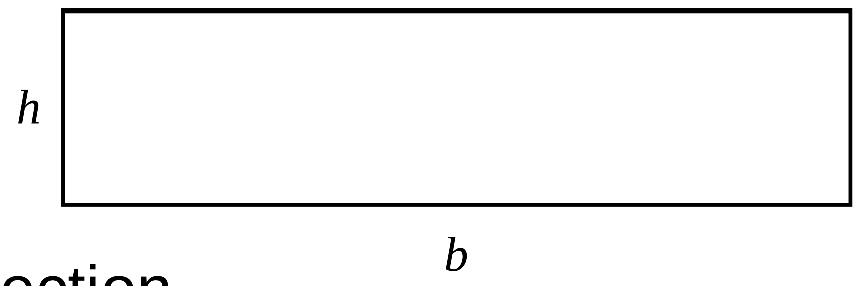
$$G = \frac{E}{2(1+\nu)}$$ (shear modulus)

K_T (cross-sectional factor)

Cross-sectional factor

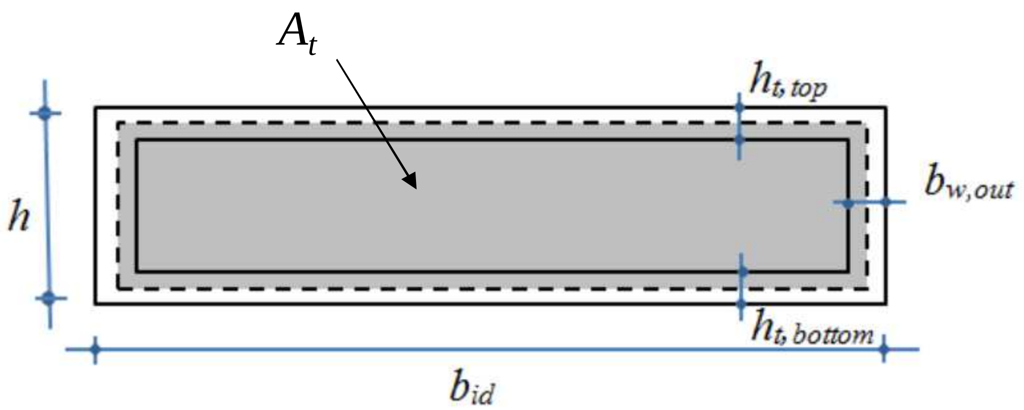
- Solid rectangular section *b* > *h*

$$K_{\mathrm{T}} = \frac{bh^3}{12}\left(1 - 0{,}63\frac{h}{b}\right)$$



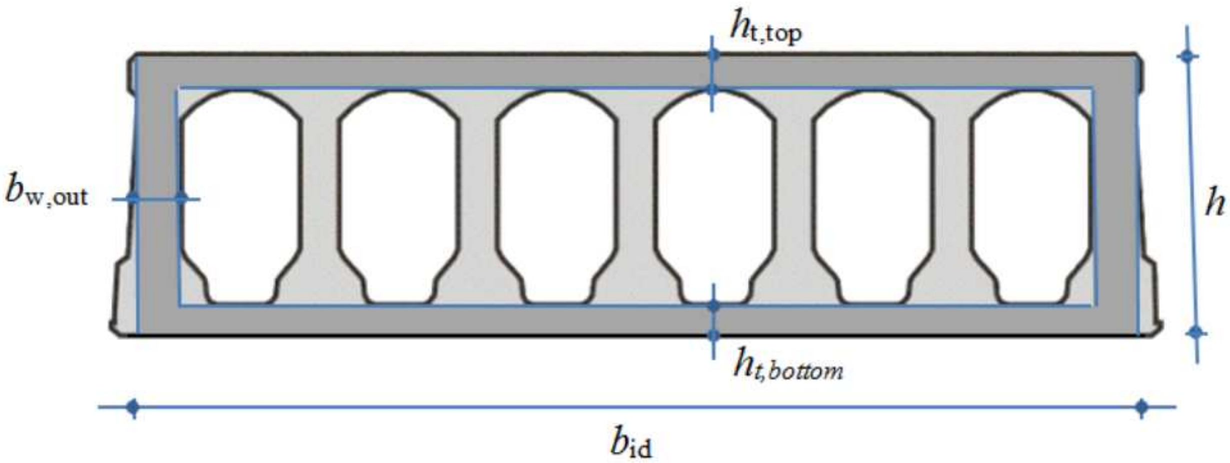
- Thin walled rectangular section

$$K_T = \frac{4A_t^2}{\sum \frac{u_i}{t_i}}$$

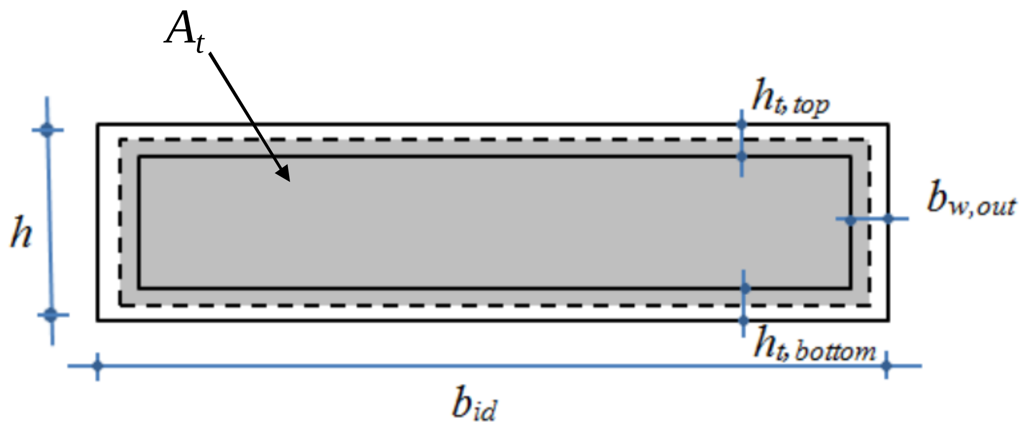


Hollow core section

- Transformation to equivalent thin walled rectangular section



Cross-sectional factor



$$K_T = \frac{4A_t^2}{\sum \frac{u_i}{t_i}}$$

$$A_t = (b_{id} - b_{w,out}) \cdot (h - 0{,}5(h_{t,top} + h_{t,bottom}))$$

$$\sum \frac{u_i}{t_i} = \frac{b_{id} - b_{w,out}}{h_{t,top}} + \frac{b_{id} - b_{w,out}}{h_{t,bottom}} + \frac{2(h - 0{,}5(h_{t,top} + h_{t,bottom}))}{b_{w,out}}$$

Cracking due to torsion

- Load case:
  - Normal stress due to prestress and bending moment (eccentricity of prestress and load)
  - Shear stress due to vertical shear (load) and torsion

- Different conditions in webs and flanges

- Crack occurs when the principal stress reaches the concrete tensile stress

Principal tensile stress in flange

$$\sigma_{\mathrm{I}} = \frac{\sigma_{ct}}{2} + \sqrt{\left(\frac{\sigma_{ct}}{2}\right)^2 + \tau_T}$$

Normal stress due to prestress and bending moment:

$$\sigma_{\mathrm{c}t} = \frac{-P(x)}{A_c} + \frac{-P(x)\cdot e + M(x)}{I_c}\cdot(-x_c)$$

Torsional crack in top flange

$$\underset{f_{ctd}}{\sigma_{\mathrm{I}}} = \frac{\sigma_{ct}}{2} + \sqrt{\left(\frac{\sigma_{ct}}{2}\right)^2 + \tau_T}$$

- Calculate normal stress in top flange σ_{ct}
- Assume that the principal stress equals the concrete tensile stress
- Solve the torsional shear stress that creates a skew crack in the top flange

$$\tau_{T,top} = f_{ctd}\sqrt{1 - \frac{\sigma_{ct}}{f_{ctd}}}$$

- Torsional moment that results in the crack

$$T_{cr,top} = W_{T,top} \cdot \tau_{T,top} = W_{T,top} \cdot f_{ctd}\sqrt{1 - \frac{\sigma_{ct,top}}{f_{ctd}}}$$

Torsional modulus of section W_t

For section with constant wall thickness

$$\tau = \frac{T}{2A_t \cdot t} = \frac{T}{W_t} \qquad W_t = 2A_t \cdot t \qquad A_t = b_t \cdot h_t = (b-t)\cdot(h-t)$$

For hollow core section – different wall thicknesses

$$A_t = \left(b_{id} - b_{w,out}\right)\cdot\left(h - 0,5(h_{t,top} + h_{t,bottom})\right)$$

$$\left.\begin{aligned} W_{T,top} &= W_T\left(t_{top}\right) \\ W_{T,web} &= W_T\left(b_{w,out}\right) \\ W_{T,bottom} &= W_T\left(t_{bottom}\right) \end{aligned}\right\}$$

Different values depending on actual wall thickness

Principal tensile stress in web

$$\sigma_{\mathrm{I,web}} = \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau_T + \tau_V)}$$

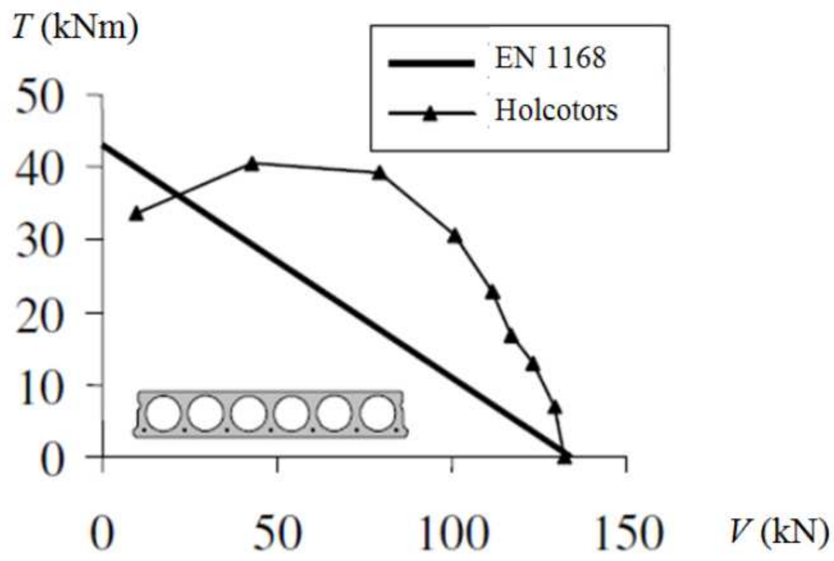
Shear stress due to vertical shear:

$$\tau_V = \frac{1}{b_w}\left[\left(\frac{A_{cp}}{A_c} - \frac{S_{cp}\cdot e}{I_c}\right)\frac{dP}{dx} + \frac{S_{cp}}{I_c}\cdot V(x)\right]$$ (inside transfer length)

$$\tau_V = \frac{S_{cp}\cdot V(x)}{I_c \cdot b_w}$$ (outside transfer length)

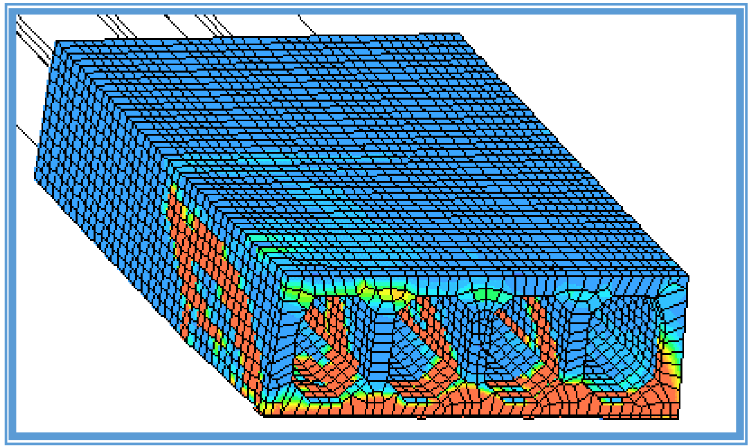
Shear and torsion interaction

- Traditional design approach
  - Stresses from vertical shear and torsion are superimposed
  - The maximum principal stress creates a crack which causes failure
  - One point in web considered
  - Linear interaction assumed
- Holcotors
  - Non-linear analysis
  - Favourable stress redistribution in cracking concrete and influence of restraint from boundaries



Increase with up to 55 % for 200 mm units and up to 30% for 400 mm units

Shear and torsion in hollow core slabs
Holcotors



Financiers and collaboration partners

- European Commission
- International Prestressed Hollow Core Association
- Bundesverband Spannbeton-Hohlplatten

- Castelo
- Consolis
- Echo
- A. Van Acker

- Strängbetong
- VTT
- Chalmers

Holcotors

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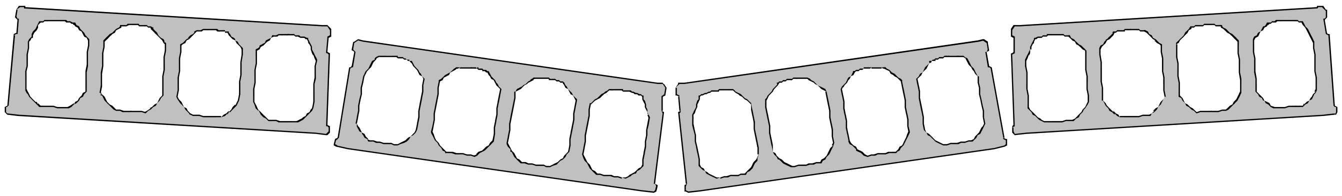
D.Sc. (tech.)
VTT

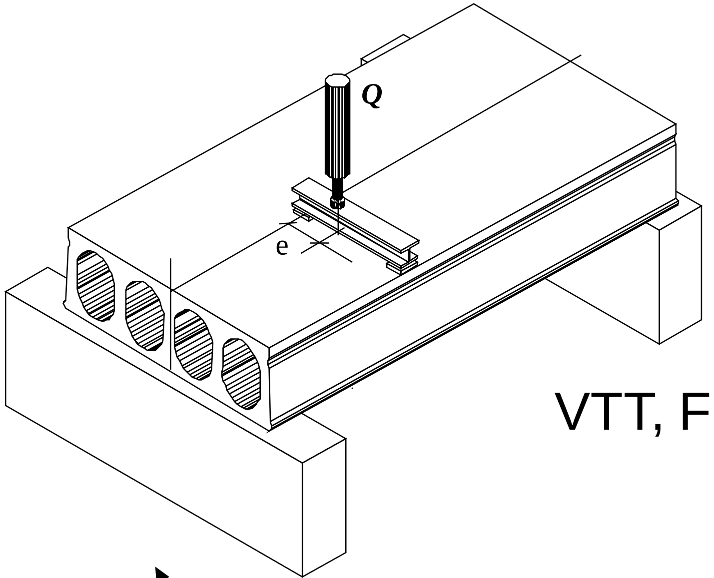
Project started January 1, 2002 and ended December 31, 2004

Aim of project

- To use the capacity of hollow core slabs better
- To develop methods to design for combined shear and torsion in hollow core slabs
  - Single units
  - Whole floors

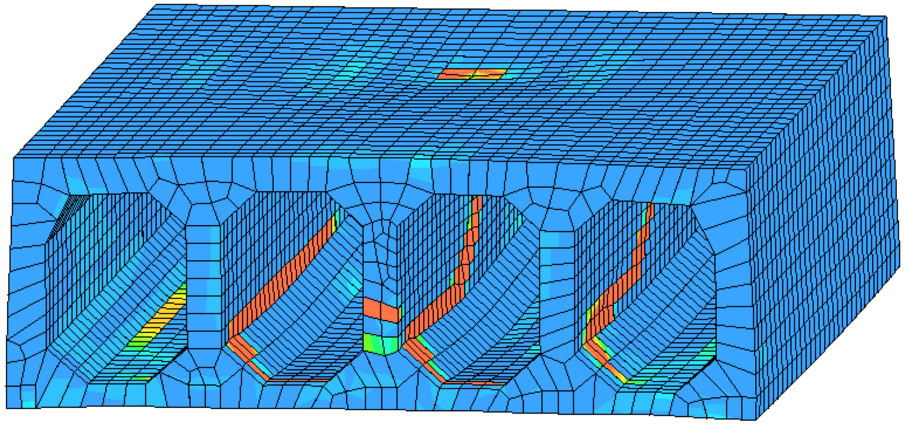


Holcotors, 2002 – 2004

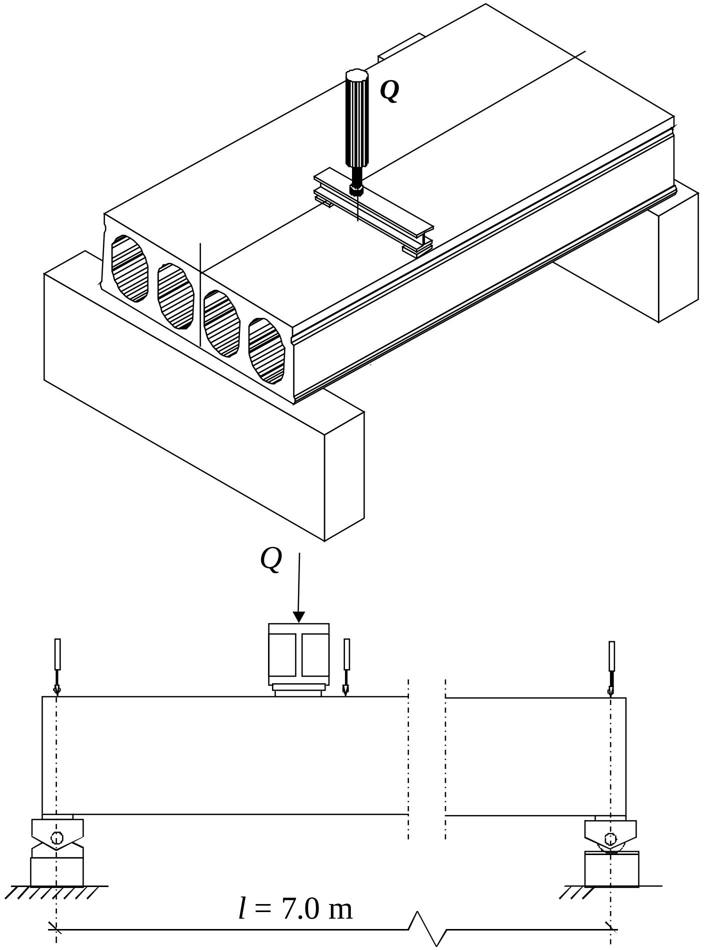


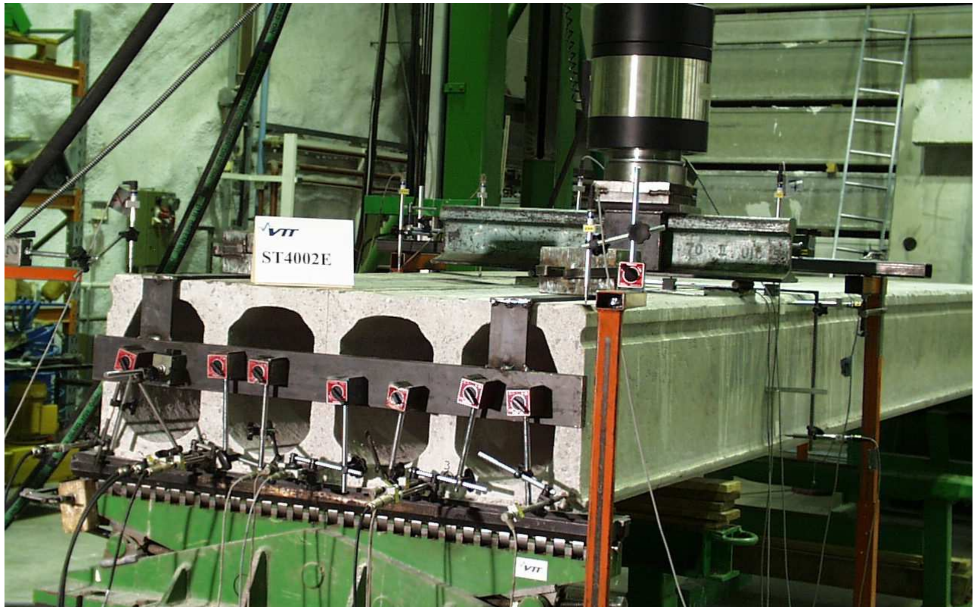
VTT, Finland

Chalmers University, Sweden

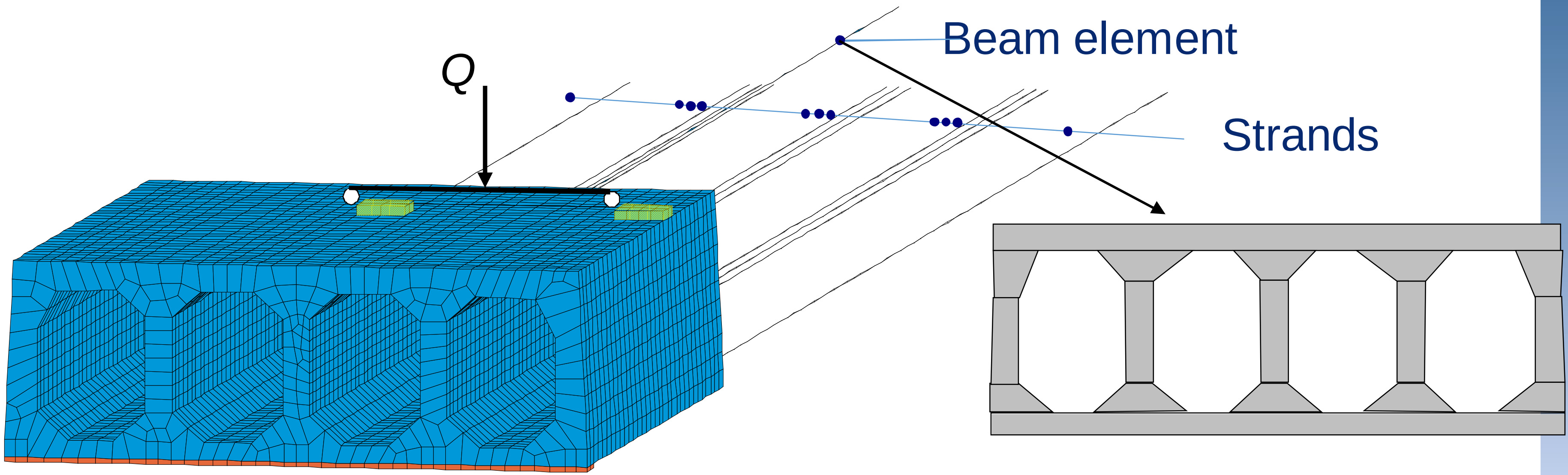


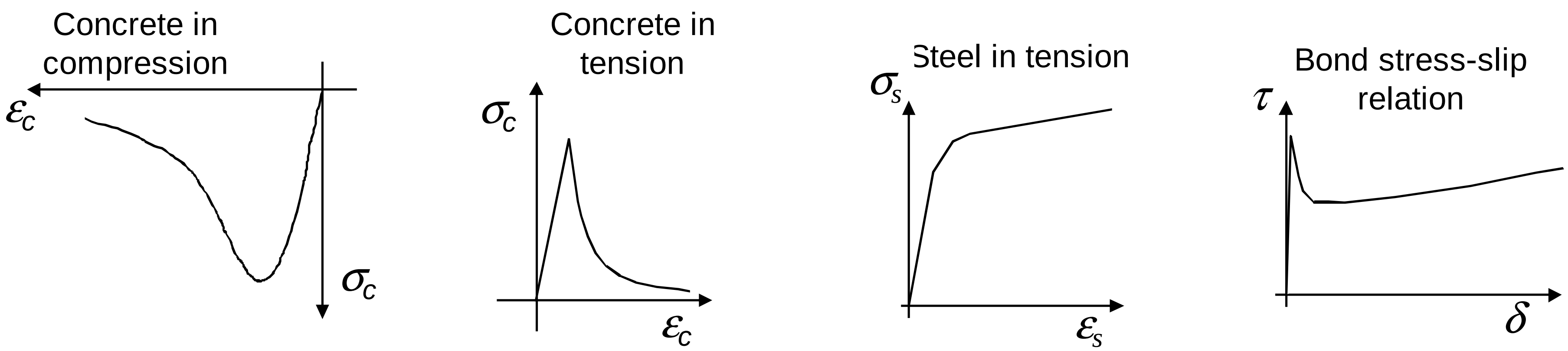
Tests on hollow core units



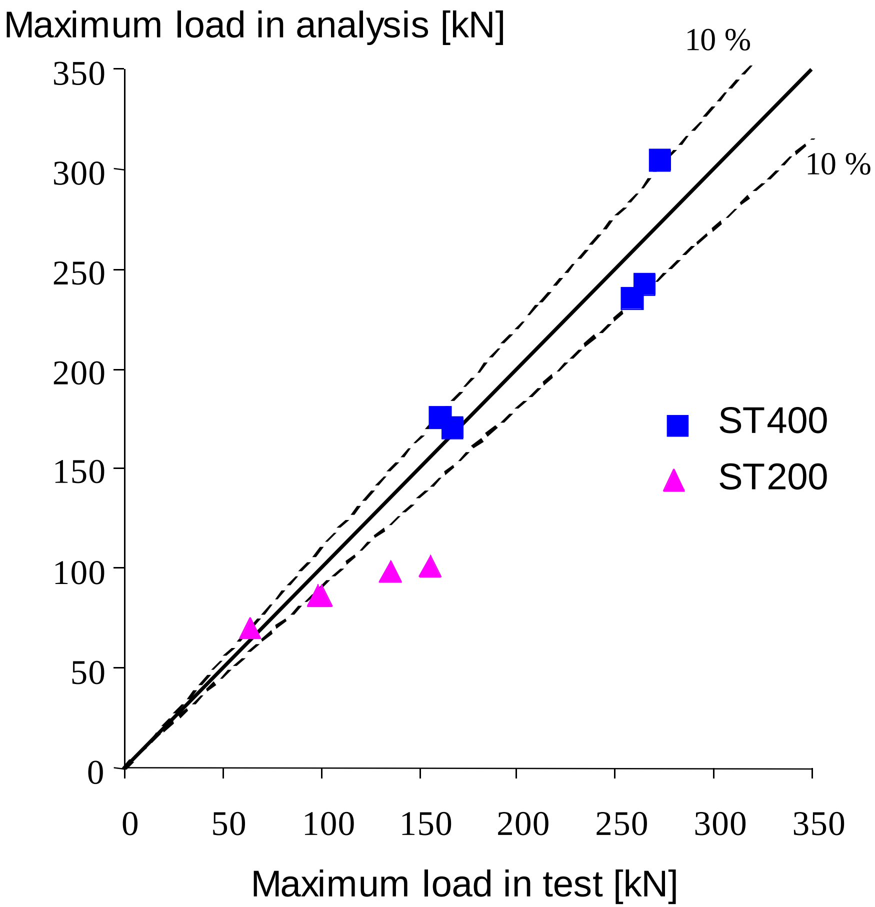


FE-model of hollow core unit

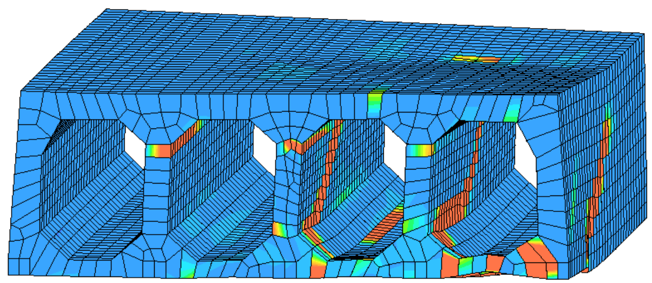




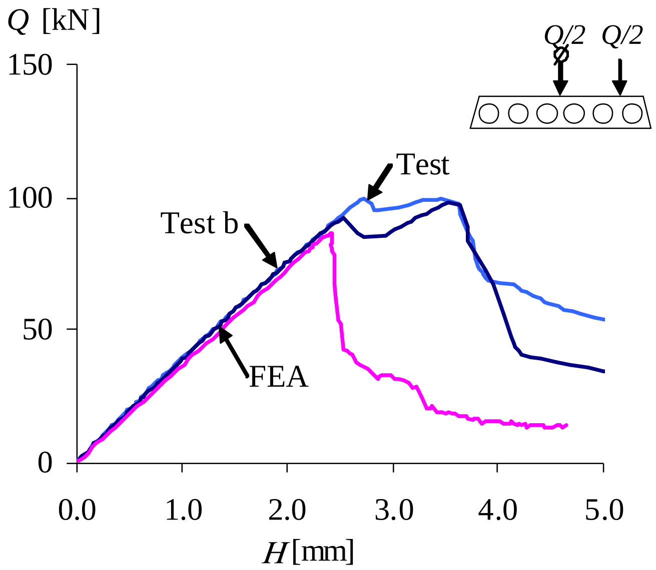
Comparison of results

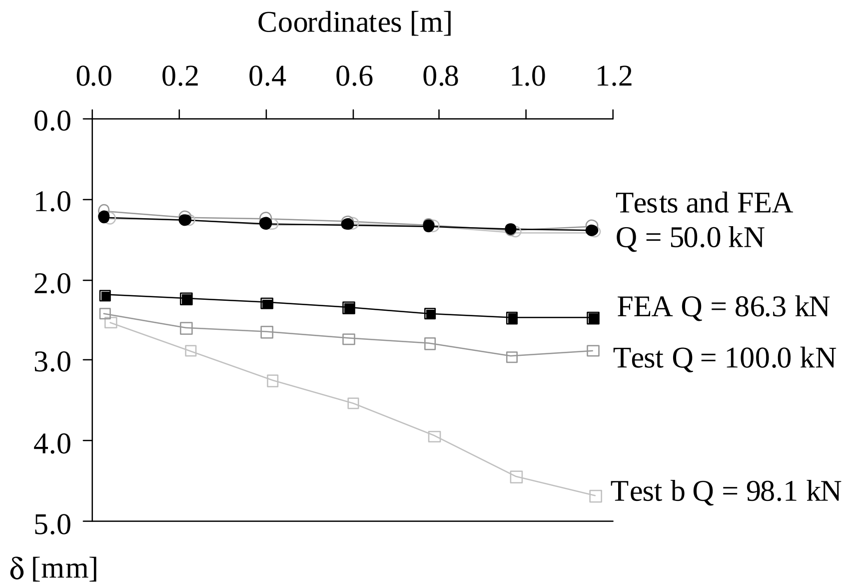


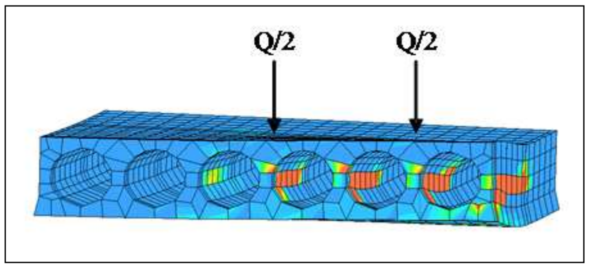
- Maximum load
- Load versus deflection
- Failure mode
- Crack pattern



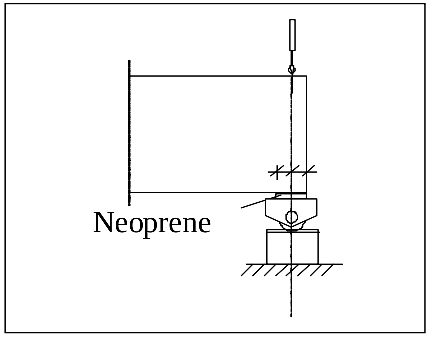
Comparison of results

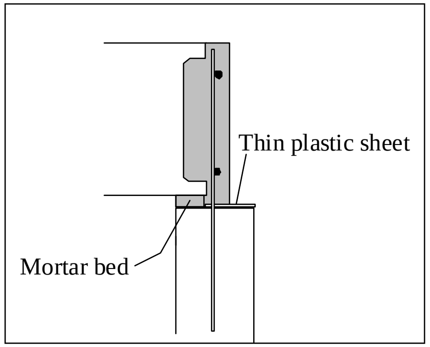


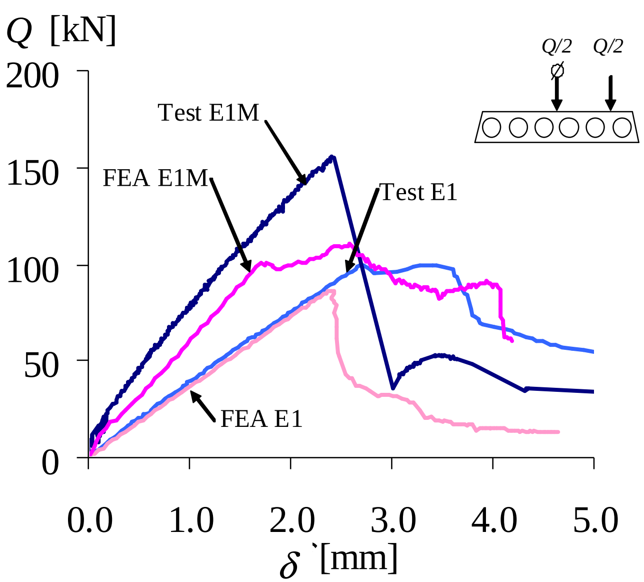


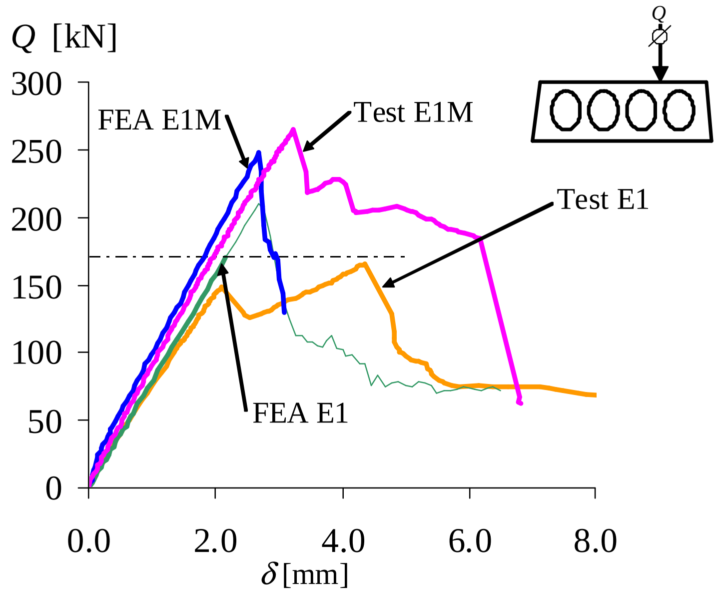


Comparison of results

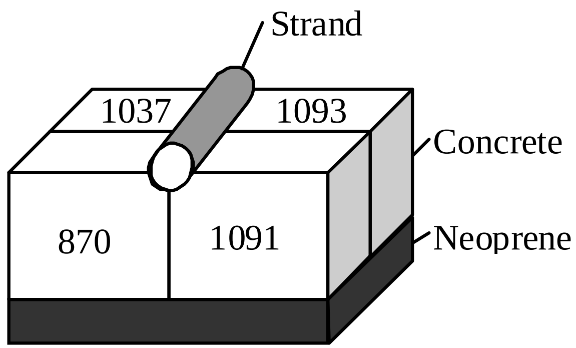


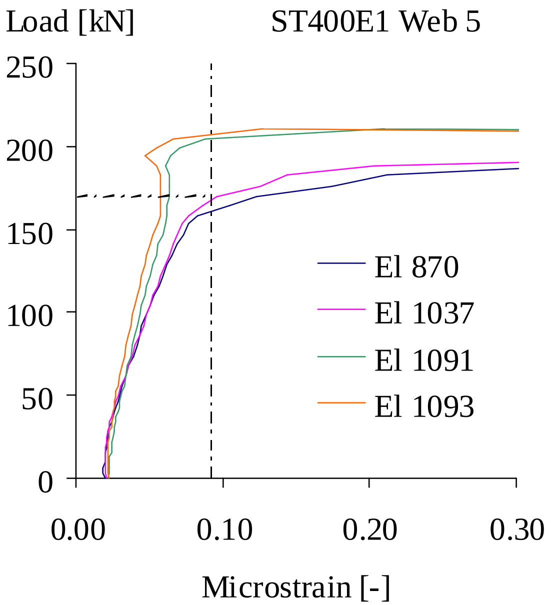


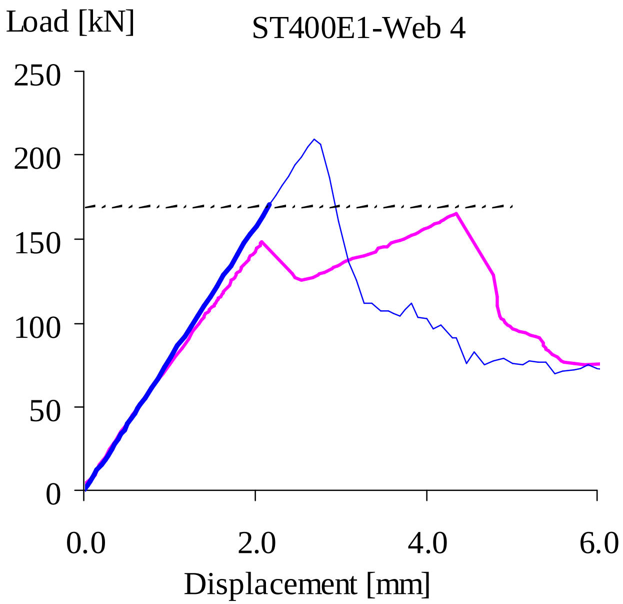




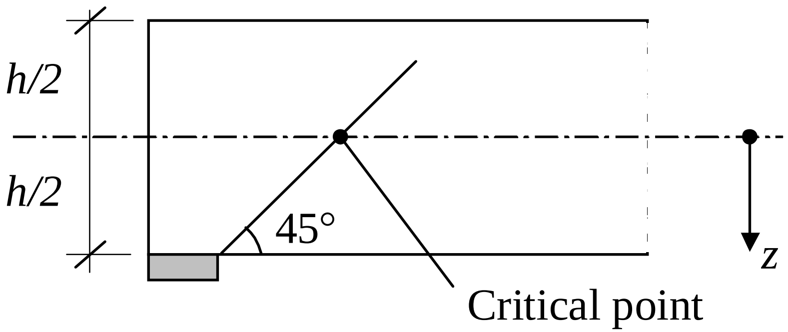
Effect of neoprene bearing

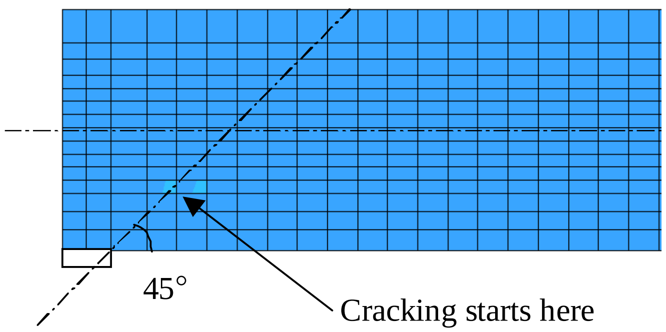






Critical section for shear tension crack in 400 mm units





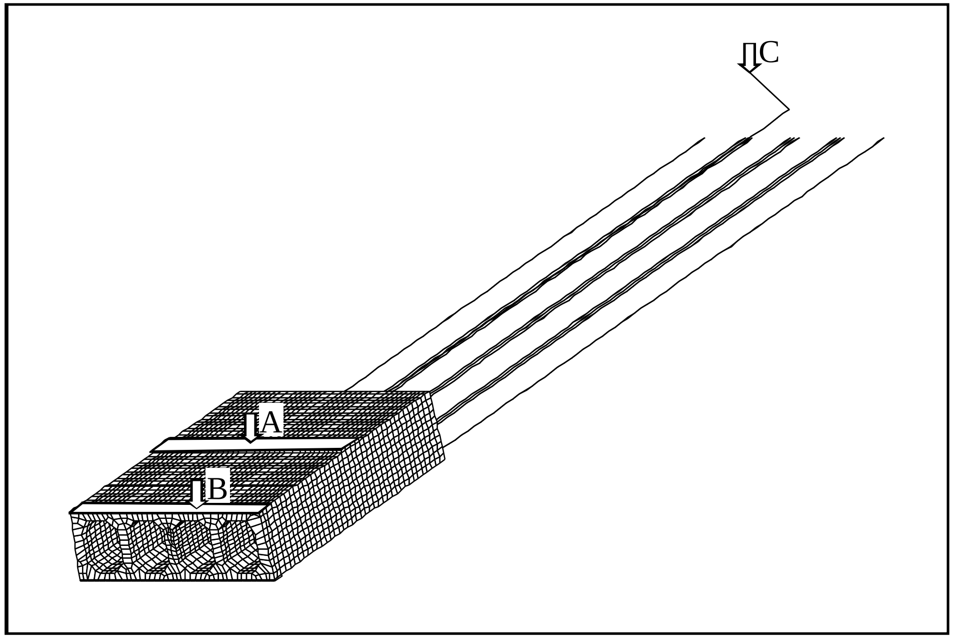
Analytical model

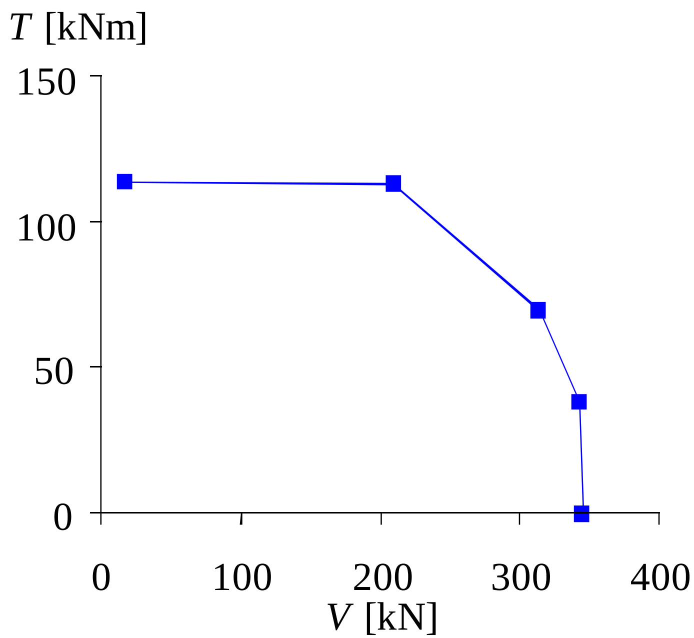
FE-analysis, pure shear

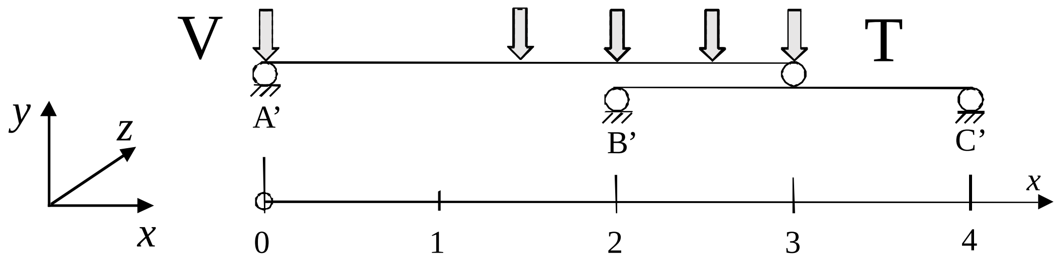
Conclusions

- FE-analyses are able to capture the overall behaviour in tests
  - Failure mode
  - Maximum load
  - Crack pattern
  - Vertical deflection (until first crack)
- Large difference in capacity due to support condition

FE-analyses for V-T interaction





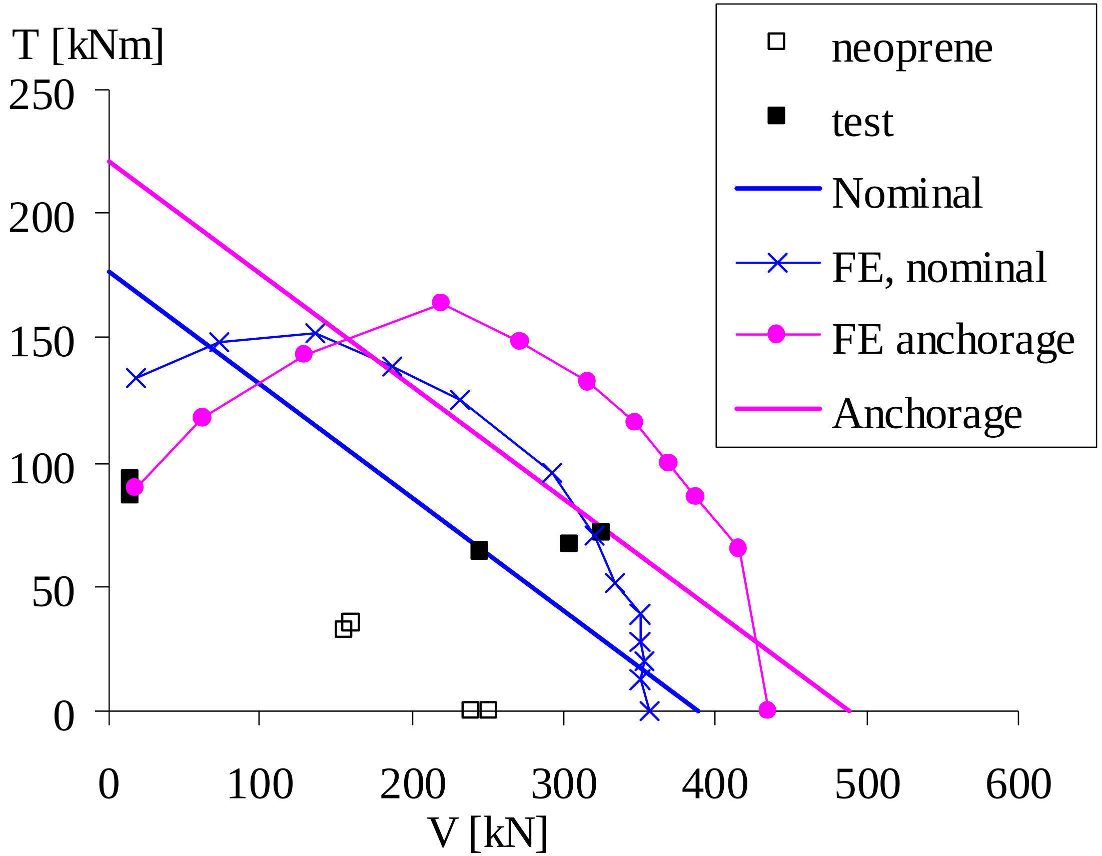


$$\delta_y^A = \delta_y^{A'}$$

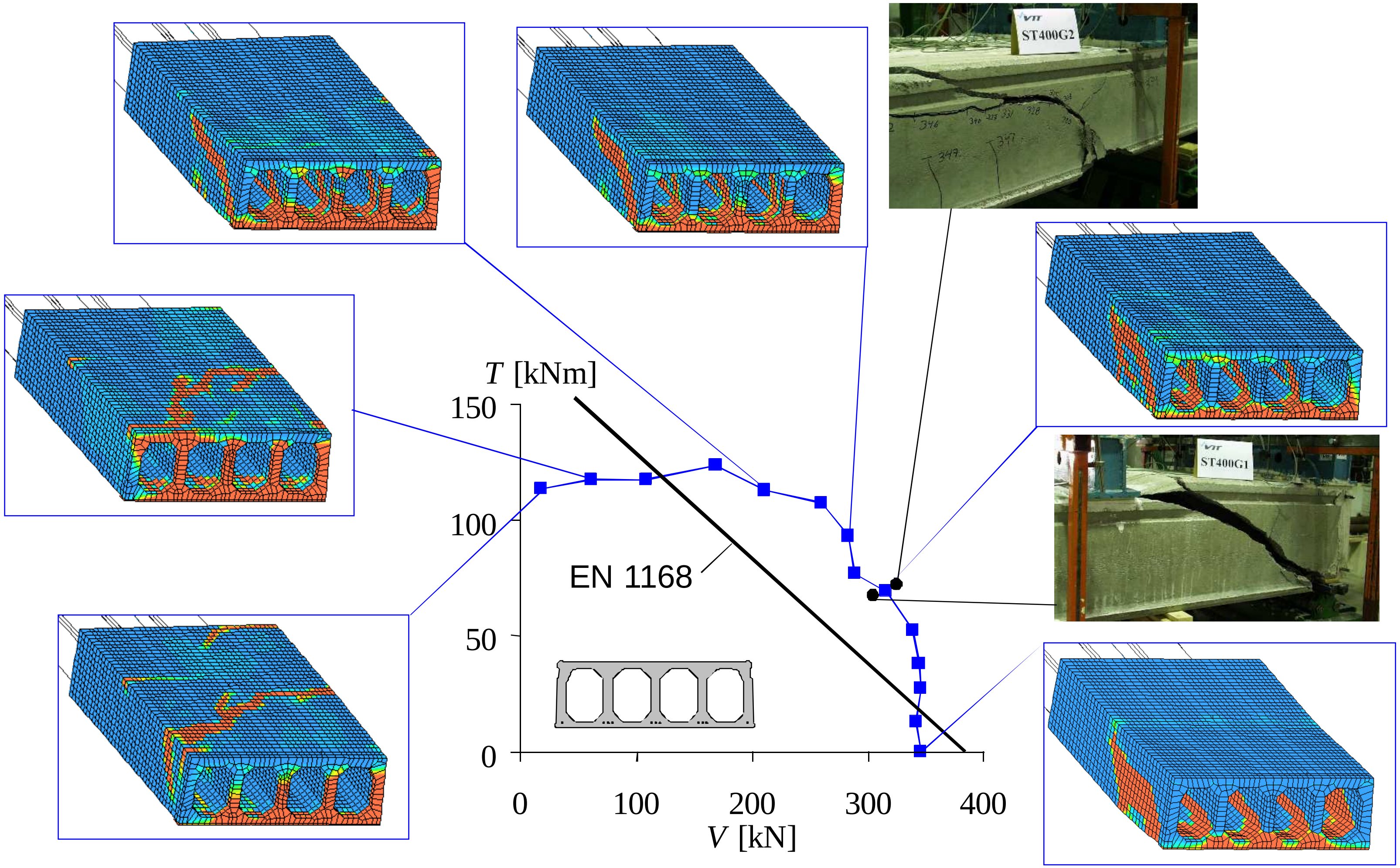
$$\delta_y^B = \delta_y^{B'}$$

$$\delta_y^C = \delta_y^{C'}$$

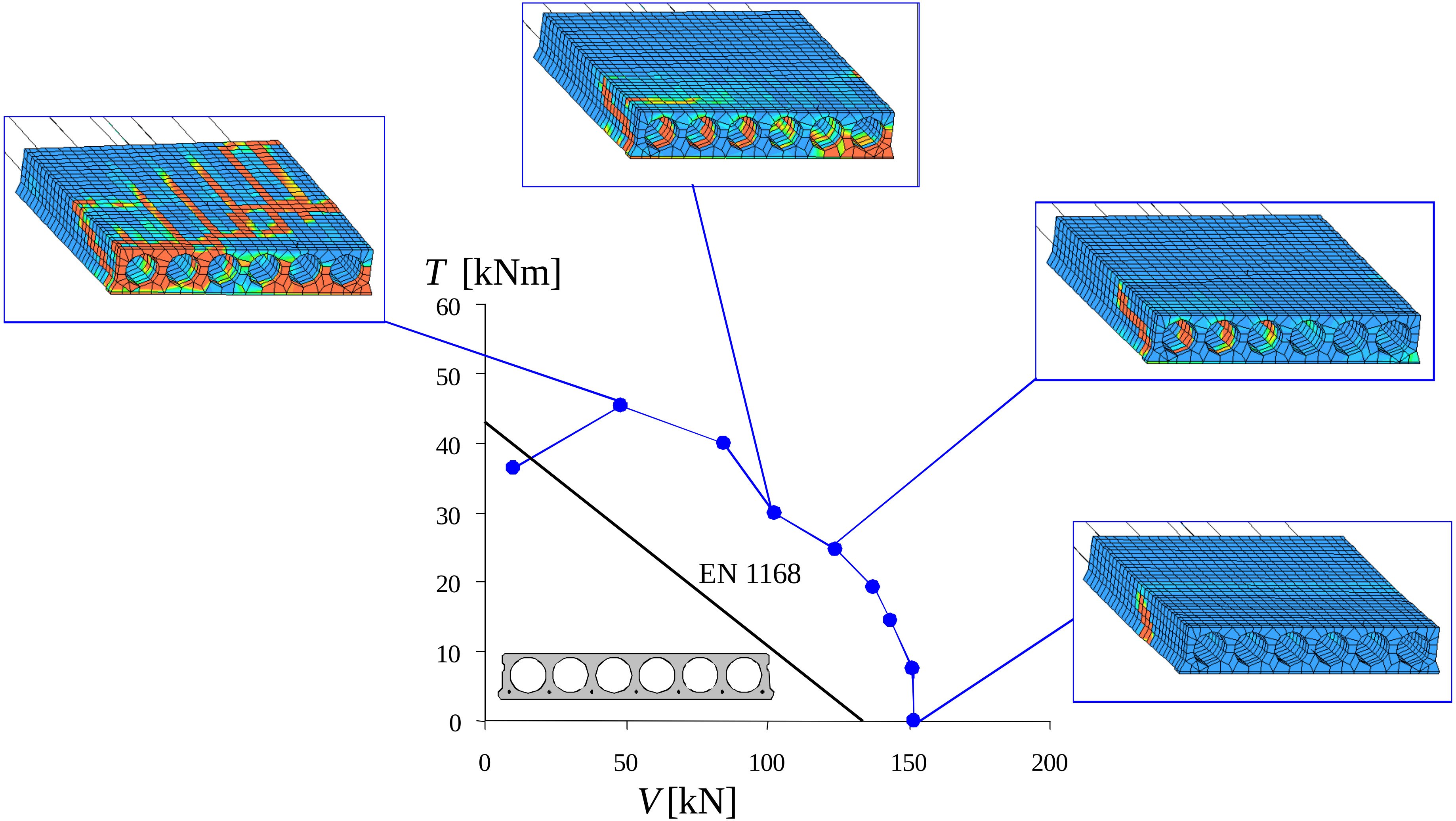
Shear torsion interaction 400 mm



Shear torsion interaction 400 mm



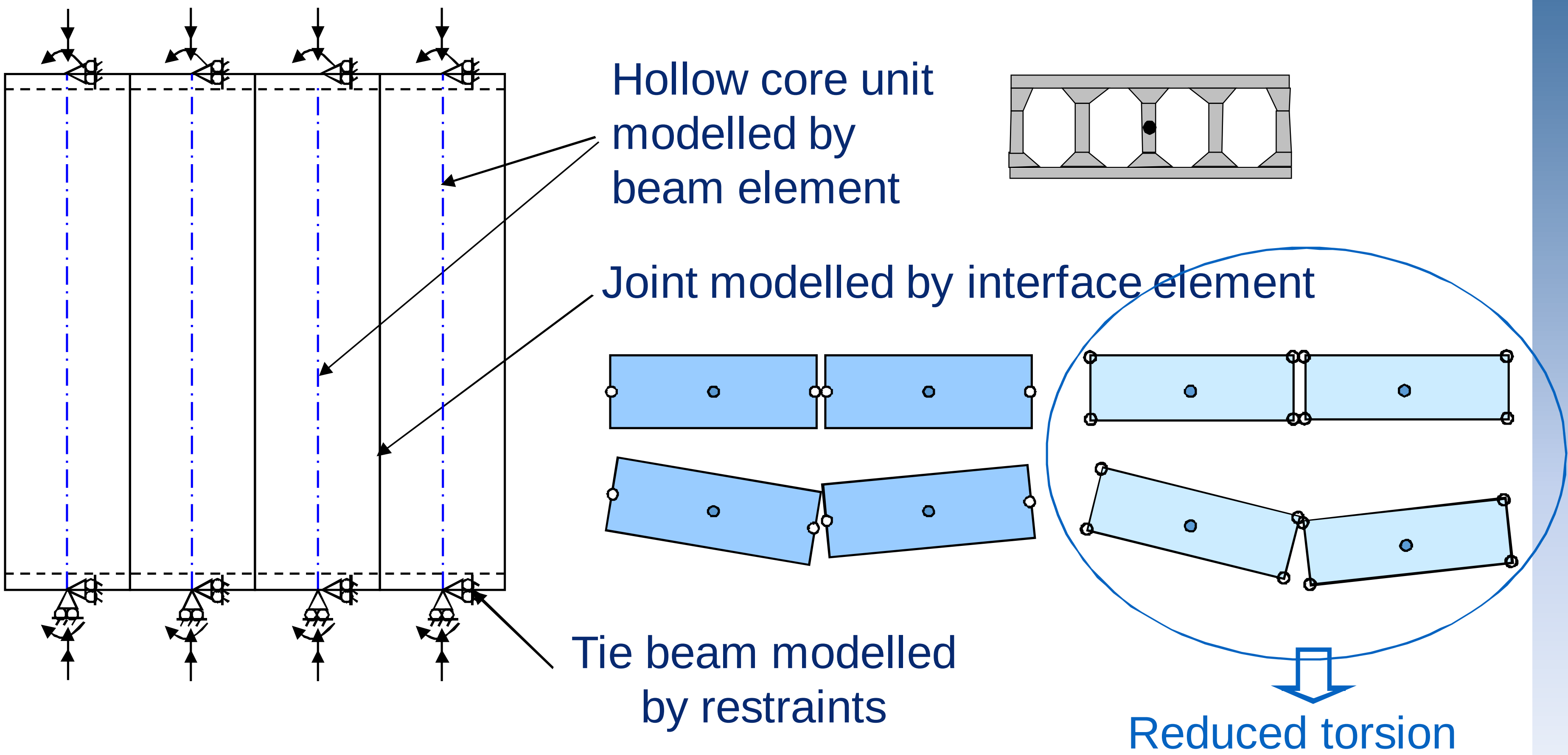
Shear tension interaction 200 mm



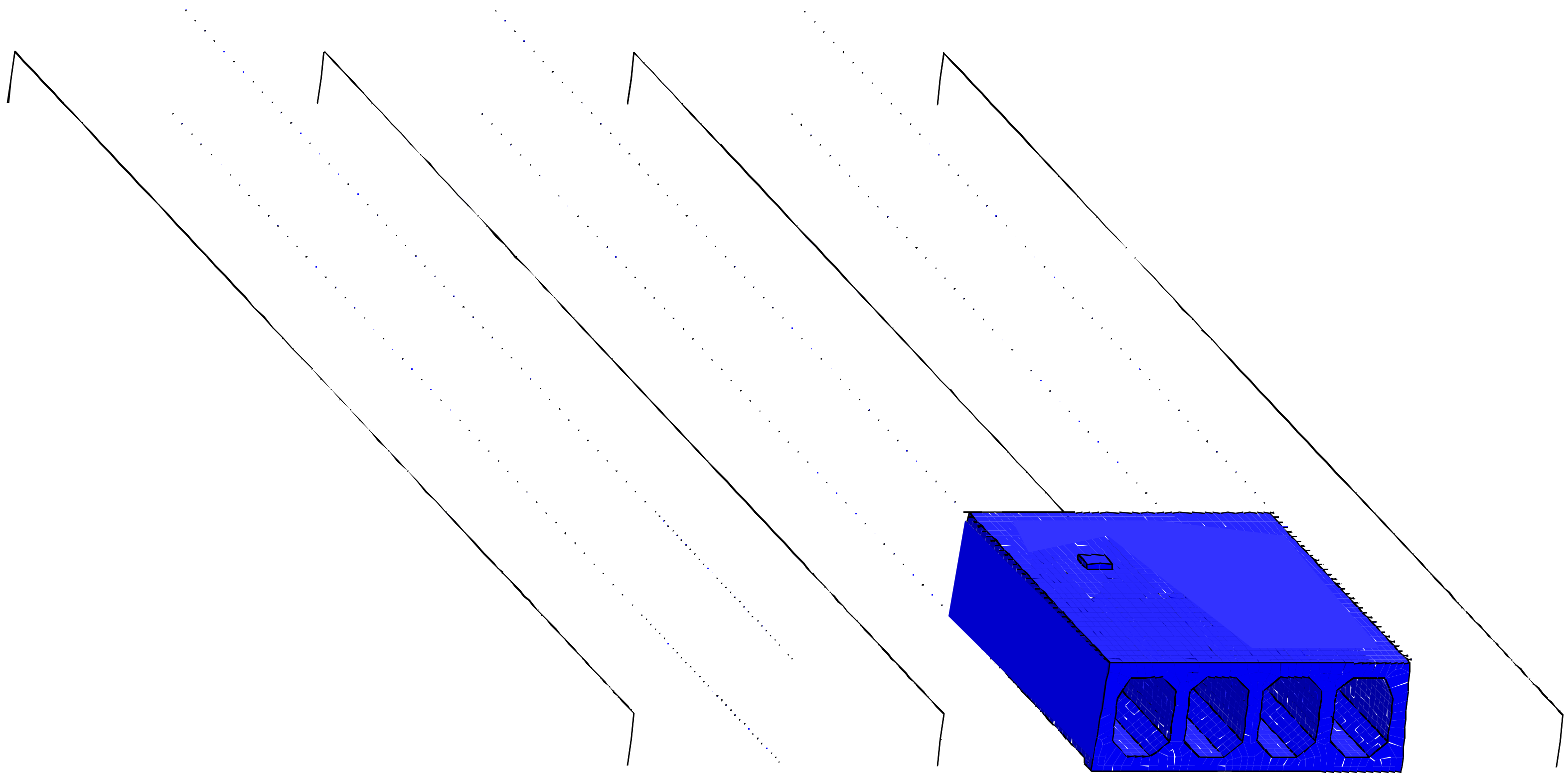
Failure modes

- Torsion dominates
  - Diagonal crack in top flange
- Vertical shear dominates
  - Web shear crack in most loaded web and bending crack
- Intermediate situations
  - Mixed mode

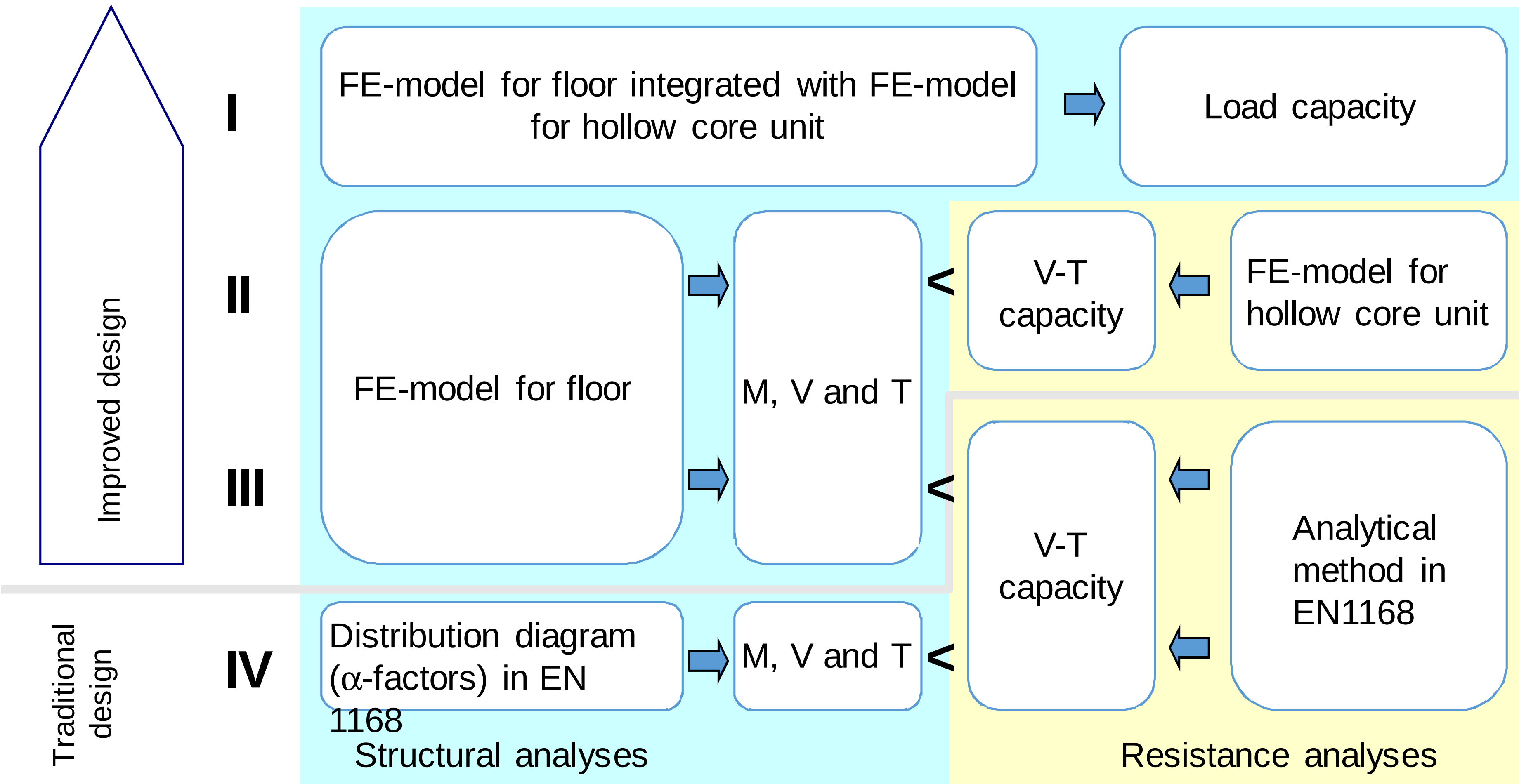
FE-model of hollow core floor



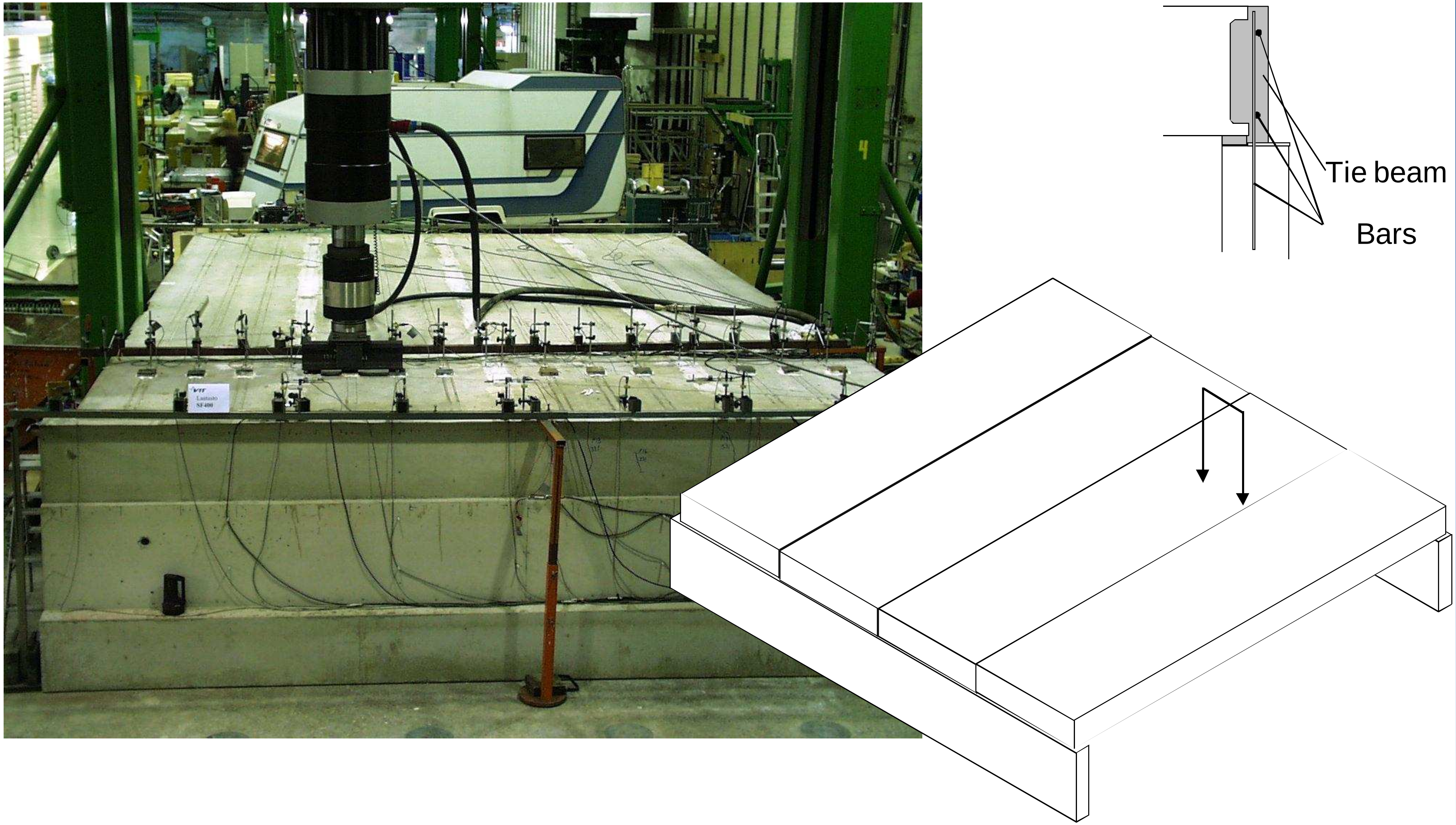
Integrated model for complete floor



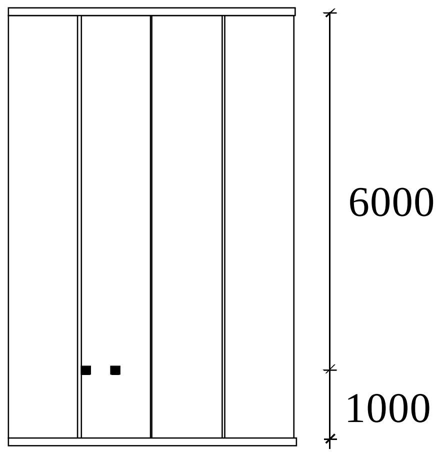
Design of hollow core slabs

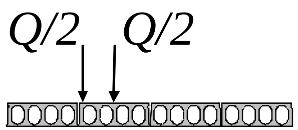


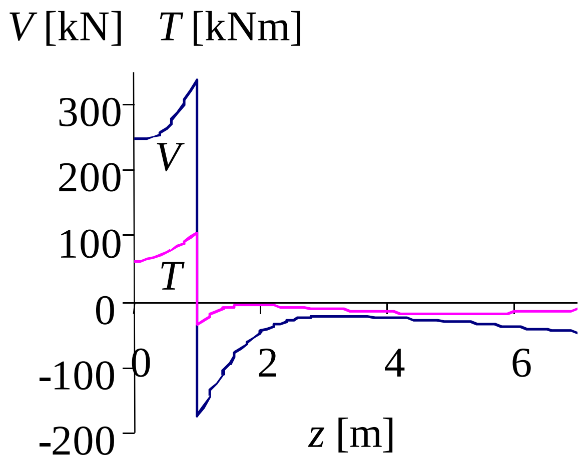
Test on complete floor

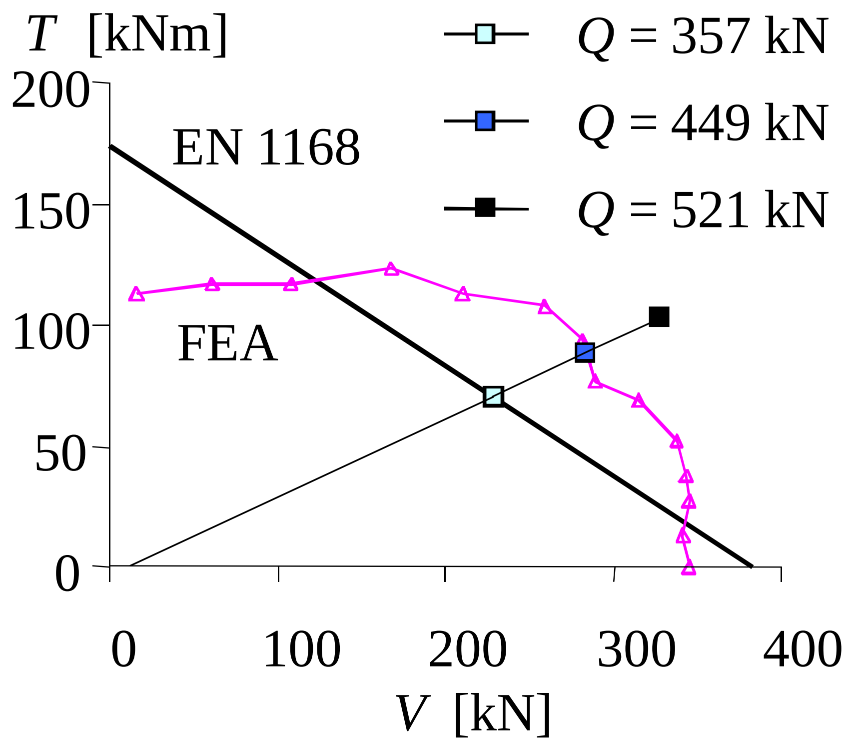


Design of floor – level III and II

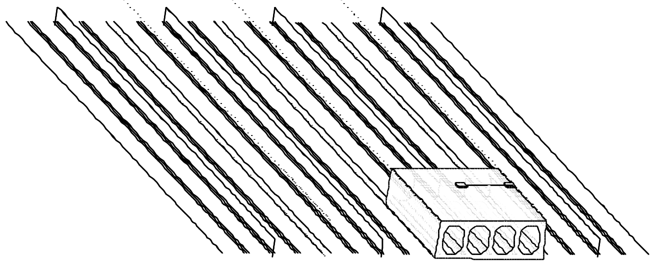


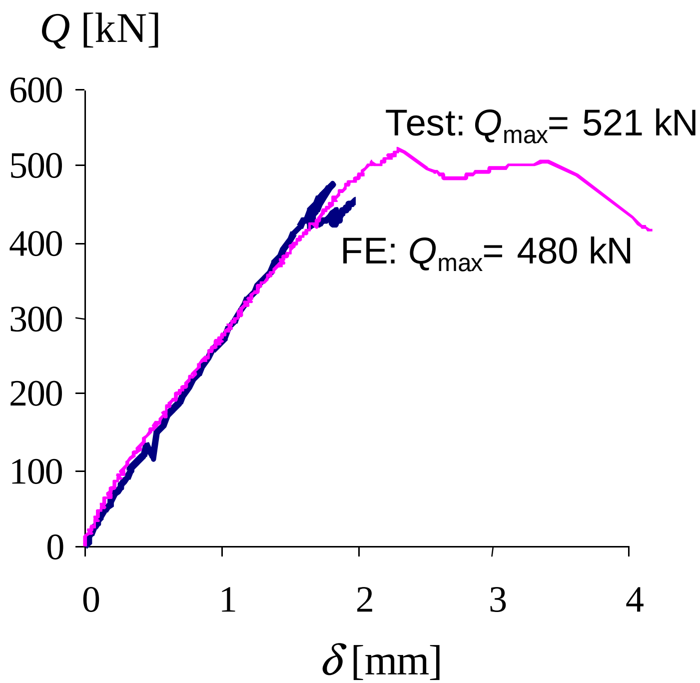


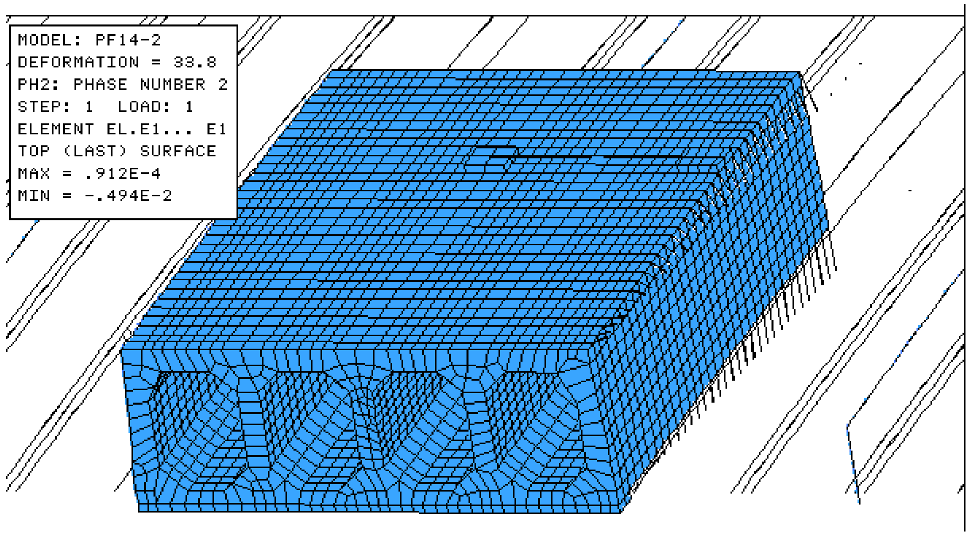


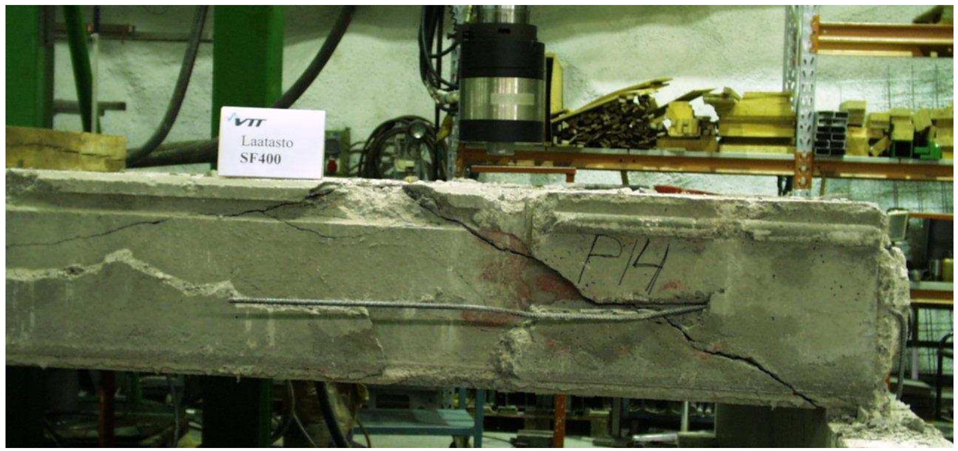


Simulation of floor test – level I

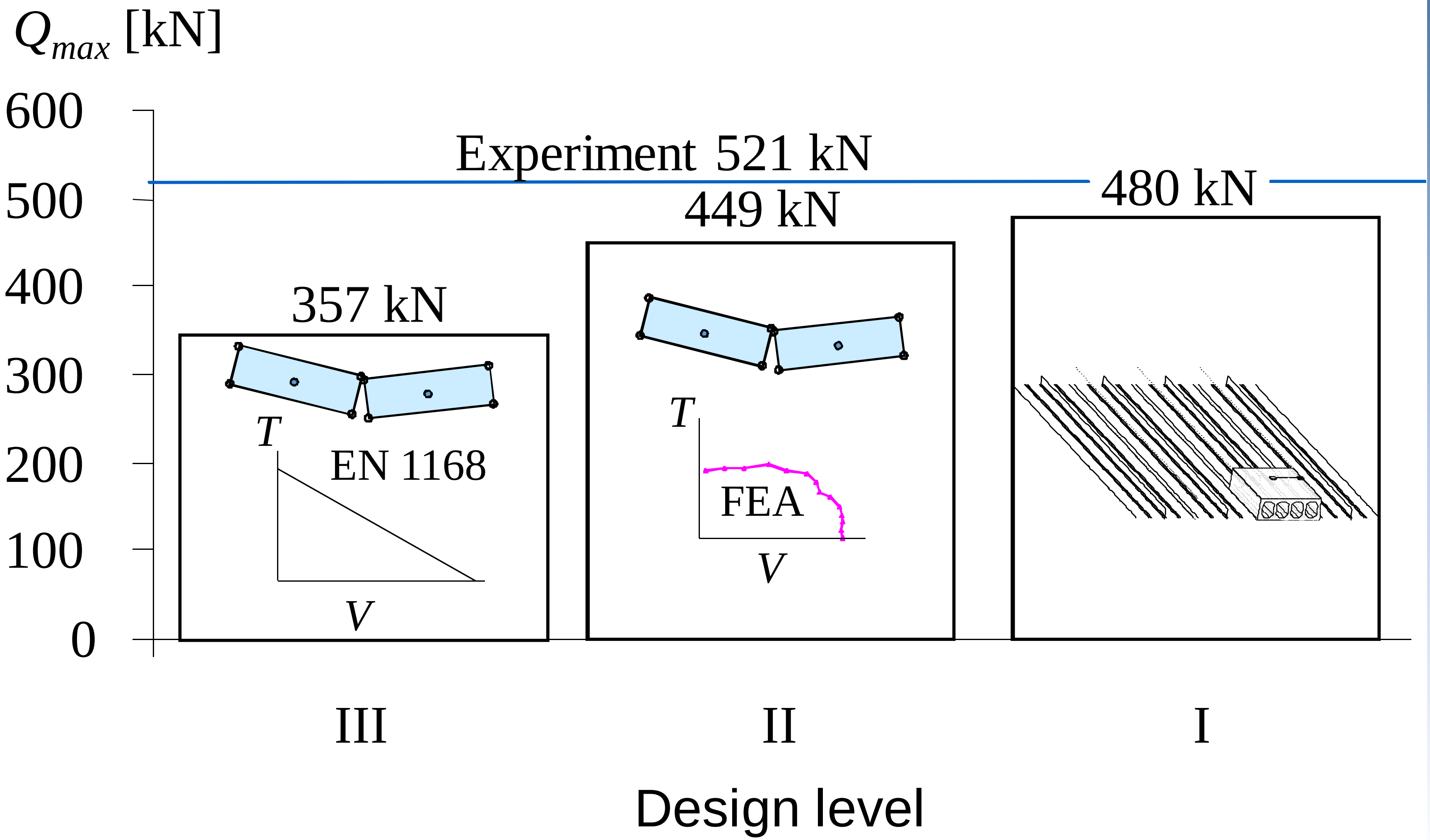








Floor design, example



Conclusions

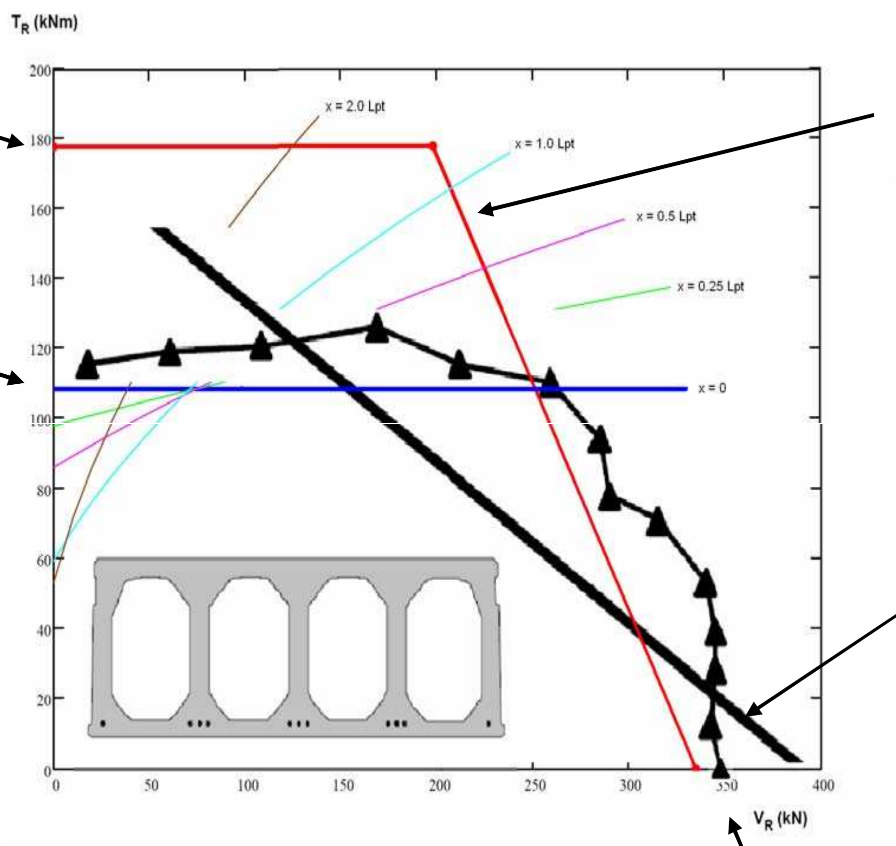
- Modelling levels for hollow core slabs were developed
  - Hollow core floor $\Rightarrow$ Sectional forces M, V, T
    - *Reduced torsional moment*
    - *Arbitrary geometry and loading*
  - Hollow core unit $\Rightarrow$ Shear-torsion capacity
    - Higher resistance
- The capacity of hollow core units can be used better

Development of level IV method

Failure in outer web

Failure in top flange

Mixed mode shear failure

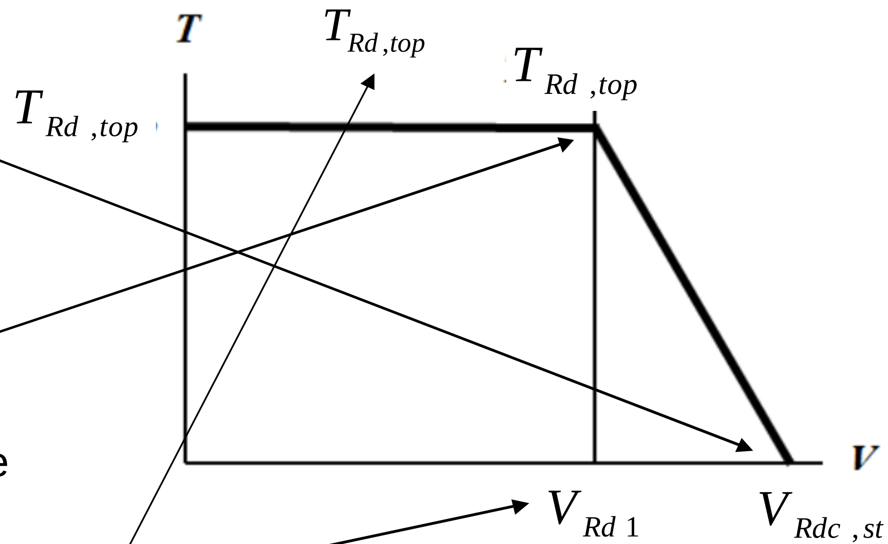


Capacity according to EN 1168

Pure shear failure in web (web shear tension)

Simplified interaction diagram

1) Select section
2) Resistance to web shear tension failure in outer web (pure shear failure) $V_{Rdc,st}$
3) Torsional capacity due to cracking in outer web
   The vertical shear is carried by the internal web only (full redistribution)
4) Resistance to web shear tension failure in internal web
   To be combined with 3)
5) Check torsional capacity due to cracking in top flange

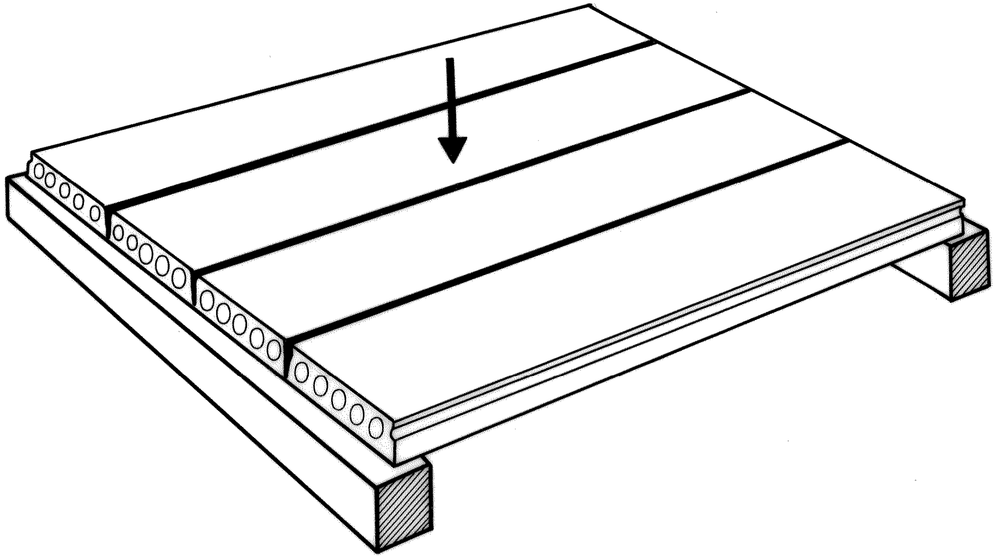


$$T_{Rd,web} = W_{T,web} \cdot \tau_V(z_0)$$

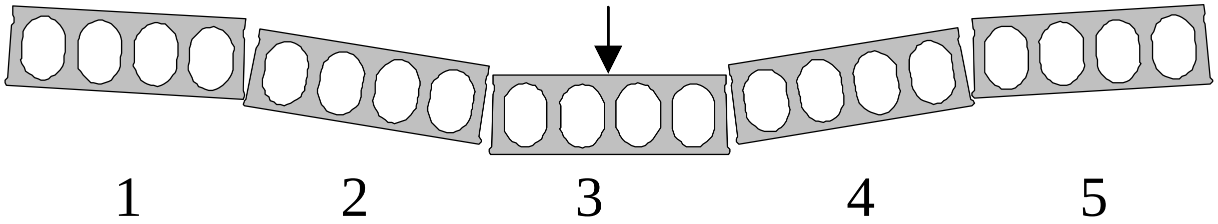
$$\tau_V(z_0) = \frac{V_{Rd1} \cdot S_{cp}(z_0)}{I \cdot b_{w,inner}(z_0)}$$

$$T_{Rd,top} = W_{T,top} \cdot \tau_T$$

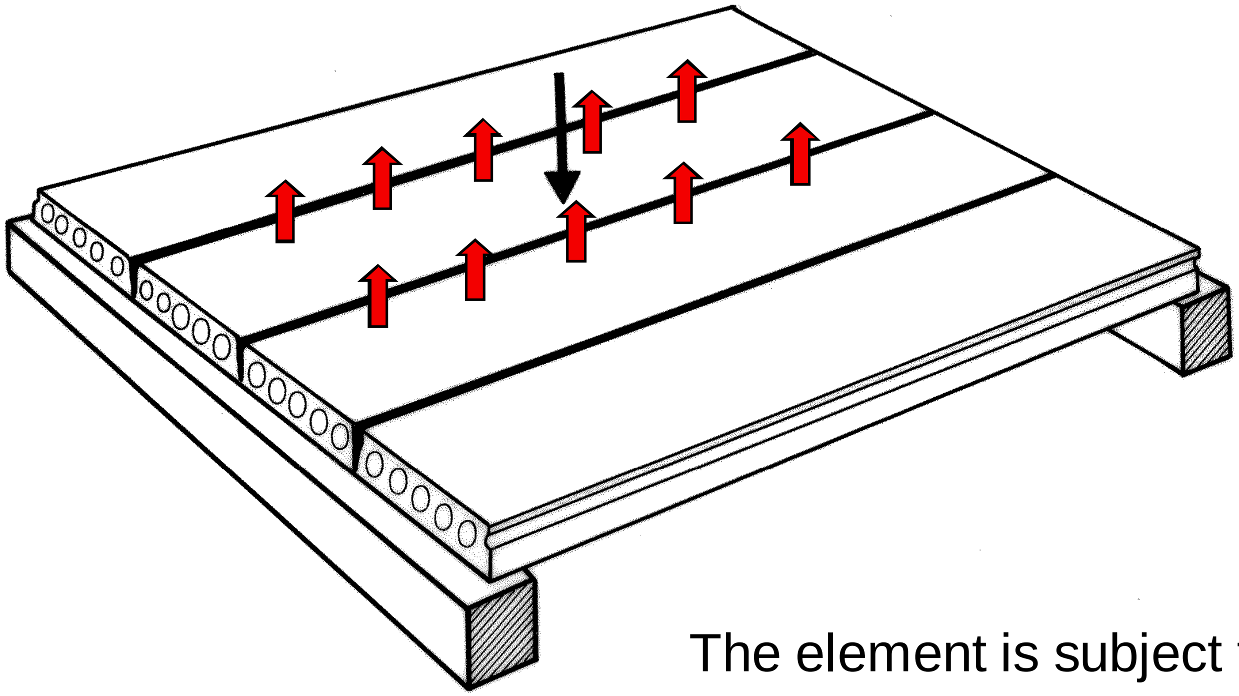
Transverse distribution of load effects



- Transfer of vertical shear
- Lateral restraint

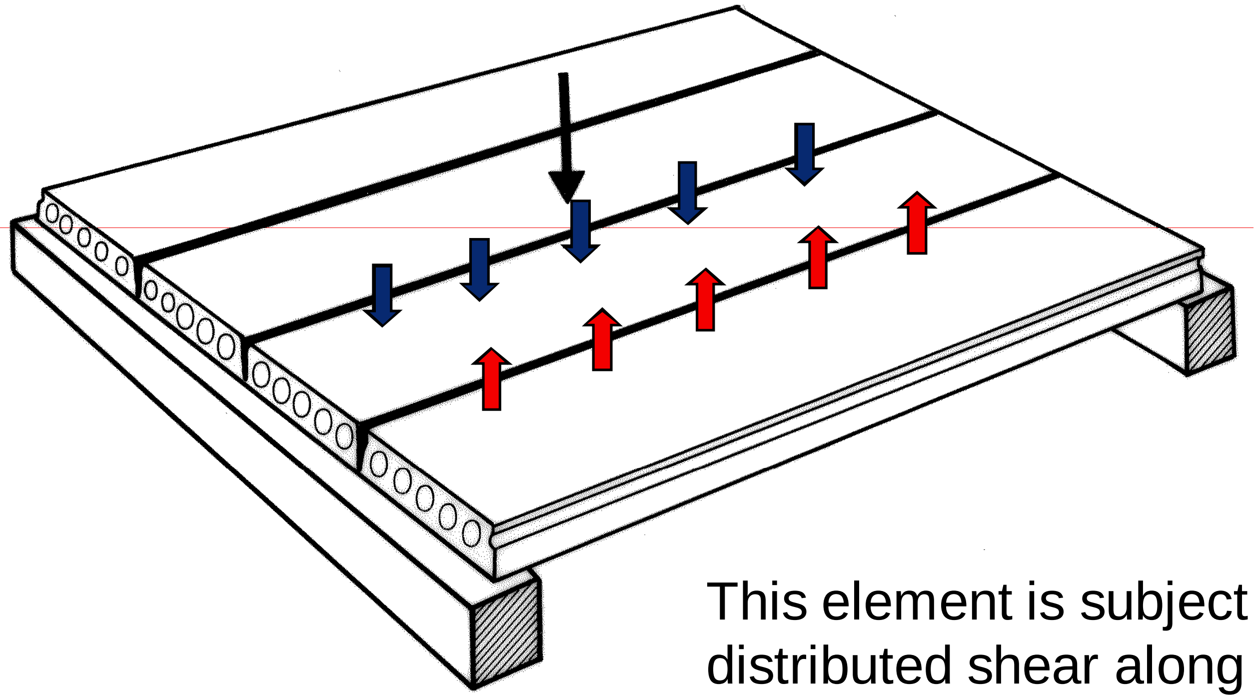


Forces on the loaded element



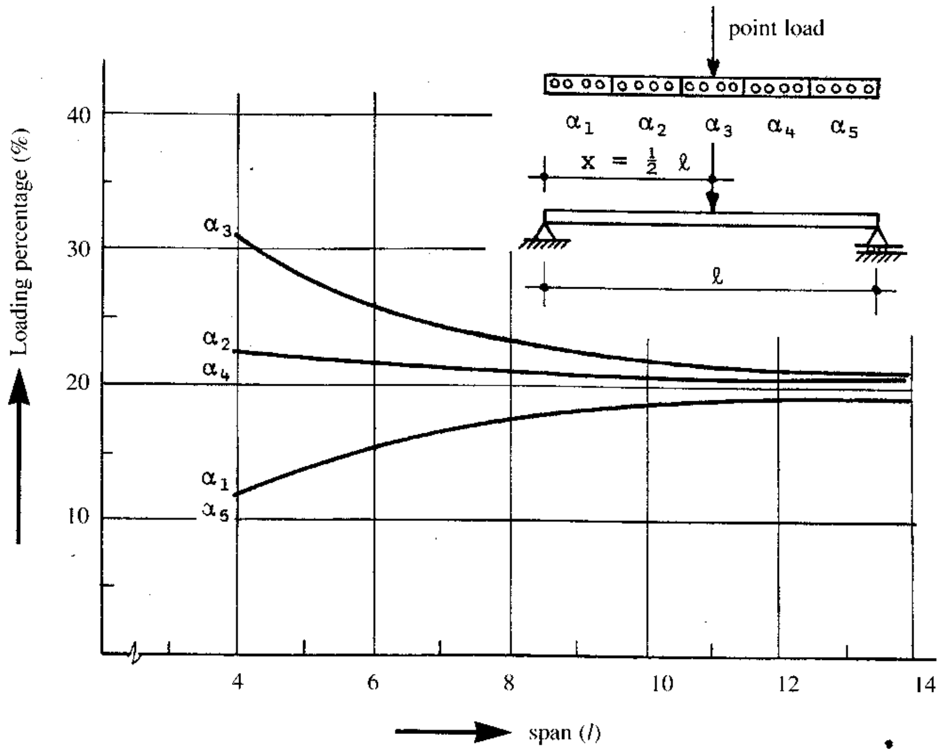
The element is subject to a concentrated force and distributed shear along the edges

Forces on the adjacent element



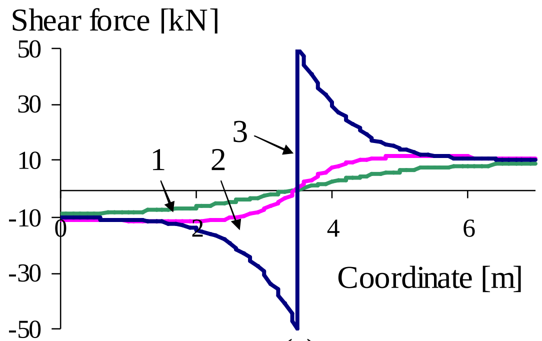
This element is subject to a distributed shear along one edge downwards, and upwards shear at the other edge – means torsion

Load distribution factors

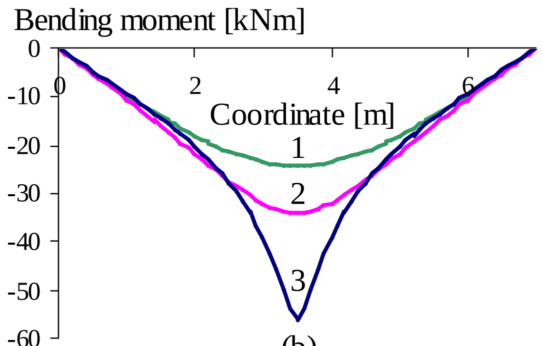


Note! It is not the load that is distributed, but the load effect.
Different factors for bending moment, shear and torsion.

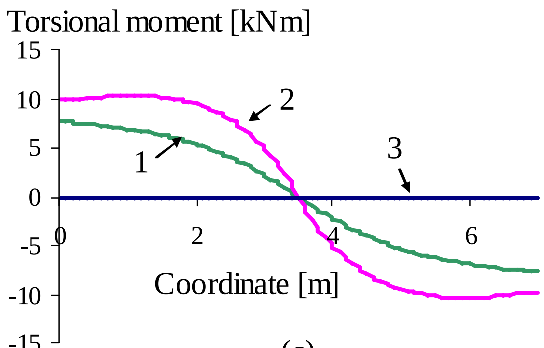
Distribution of shear, bending moment and torsion



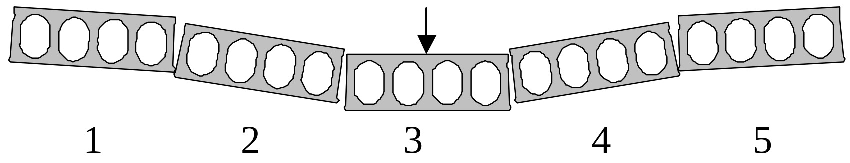
(a)



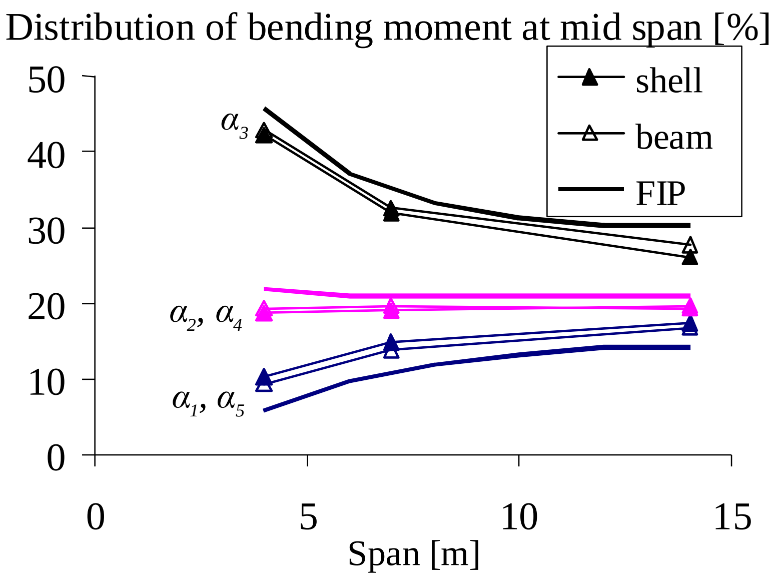
(b)



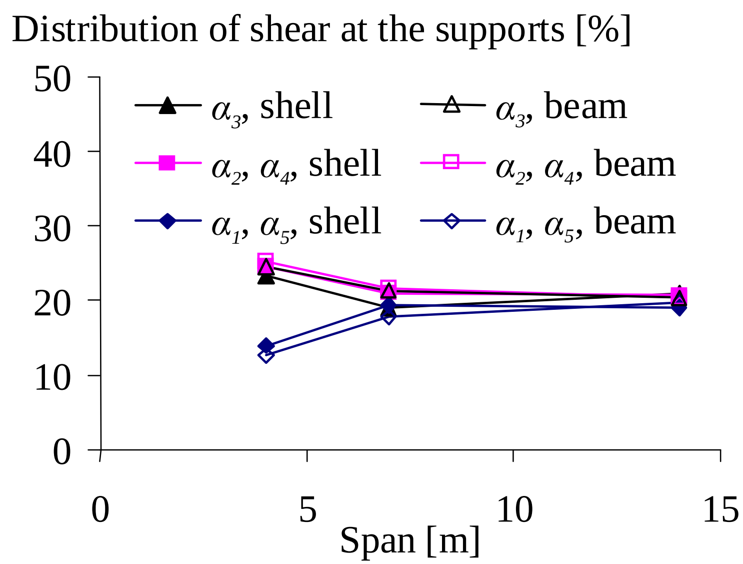
(c)



Distribution of maximum moment and maximum shear



(a)



(b)