IPHA TECHNICAL SEMINAR 2017

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Shear and anchorage resistance

Shear and bending

Ronald Klein-Holte

Consolis VBI | The Netherlands

Organized by

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INTERNATIONAL PRESTRESSED HOLLOWCORE ASSOCIATION

in cooperation with

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Design Course
Hollow core slab and Floor design

Shear and anchorage resistance

INTERNATIONAL PRESTRESSED HOLLOWCORE ASSOCIATION

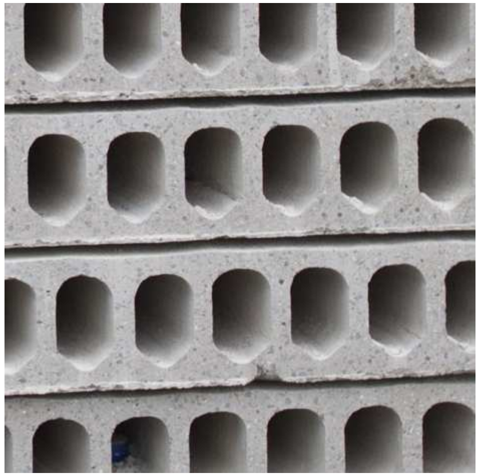
Ronald KLEIN-HOLTE

Hollow core slab

CONSOLIS VBI

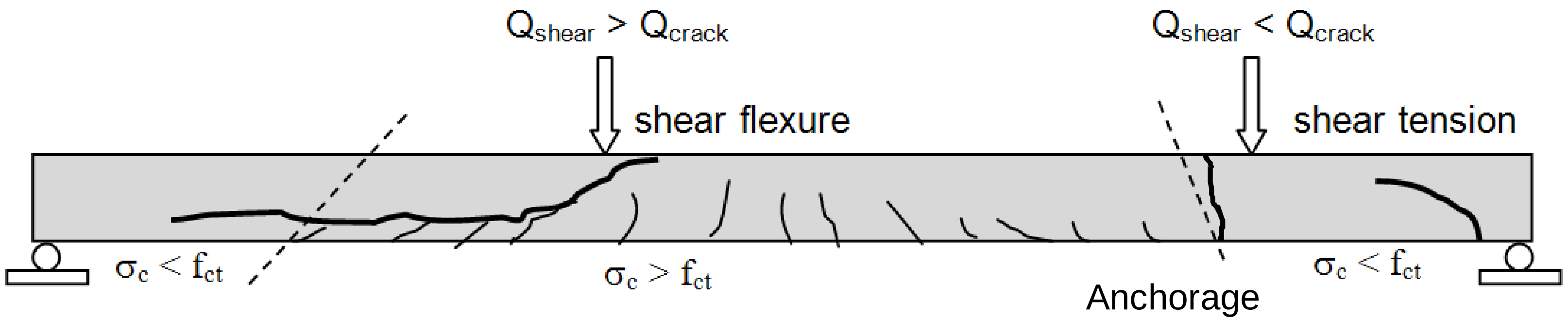
Hollow core slab

- cores in longitudinal direction:
  - to reduce use material and
  - to reduce self weight and
  - to increase the effect of pre-stressing;
- one way pre-stressed element;
- no transverse reinforcement;
- no spalling reinforcement,
- mostly produced on beds with extruder or slipform technology.
- Widely used over the world. In Europe there are about 1000 million m^2 hollow core floors applied.

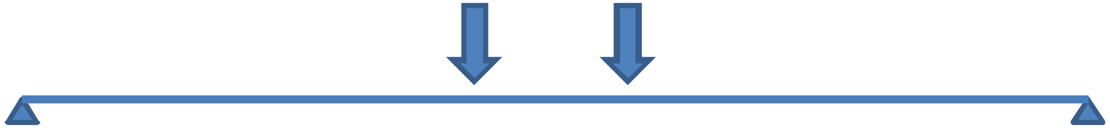


Mechanical behaviour

Failure modes



Shear flexural resistance



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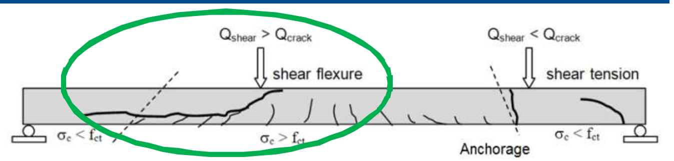
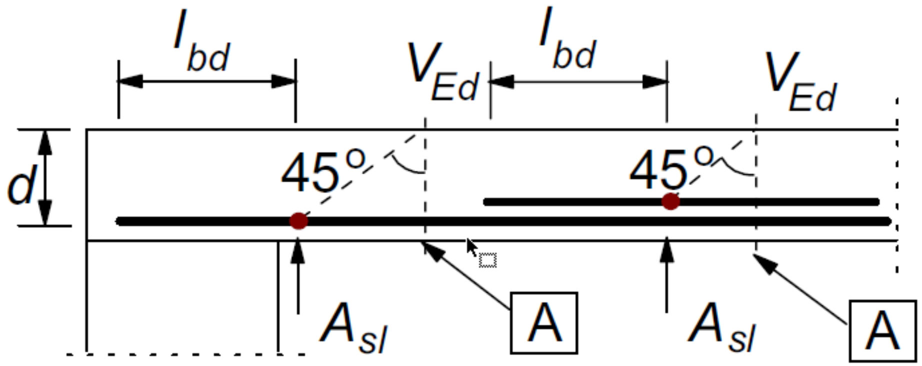


$$V_{Rd,c} = [C_{Rd,c}k(100\ \rho_l f_{ck})^{1/3} + k_1\ \sigma_{cp}]\ b_w d$$

with a minimum of

$$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\ b_w d$$

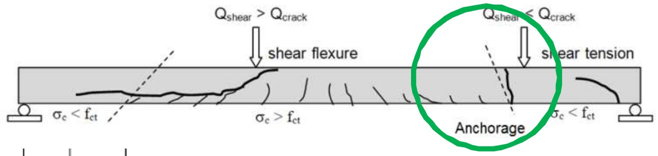


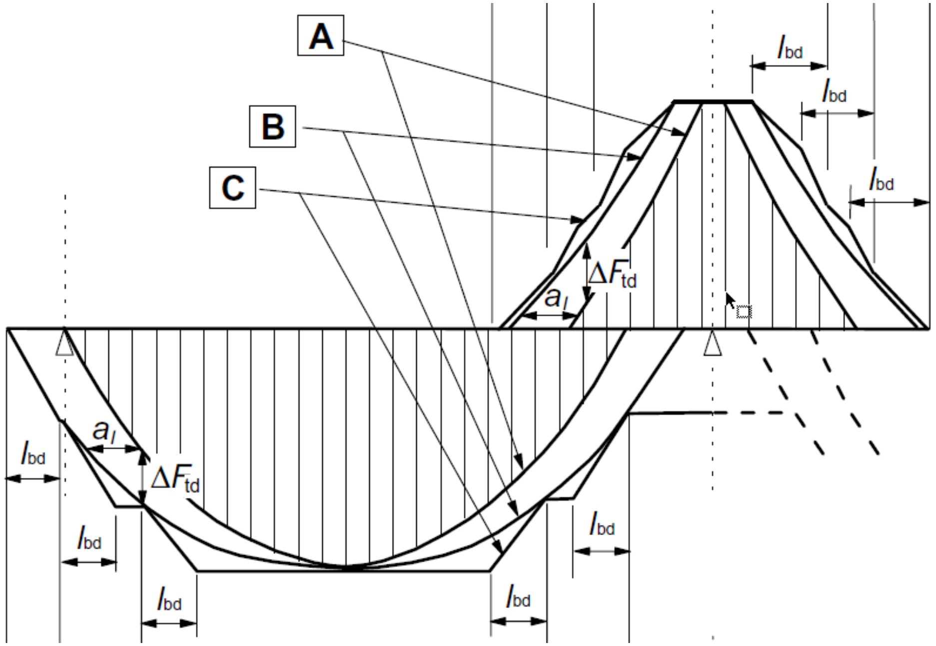
Mechanical behaviour

Anchorage failure

Design following EC2

Anchorage failure





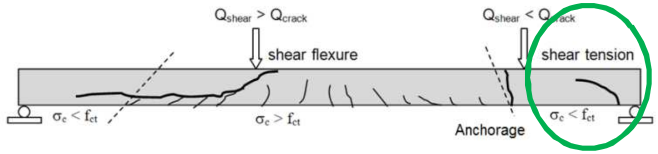
For members with shear reinforcement the additional tensile force, ΔF_{td}, should be calculated according to 6.2.3 (7). For members without shear reinforcement ΔF_{td} may be estimated by shifting the moment curve a distance $a_l = d$ according to 6.2.2 (5). This "shift rule" may also be used as an alternative for members with shear reinforcement.

Mechanical behaviour

Shear tension resistance

Design following EC2

Shear tension resistance



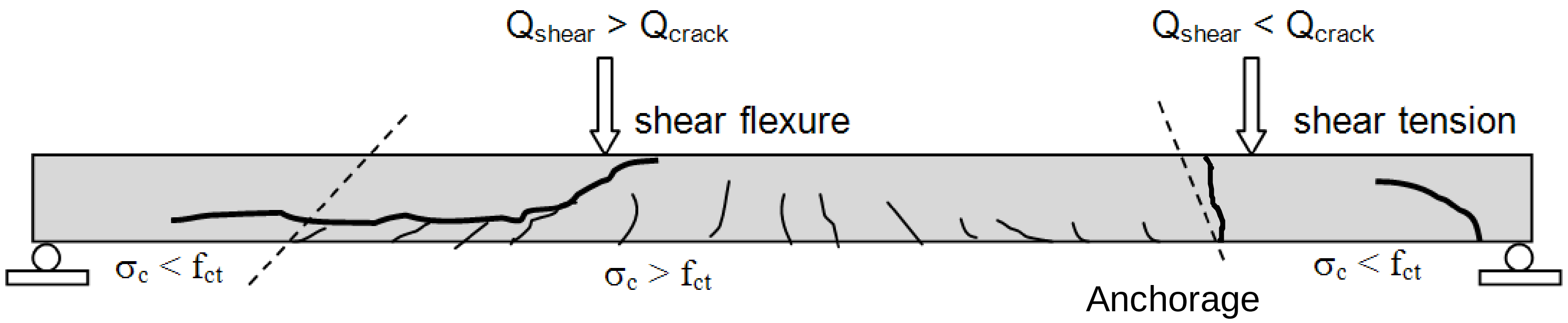
$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \sigma_{cp} f_{ctd}}$$

In regions uncracked in bending (where the flexural tensile stress is smaller than $f_{ctk,0,05}/\gamma_c$) the shear resistance should be limited by the tensile strength of the concrete.

For cross-sections where the width varies over the height, the maximum principal stress may occur on an axis other than the centroidal axis. In such a case the minimum value of the shear resistance should be found by calculating $V_{Rd,c}$ at various axes in the cross-section.

Mechanical behaviour

Failure modes

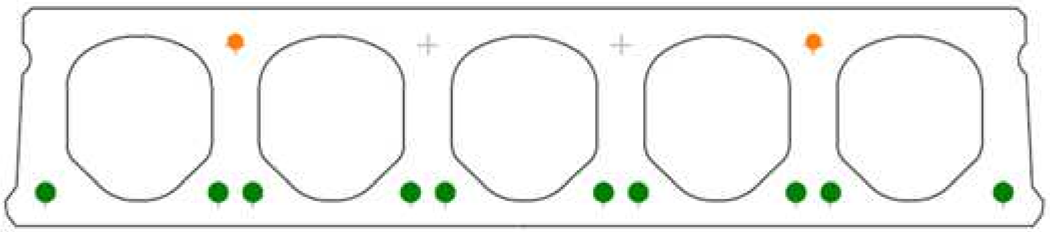


- ❑ Shear flexure failure;
- ❑ Anchorage failure;
- ❑ Shear tension failure;

We go more into detail ……

Shear flexural resistance

Example calculation



Slab depth:		$h_{c1} = 265$	mm	
Selfweight:		$p_{g1} = 3.60$	kN/m²	
Thickness of the top flange:		$h_{sl.tf} = 34$	mm	
Thickness of the bottom flange:		$h_{sl.uf} = 31$	mm	
Perimeter of the cores:		$u_{cores} = 3007$	mm	
Concrete strength:	C45/55	$f_{ck1} = 45$	MPa	char cylinderstrength
Concrete strength at time of release:		$f_{cmp} = 28$	MPa	mean cylinderstrength

Shear flexural resistance

Prestressing pattern: **10 Strands**

tendon	number n_p	diameter $Ø_k$	area A_p	distance Y_p	prestress σ_p	strength $f_{p0.1k}$	strength f_{pk}	strain ε_{uk}
strand	2	9.3	52	224	700	1500	1770	0.035
strand	10	12.5	93	41	900	1500	1770	0.035

Compressive stress in the concrete due to prestressing: $\sigma_{cpt} = 4.0$ MPa

Tendon stresses in N/mm^2:

	σ_{pm0}	σ_{pmt}
2 strand Ø9.3	694	483
10 strand Ø12.5	841	660

Shear flexural resistance

Eurocode 2 6.2.2 (1):

$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l\,f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$ (6.2.a)

with a minimum of

$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\,b_w d$ (6.2.b)

Concrete strength at the compressed side $f_{ck} = 45$ MPa

Parameter according to National Annex: $k_1 = 0.15$

Effective depth: $d = h_{c1} - Y_p$ $d = 224.0$ mm

$k = \min\left(1 + \sqrt{\frac{200}{d}}, 2\right)$ $k = 1.94$

Shear flexural resistance

Eurocode 2 6.2.2 (1):

$$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d \qquad (6.2.a)$$

with a minimum of

$$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\,b_w d \qquad (6.2.b)$$

The recommended value for V_{min} is: $C_{v.min} = 0.035$

$V_{min} = (\,C_{v.min}.k^{3/2} \cdot f_{ck}{}^{1/2})$ $\quad V_{min} = 0.64$

The smallest width in the tensile area: $b_{min} = 324$ mm

The design value for the shear capacity according to formula (6.2b):

$V_{Rdc.62b} = [(V_{min} + k_1 \cdot \sigma_{cpt}) \cdot b_{min} \cdot \mathrm{d}\,] \cdot 10^{-3}$ $\quad V_{rdc.62b} = 89.34$ kN

Shear flexural resistance

Eurocode 2 6.2.2 (1):

$$V_{Rd,c} = [C_{Rd,c} k(100\,\rho_1 f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d \qquad (6.2.a)$$

with a minimum of

$$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\,b_w d \qquad (6.2.b)$$

The recommended value for $C_{Rd.c}$ is: $C_{R.c} = 0.18$

$$C_{Rd.c} = \frac{C_{R.c}}{\gamma_c} = 0.12$$

Only the reinforcement on half the height at the tensile side will be taken into account:

10 strands Ø12.5: $A_{s1} = 930\ \mathrm{mm}^2$

Reinforcement degree: $\rho_1 = \min\left(0.02\,, \dfrac{A_{sl}}{b_{min}\cdot d}\right)$ $\qquad \rho_1 = 0.0128$

Concrete strength at the compressed side $\qquad f_{ck} = 45\ \mathrm{MPa}$

Shear flexural resistance

Eurocode 2 6.2.2 (1):

$V_{Rd,c} = [C_{Rd,c}k(100\,\rho_l f_{ck})^{1/3} + k_1\,\sigma_{cp}]\,b_w d$ (6.2.a)

with a minimum of

$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})\,b_w d$ (6.2.b)

The design value for the shear capacity: (6.2a)

$$V_{Rdc.62a} = [\ [C_{Rd.c}\cdot k\cdot(100\cdot\rho_1\cdot f_{ck})^{1/3} + k_1\sigma_{cpt}\ ]\cdot\frac{b_{min}\cdot d}{1000}\ ]$$

$$V_{Rdc.62a} = [\ [0.12\cdot 1.94\cdot(100\cdot 0.0128\cdot 45)^{1/3} + 0.15\cdot 4.0\ ]\cdot\frac{324\cdot 224}{1000}\ ]$$

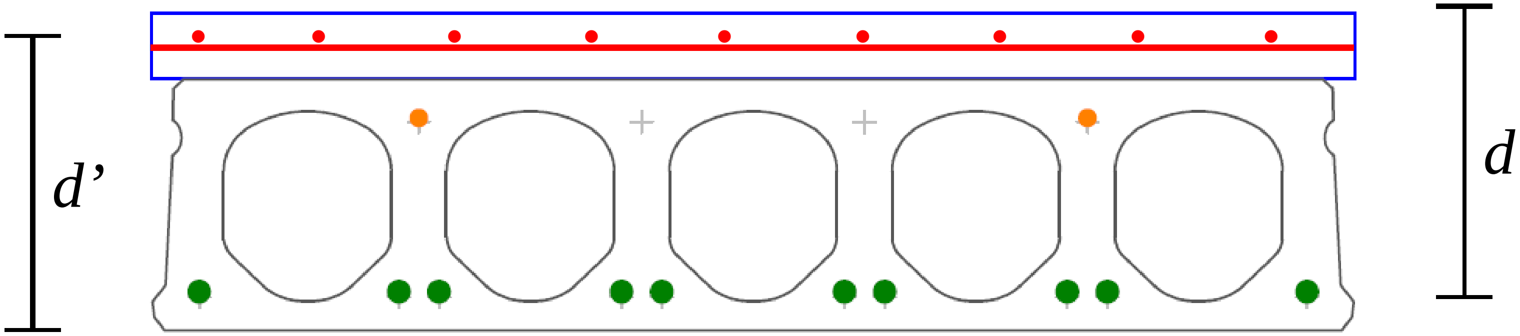
$$V_{rdc.62a} = 108.57\text{ kN}$$

The design value for the shear resistance is maximum value according to formula 6.2a and 6.2b:

$V_{Rdc.62} = \max(\ V_{Rdc.62a}\ ,\ V_{Rdc.62b}\ ) = \ \max(\ 108.57\ ,\ 89.34\ ) =$ **108.57 kN**

Shear flexural resistance

Structural topping



- Increase the effective depth;
- Check the shear stress at the interface between slab and topping
- Be aware of the compression side and its concrete strength;
- Be aware of the reinforcement at the tensile zone.

Design following EC2

8.10.2.3 Anchorage of tensile force for the ultimate limit state

(1) The anchorage of tendons should be checked in sections where the concrete tensile stress exceeds $f_{ctk,0,05}$. The tendon force should be calculated for a cracked section, including the effect of shear according to 6.2.3 (6); see also 9.2.1.3. Where the concrete tensile stress is less than $f_{ctk,0,05}$, no anchorage check is necessary.

(2) The bond strength for anchorage in the ultimate limit state is:

$$f_{bpd} = \eta_{p2}\, \eta_1\, f_{ctd} \tag{8.20}$$

where:

η_{p2} is a coefficient that takes into account the type of tendon and the bond situation at anchorage

η_{p2} = 1,4 for indented wires or

η_{p2} = 1,2 for 7-wire strands

η_1 is as defined in 8.10.2.2 (1)

Note : Values of η_{p2} for types of tendons other than those given above may be used subject to a European Technical Approval.

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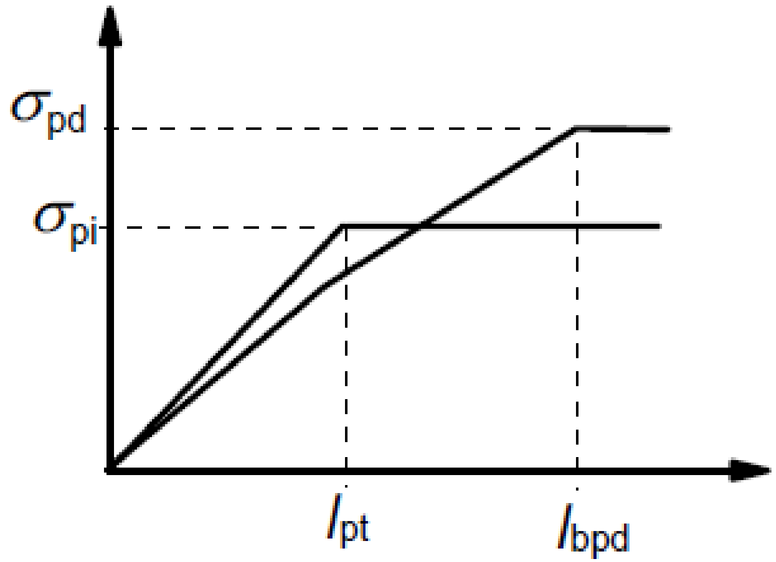
Anchorage failure

Design following EC2



(4) The total anchorage length for anchoring a tendon with stress σ_{pd} is:

$$l_{bpd} = l_{pt2} + \alpha_2 \phi (\sigma_{pd} - \sigma_{pm\infty})/f_{bpd}$$

where

l_{pt2} is the upper design value of transmission length, see 8.10.2.2 (3)

α_2 as defined in 8.10.2.2 (2)

σ_{pd} is the tendon stress corresponding to the force described in (1)

$\sigma_{pm\infty}$ is the prestress after all losses

Anchorage failure

Coefficient for strands: α_2 = 0.19

Good bond conditions: η_1 = 1.0

Coefficient for the bond situation for strands: η_{p2} = 1.2

The bond strength for anchorage in the ultimate limit state is:

$$f_{bpd} = \eta_{p2} \cdot \eta_1 \cdot f_{ctd}$$

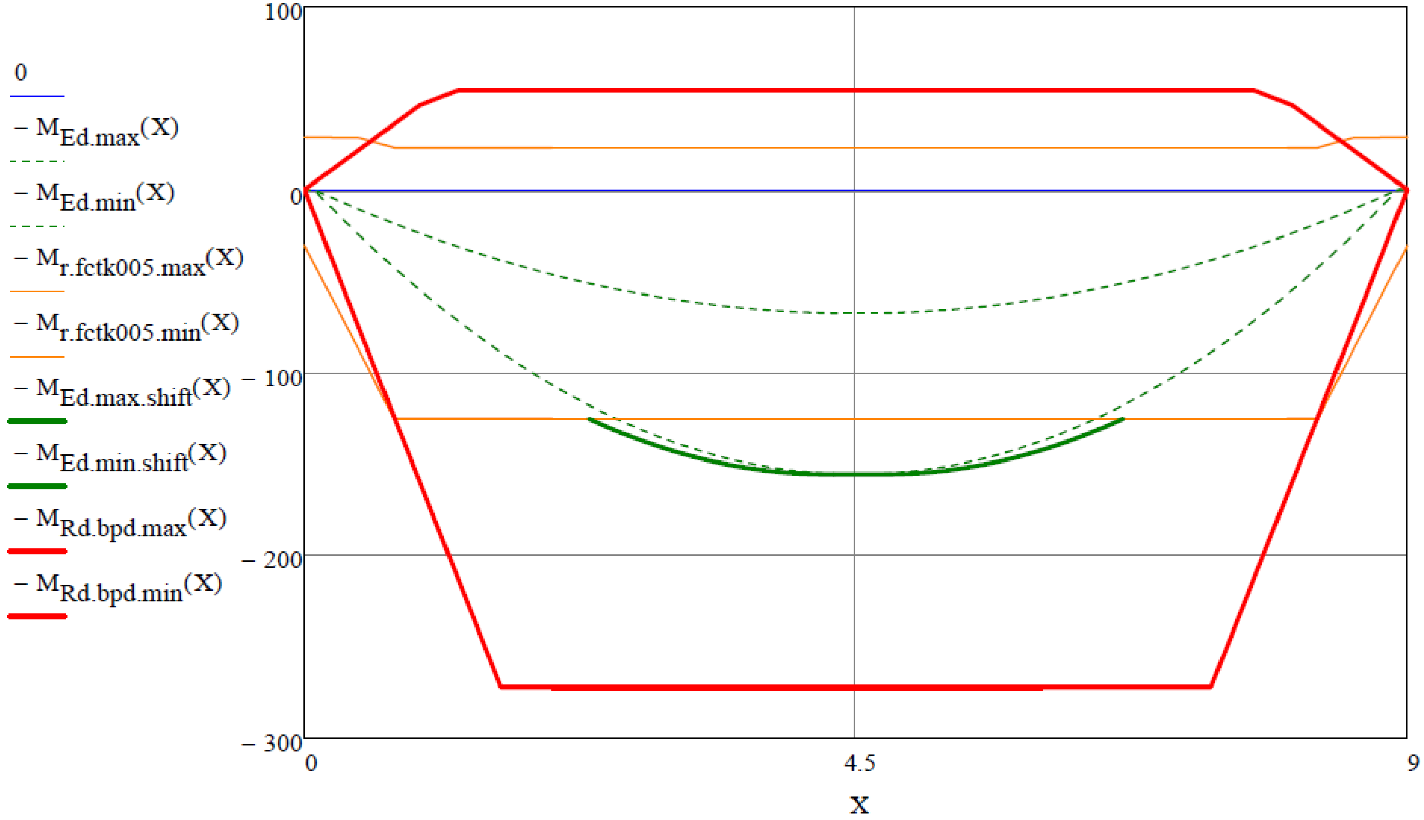
$$f_{bpd} = 1.2 \cdot 1.0 \cdot 1.77 = 2.1 \quad N/mm^2$$

$$F_{bpd} = A_p \cdot \frac{f_{bpd}}{\alpha_2 \cdot \text{Ø}}$$

For strands Ø9.3 the ULS bond is: $F_{bpd} = 2 \cdot 52 \cdot \frac{2.1}{0.19 \cdot 9.3} = 123\ N/mm$

For strands Ø12.5 the ULS bond is: $F_{bpd} = 10 \cdot 93 \cdot \frac{2.1}{0.19 \cdot 12.5} = 822\ N/mm$

Anchorage failure



Requirement(Anchorage) = "is fulfilled"

Shear tension resistance

Principal tensile stress = Concrete tensile strength

$$\sigma_I = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \qquad \sigma_I = f_{ct}$$

- In the web of a pre-stressed hollow core slab, the stress state is essentially two-dimensional.
- The compressive principal stress in the web is so small, that its effect on the transverse tensile strength is small.

Shear stress due to the load resp. the shear force:

$$\tau = V \cdot \frac{S}{b \cdot I}$$

Shear tension resistance

some mathematics

$$f_{ct} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\left(f_{ct} - \frac{\sigma}{2}\right)^2 = \left(\frac{\sigma}{2}\right)^2 + \left(V \cdot \frac{S}{b \cdot I}\right)^2$$

$$f_{ct}{}^2 - 2 \cdot \frac{\sigma}{2} \cdot f_{ct} + \left(\frac{\sigma}{2}\right)^2 - \left(\frac{\sigma}{2}\right)^2 = \left(V \cdot \frac{S}{b \cdot I}\right)^2$$

$$\sqrt{f_{ct}{}^2 - f_{ct} \cdot \sigma} = V \cdot \frac{S}{b \cdot I}$$

$$\boxed{V = \frac{b \cdot I}{S} \sqrt{f_{ct}{}^2 - f_{ct} \cdot \sigma}}$$

Remark: physically the sign for the stress component is positive for tension and negative for compression.
When supposing compressive stress as positive the minus sign changes

Shear tension resistance

the stress components

Stress due to pre-stressing and due to actions:

$$\sigma = P \cdot \left(\frac{1}{A} + \frac{e \cdot z}{I} \right) + M \cdot \frac{z}{I}$$

Additional: shear stress due to transmission of the pre-stressing force (Yang)

$$\tau = \frac{1}{b_w} \cdot \left(\frac{A_{cp}}{A} - \frac{S_{cp} \cdot e}{I} \right) \cdot \frac{dP}{dx}$$

dP/dx is the gradient of the tendon forces, *e* is the eccentricity of tendon force and A_{cp} and S_{cp} the cross-sectional area and first moment of area above the considered axis, respectively.

Shear tension resistance

extended formula in product standard EN 1168

$$V_{\mathrm{Rdc}} = \frac{I b_{\mathrm{w}}(y)}{S_{\mathrm{c}}(y)} \left(\sqrt{(f_{\mathrm{ctd}})^2 + \sigma_{\mathrm{cp}}(y) f_{\mathrm{ctd}}} - \tau_{\mathrm{cp}}(y) \right)$$

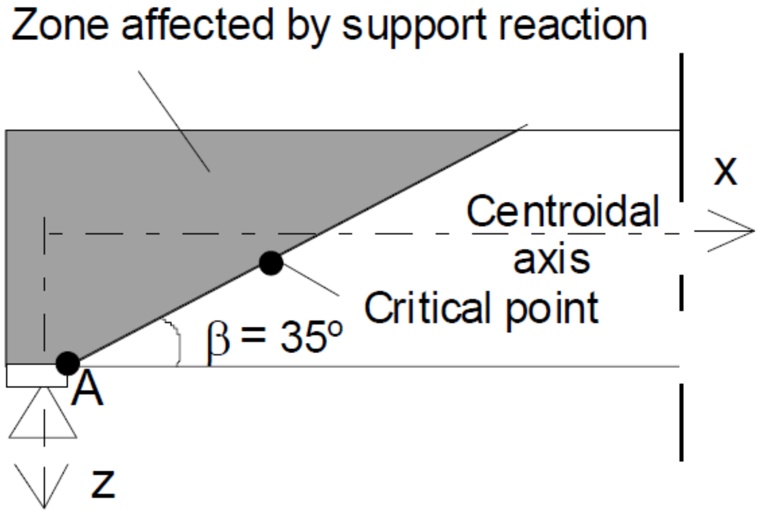
where

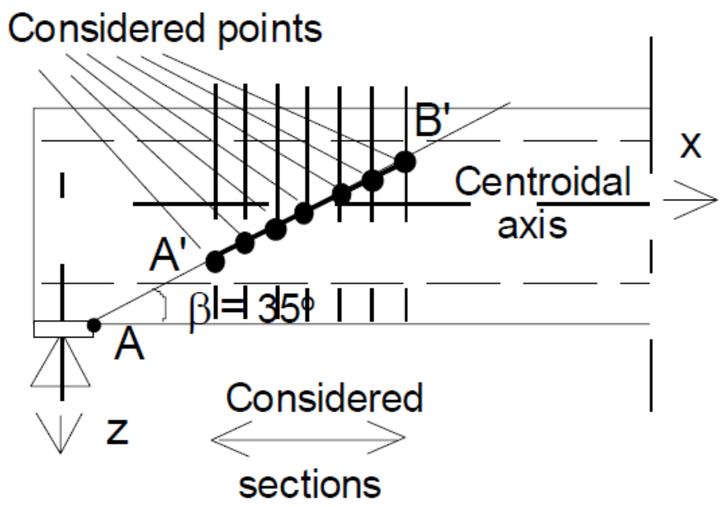
$$\sigma_{\mathrm{cp}}(y) = \sum_{t=1}^{\mathrm{n}} \left\{ \left[\frac{1}{A} + \frac{(Y_{\mathrm{c}} - y)(Y_{\mathrm{c}} - Yp_{\mathrm{t}})}{I} \right] \times P_{\mathrm{t}}(l_{\mathrm{x}}) \right\} - \frac{M_{\mathrm{Ed}}}{I} \times (Y_{\mathrm{c}} - y)$$ (positive if compressive)

$$\tau_{\mathrm{cp}}(y) = \frac{1}{b_{\mathrm{w}}(y)} \times \sum_{t=1}^{\mathrm{n}} \left\{ \left[\frac{A_{\mathrm{c}}(y)}{A} - \frac{S_{\mathrm{c}}(y) \times (Y_{\mathrm{c}} - Yp_{\mathrm{t}})}{I} + Cp_{\mathrm{t}}(y) \right] \times \frac{dP_{\mathrm{t}}(l_{\mathrm{x}})}{dx} \right\}$$

Shear tension resistance

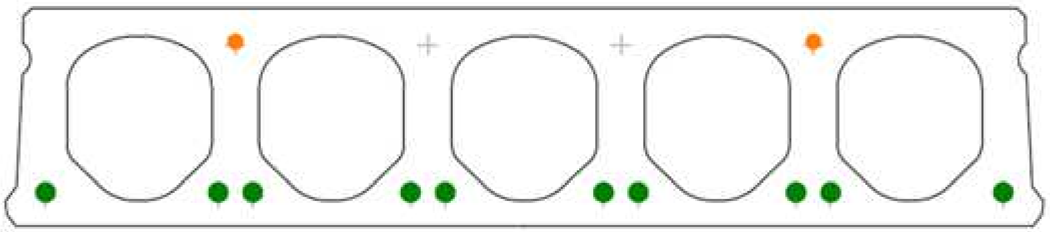
extended formula in product standard EN 1168





Shear tension resistance

Example calculation



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Shear tension resistance

Prestressing pattern: **10 Strands**

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Compressive stress in the concrete due to prestressing: σ_{cpt} = 4.0 MPa

Tendon stresses in N/mm^2:

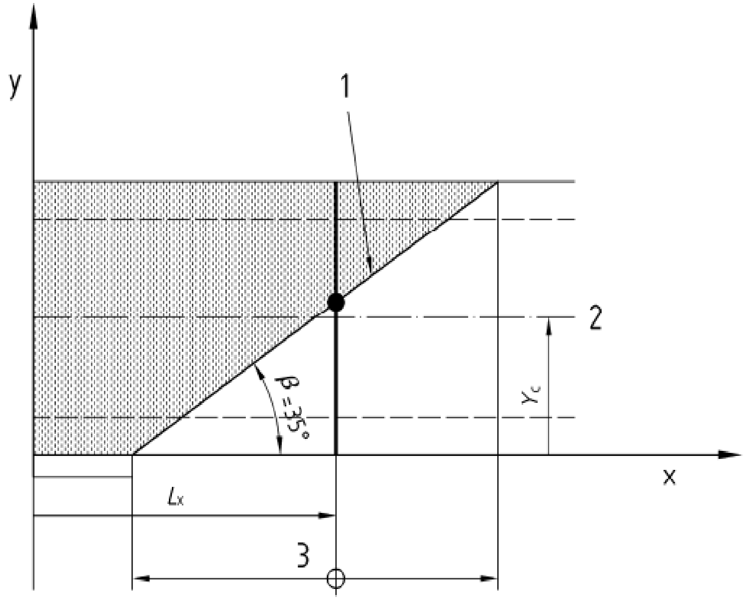
	σ_{pm0}	σ_{pmt}
2 strand Ø9.3	694	483
10 strand Ø12.5	841	660

Shear tension resistance

General data:

Support length:	$a =$	100	mm
Area of the concrete section	$A =$	172989	mm^2
2^{nd} moment of area:	$I_{ci} =$	$1481.7 \cdot 10^6$	mm^4
Height of the centroidal axis:	$y_{ci} =$	128.1	mm
The design value of the concrete tensile strength:	$f_{ctd} =$	1.77	N/mm^2
Design value of the acting shear force:	$V_{Ed} =$	135.5	kN

Shear tension resistance



This example calculation is given for
the critical point at: y = 96 mm

The distance of the considered point on the line of failure
from the starting point of the transmission length:

$$L_x = a + y / \tan(35°) = 100 + 96 / 0.7 \qquad L_x = 237 \text{ mm}$$

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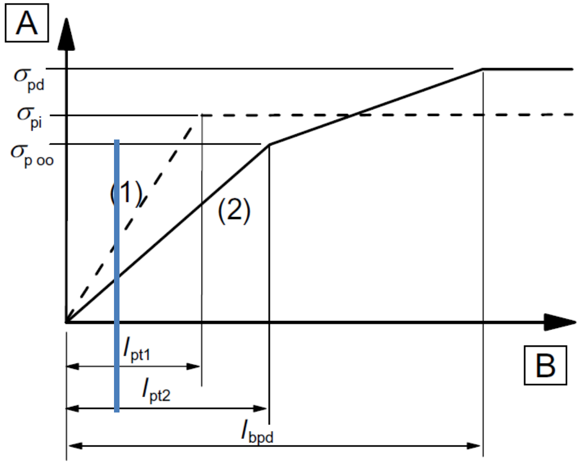
Shear tension resistance



The basic value of the transmission length:

$$l_{pt} = \frac{\alpha_1 \cdot \alpha_2 \cdot \phi_k \cdot \sigma_{pm0}}{f_{bpt}}$$

The basic transmission length for strands Ø 9.3: $l_{pt.Ø9.3}$ = 371 mm

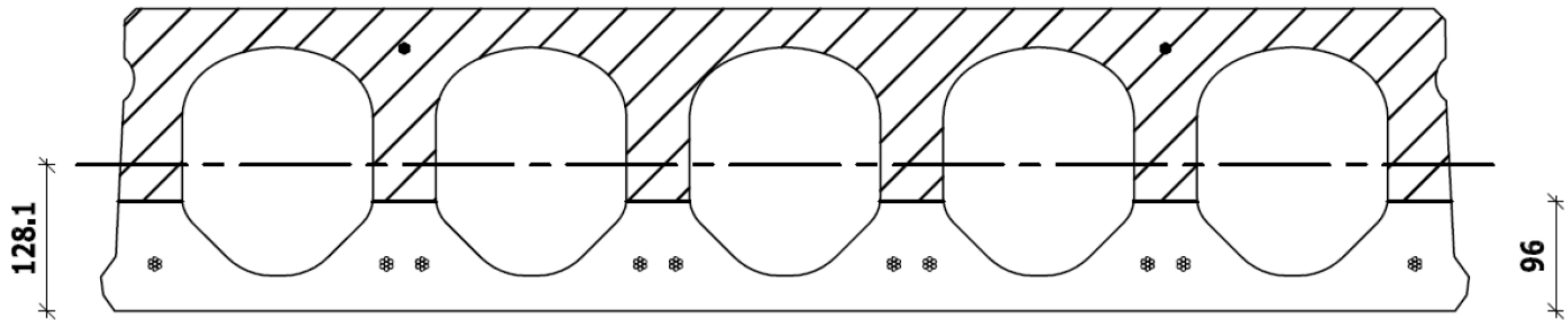
The basic transmission length for strands Ø12.5: $l_{pt.Ø12.5}$ = 605 mm

The design transmission length for strands Ø 9.3: $l_{pt2.Ø9.3}$ = 445 mm

The design transmission length for strands Ø12.5: $l_{pt2.Ø12.5}$ = 726 mm

Shear tension resistance

<u>Statical properties of the area above height y:</u>



The total web width at the height *y*:	$b_w(y)$ =	333	mm
The area above height *y*:	$A_{ci}(y)$ =	93627	mm^2
The first moment of the area above height *y* and about the centroidal axis:	$S_{ci}(y)$ =	7187253	mm^3

Shear tension resistance

$$\sigma_{cp}(y) = \sum_{t=1}^{n} \left\{ \left[\frac{1}{A} + \frac{(Y_c - y)(Y_c - Yp_t)}{I} \right] \times P_t(l_x) \right\} - \frac{M_{Ed}}{I} \times (Y_c - y) \qquad \text{(positive if compressive)}$$

Acting bending moment in the considered section M_{Ed}:

Leverarm: $z = 50 / 2 + y / \tan(35°)$	z =	162	mm
$M_{Ed} = V_{Ed} . z = 135500 . 162$	M_{Ed} =	$21.97 \cdot 10^6$	Nmm

The prestressing force at distance L_x:

$P_{t.Lx} = \sigma_{pmt} . A_p . n_p . L_x / l_{pt}$

For the upper strands Ø 9.3:

$P_{t.Lx} = 483 . 52 . 2 . 237 / 445$	$P_{t.Lx}$ =	26753	N

For the lower strands Ø 12.5:

$P_{t.Lx} = 660 . 93 . 10 . 237 / 726$	$P_{t.Lx}$ =	200373	N

Shear tension resistance

$$\sigma_{cp}(y) = \sum_{t=1}^{n} \left\{ \left[\frac{1}{A} + \frac{(Y_c - y)(Y_c - Yp_t)}{I} \right] \times P_t(l_x) \right\} - \frac{M_{Ed}}{I} \times (Y_c - y)$$ (positive if compressive)

Stress due to the upper strands Ø 9.3:

$$\sigma_{cp}(y) = \left(\frac{1}{172989} + \frac{(128.1 - 96) \cdot (128.1 - 224)}{1481.7 \cdot 10^6} \right) \cdot 26753$$ $\sigma_{cp}(y) =$ 0.10 N/mm^2

Stress due to the lower strands Ø 12.5:

$$\sigma_{cp}(y) = \left(\frac{1}{172989} + \frac{(128.1 - 96) \cdot (128.1 - 41)}{1481.7 \cdot 10^6} \right) \cdot 200373$$ $\sigma_{cp}(y) =$ 1.54 N/mm^2

Stress due to bending moment in the considered section:

$$\sigma_{cp}(y) = -M_{Ed} \cdot \frac{y_{ci} - y}{I_{ci}}$$

$$\sigma_{cp}(y) = -21.97 \cdot 10^6 \cdot \frac{128.1 - 96}{1481.7 \cdot 10^6}$$ $\sigma_{cp}(y) =$ -0.48 N/mm^2

The total concrete compressive stress at the height y and distance L_x:

$$\sigma_{cp}(y) = 0.10 + 1.54 - 0.48$$

$\sigma_{cp}(y) =$ 1.16 N/mm^2

Shear tension resistance

$$\tau_{cp}(y)=\frac{1}{b_w(y)}\times\sum_{t=1}^{n}\left\{\left[\frac{A_c(y)}{A}-\frac{S_c(y)\times(Y_c-Yp_t)}{I}+Cp_t(y)\right]\times\frac{dP_t(l_x)}{dx}\right\}$$

Shear stress due to the upper strands Ø 9.3:

Increase of the prestressing force at distance L_x: $\frac{d}{dx}P_{t.Lx}$ = 113 N/mm

Factor for the height of the tendon: $(y \le y_p)$ C_p = -1

$$\tau_{cp}(y)=\frac{1}{333}\cdot\left(\frac{93627}{172989}-\frac{7187253\cdot(128.1-224)}{1481.7\cdot 10^6}+-1\right)\cdot 113$$

$\tau_{cp}(y)$ = 0.002 N/mm^2

Shear stress due to the lower strands Ø 12.5:

Increase of the prestressing force at distance L_x: $\frac{d}{dx}P_{t.Lx}$ = 845 N/mm

$$\tau_{cp}(y)=\frac{1}{333}\cdot\left(\frac{93627}{172989}-\frac{7187253\cdot(128.1-41)}{1481.7\cdot 10^6}+0\right)\cdot 845$$

$\tau_{cp}(y)$ = 0.30 N/mm^2

The total concrete shear stress due to transmission of prestress at height y and distance L_x: $\tau_{cp}(y) = 0.002+0.30$ $\tau_{cp}(y)$ = 0.30 N/mm^2

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Shear tension resistance

$$V_{Rdc.st}(y) = \frac{I_{ci} \cdot b_w(y)}{S_{ci}(y)} \left(\sqrt{f_{ctd}{}^2 + \sigma_{cp}(y) \cdot f_{ctd}} - \tau_{cp}(y) \right)$$

With all intermediate results complete, the shear resistance for uncracked sections for the critical point at: $L_x = 237$ mm; $y = 96$ mm; $V_{Ed} = 135.5$ kN and the higher value of the transmission length (l_{pt2}) is:

$$V_{Rdc.st}(96) = \frac{1481.7 \cdot 10^6 \cdot 333}{7187253} \left(\sqrt{1.77^2 + 1.16 \cdot 1.77} - 0.30 \right)$$

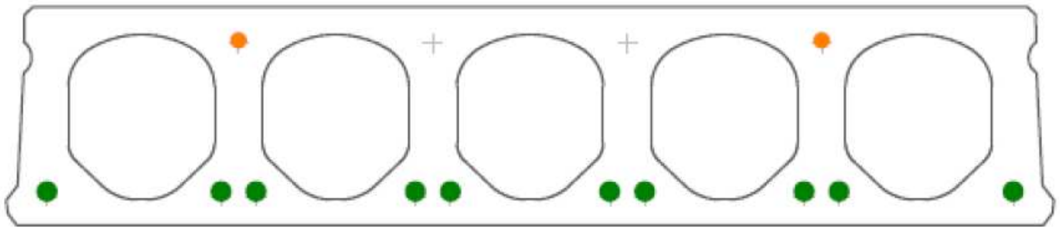
$$V_{Rdc.st}(96) = \mathbf{135.5} \cdot 10^3 \text{ N}$$

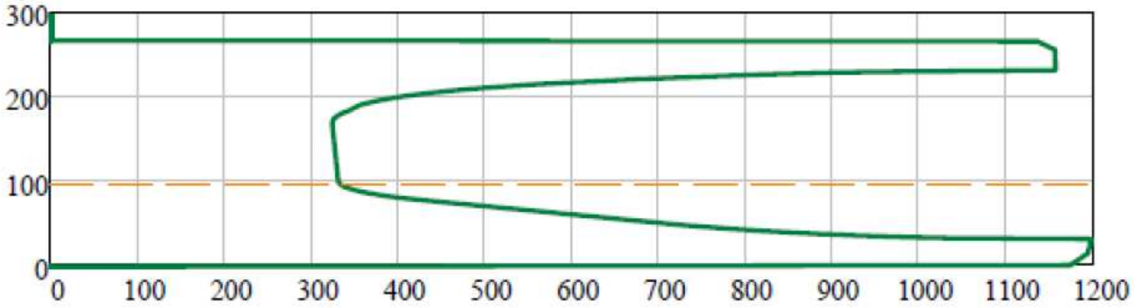
Remark: the full utilised resistance is found for this critical point because the input (V_{Ed}) and the result shear force ($V_{Rdc.st}$) have the same magnitude.

Shear tension resistance

By computing the points over the supposed line of failure with both the lower and higher value of the transmission length the intermediate results and the shear resistance over the line failure are:

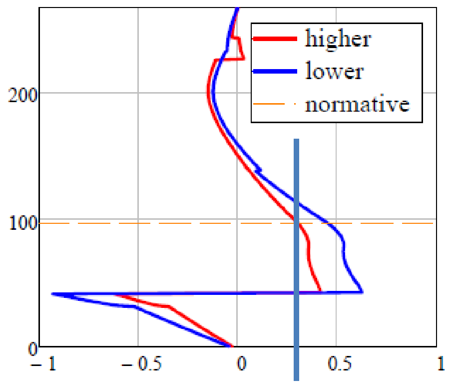
From cross section to profile of the width:





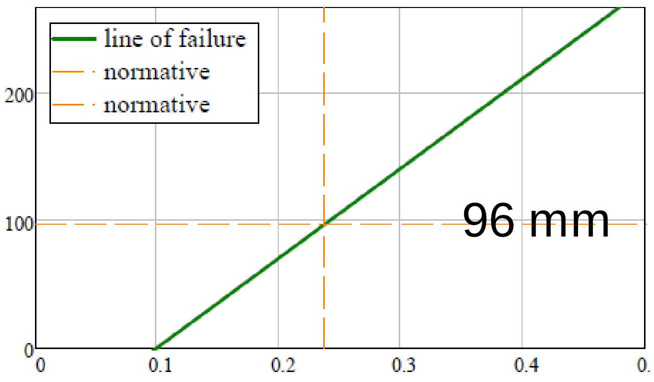
Shear tension resistance

Shear stress



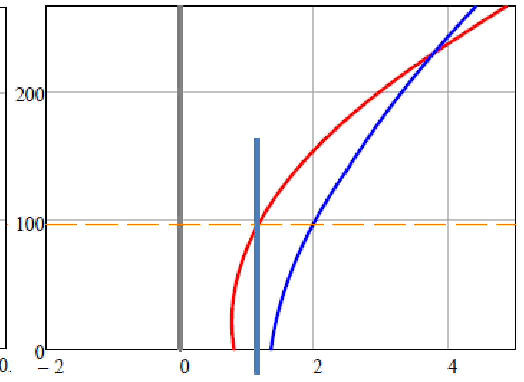
0.30 N/mm^2

Supposed line of failure

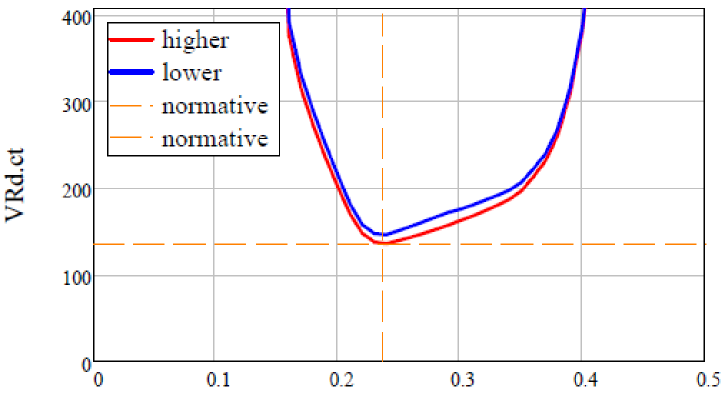


0.237 m

Compressive stress



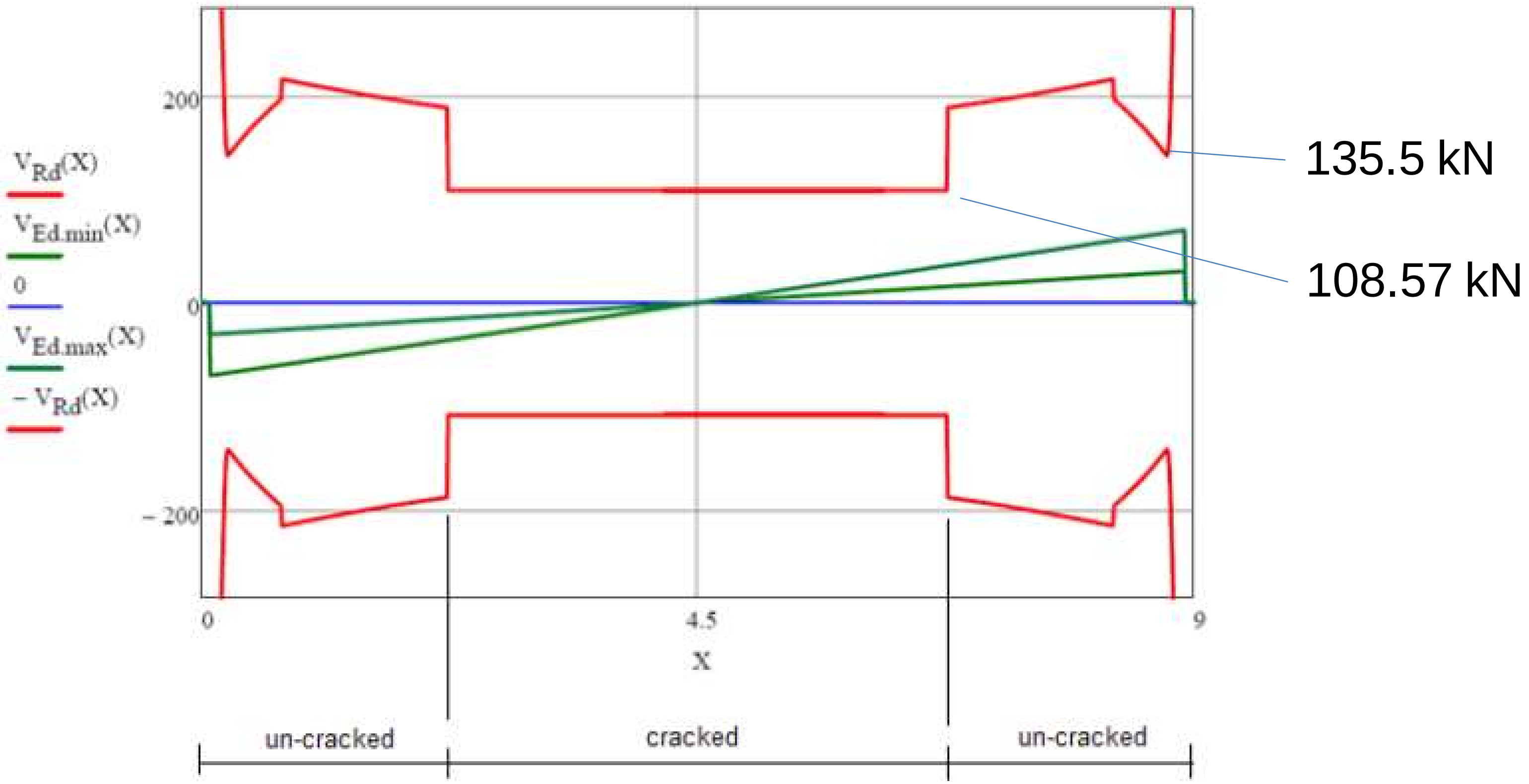
1.16 N/mm^2



135.5 kN

Shear tension resistance

The shear resistance over the length of the slab for un-cracked and cracked sections can be composed as:

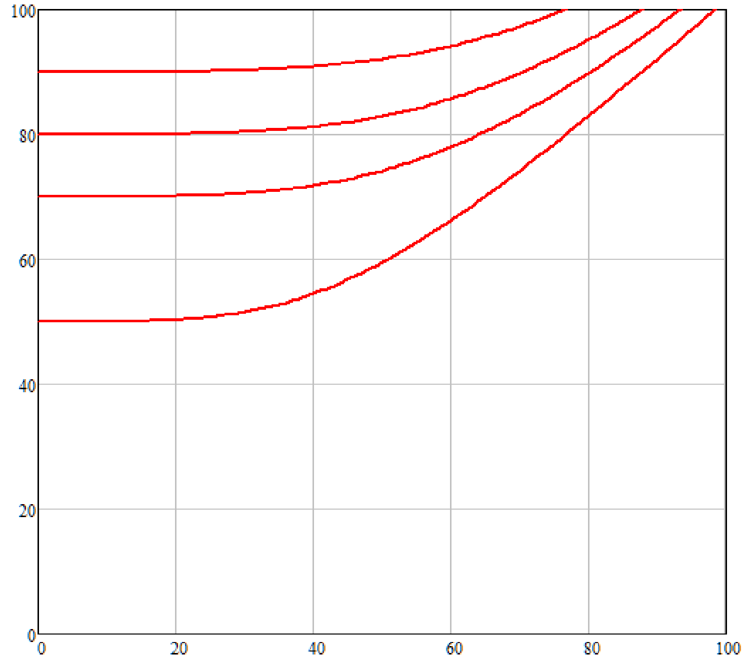


Shear and bending interaction

Where shear and bending are analysed at the same position along the span, they cannot be utilised at their respective maximum capacities.

For each position in the region cracked in bending, the combination of both should be calculated according to the following interaction formula

$$\eta_{MV} = \left(\left(\frac{V_{Ed,x}}{V_{Rd,c,SF}} \right)^4 + \left(\frac{M_{Ed,x}}{M_{Rd}} \right)^4 \right)^{\frac{1}{4}} \leq 1$$



CONSOLIS VBI

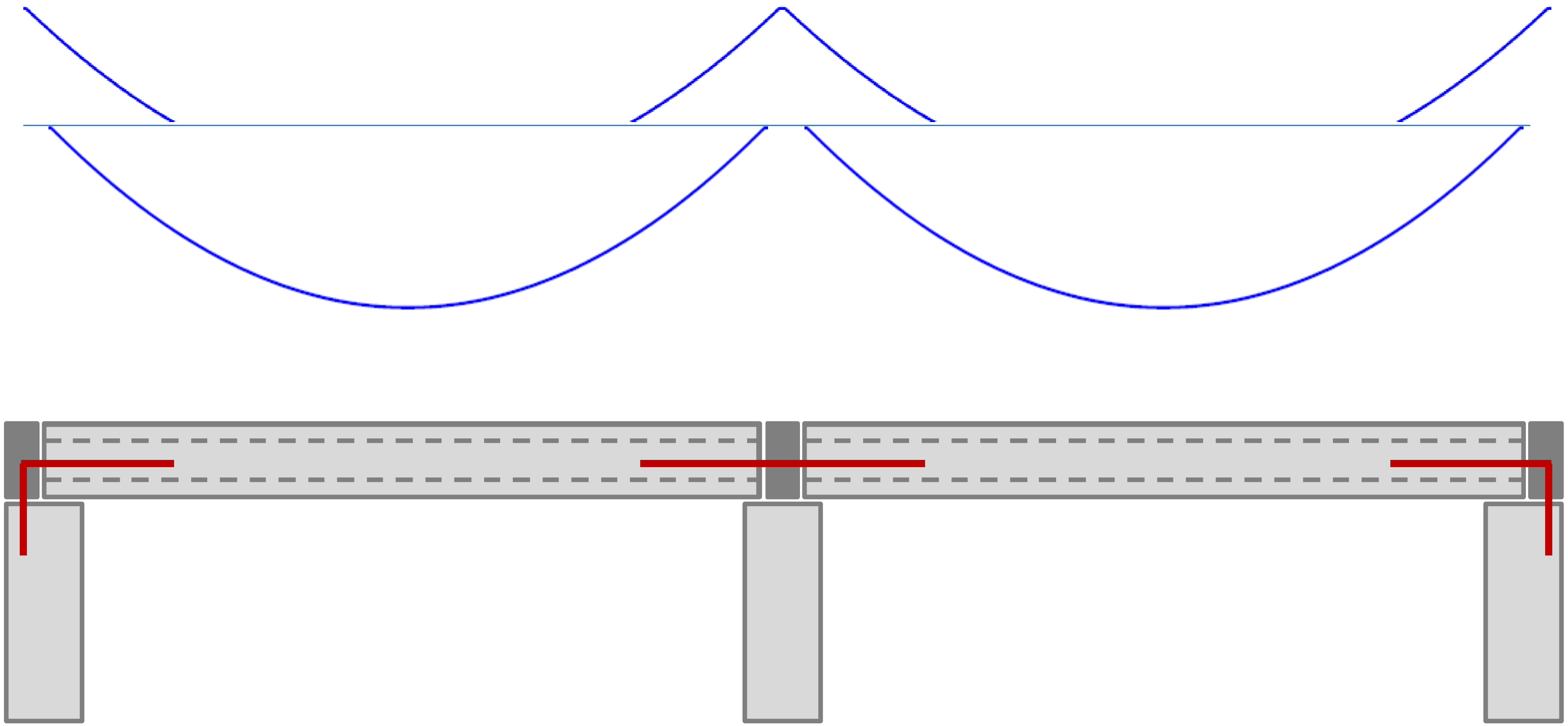
Mechanical behaviour

From simply supported slabs to a floor of hc slabs

Mechanical behaviour

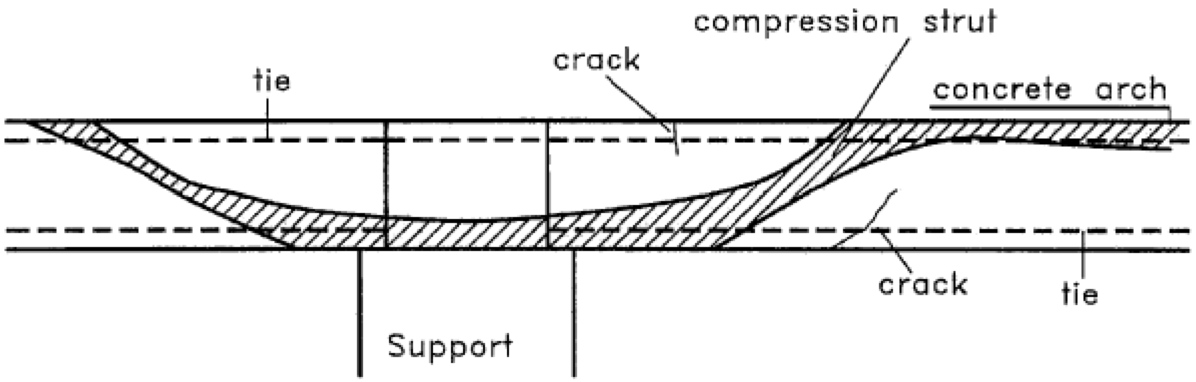
From simply supported slabs to Floor of hc slabs

Continuity…??



From simply supported slabs to Floor of hc slabs

Due to upper reinforcement ($A_s > A_{s.min}$):
Intended or non-intended continuity:
Not an isostatic beam anymore!



Thank you for your attention