IPHA TECHNICAL SEMINAR 2017

October 25–26. Tallinn, Estonia

Flexural strength and camber

Kim S. Elliott

Consultant | UK

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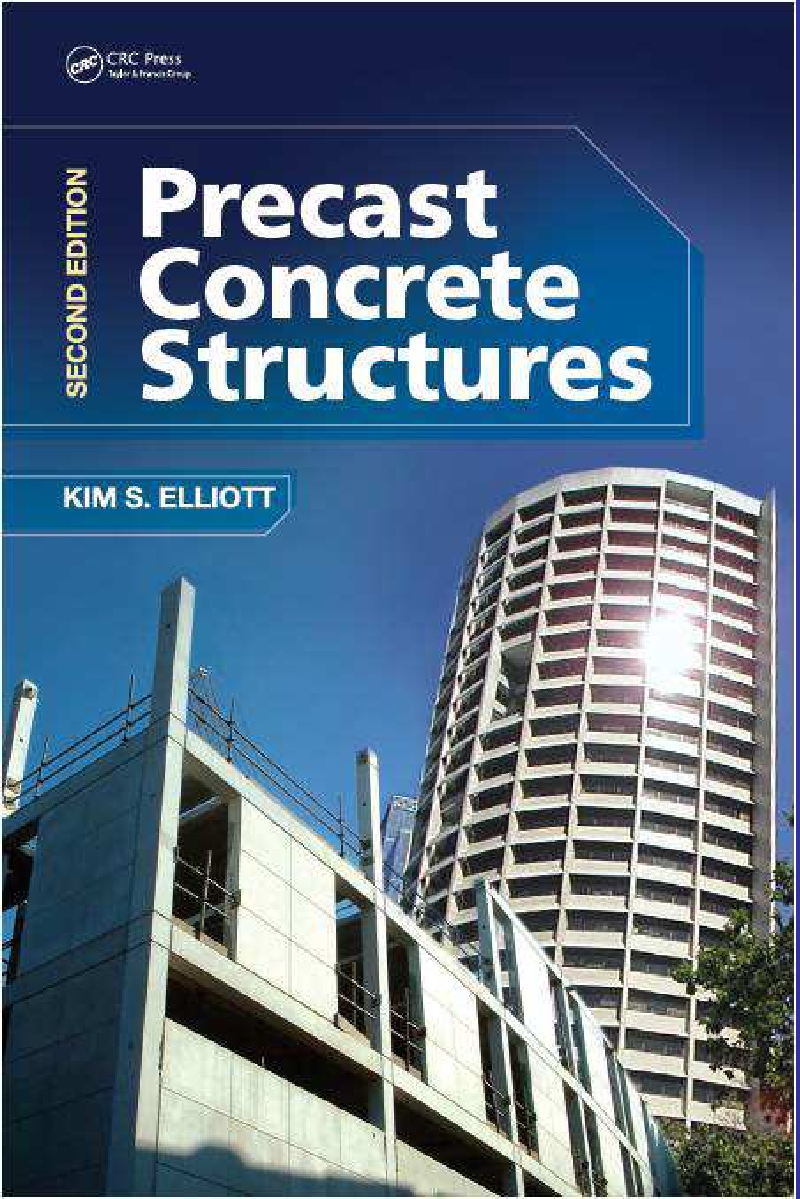
Prestressed Concrete Hollow Core Units

Flexural strength and deflections

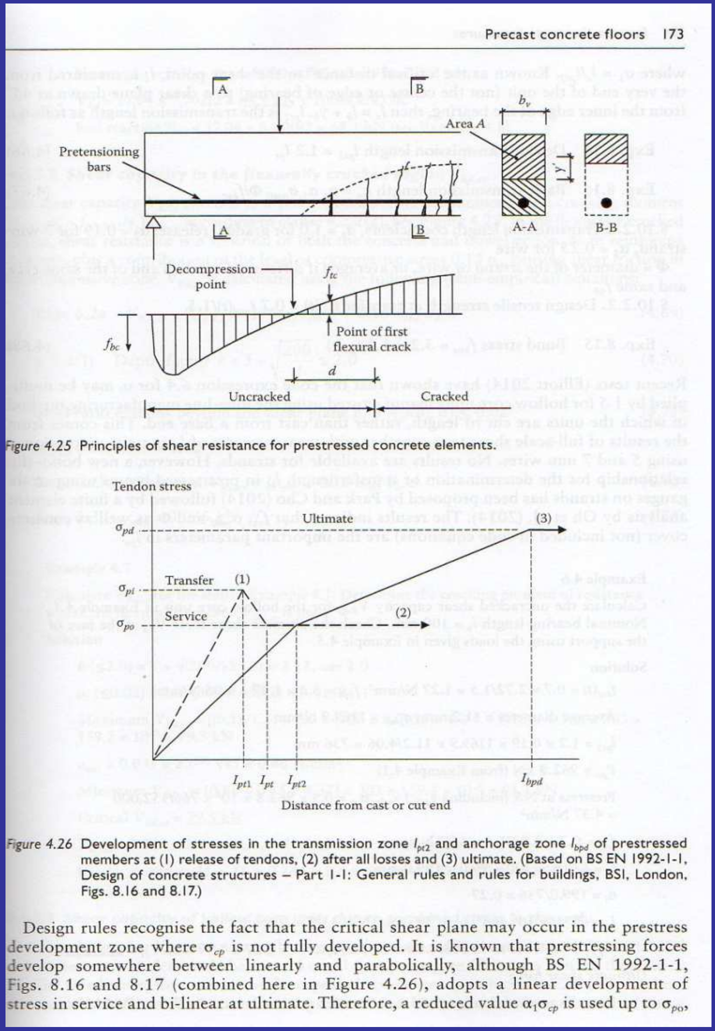
Dr Kim S Elliott, UK

IPHA Technical Seminar – Tallinn – 25-26 October 2017

Precast Concrete Structures 2^{nd} ed.



700 pages with about 200 pages on precast floors



Precast concrete floors 173

Figure 4.25 Principles of shear resistance for prestressed concrete elements.

Figure 4.26 Development of stresses in the transmission zone l_{pt2} and anchorage zone l_{bpd} of prestressed members at (1) release of tendons, (2) after all losses and (3) ultimate. (Based on BS EN 1992-1-1, Design of concrete structures – Part 1-1: General rules and rules for buildings, BSI, London, Figs. 8.16 and 8.17.)

Design rules recognise the fact that the critical shear plane may occur in the prestress development zone where σ_{cp} is not fully developed. It is known that prestressing forces develop somewhere between linearly and parabolically, although BS EN 1992-1-1, Figs. 8.16 and 8.17 (combined here in Figure 4.26), adopts a linear development of stress in service and bi-linear at ultimate. Therefore, a reduced value $\alpha_1\sigma_{cp}$ is used up to σ_{po},

Syllabus

**Definitions.
Introduction.
Concrete
and strands.
Cover.**

Syllabus

Definitions.
Introduction.
Concrete and strands.
Cover.

Prestress.
Losses.
Limit f_{ctm}.
Moment of resistance.

Syllabus

Definitions.
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Prestress.
Losses.
Limit f_{ctm}
Moment of resistance.

Ultimate strength.
Equilibrium.
Compatibility.
M_{Rd}.

Syllabus

Definitions.
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Camber.
Creep.
Deflections.
Limits.

Ultimate strength.
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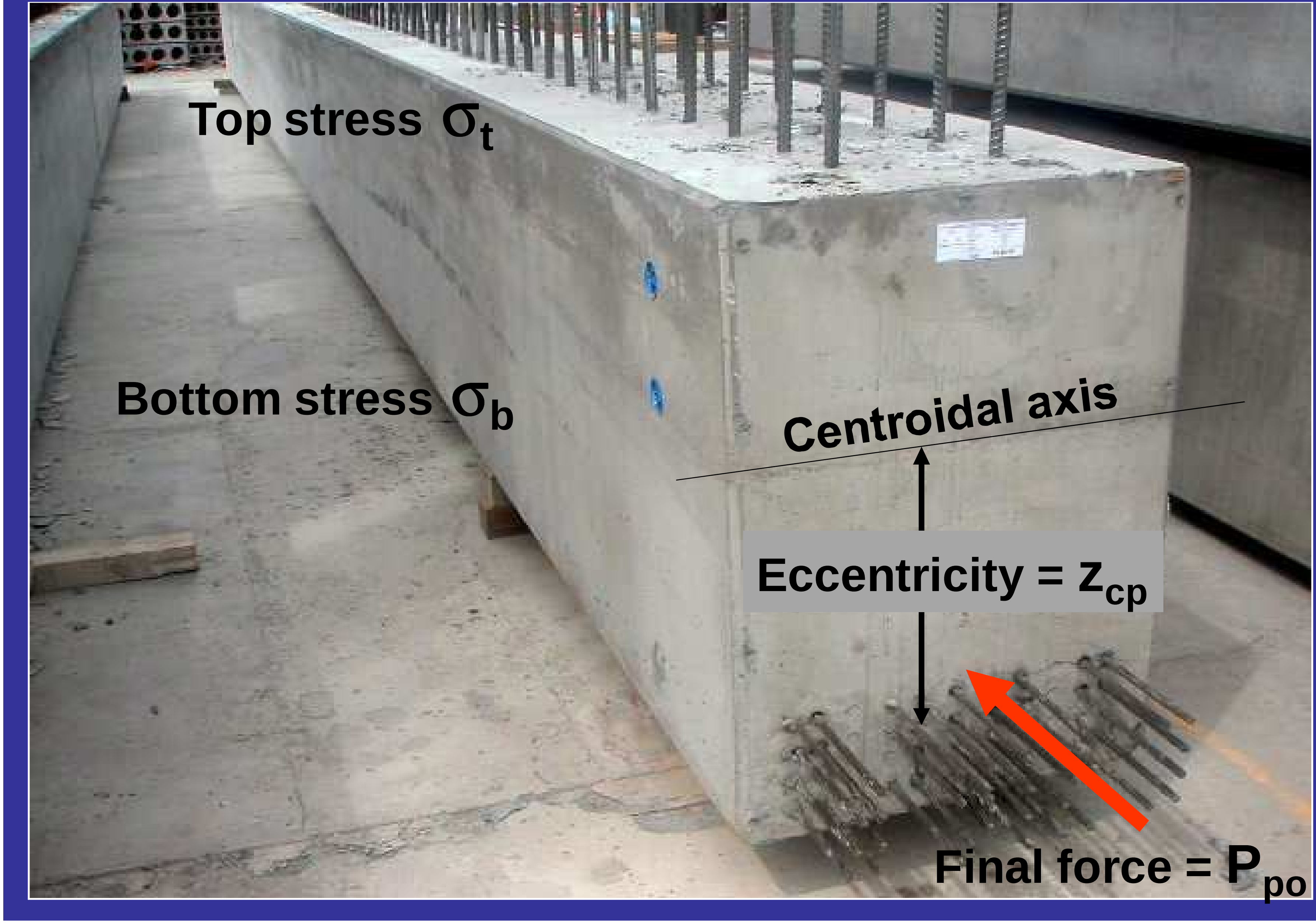
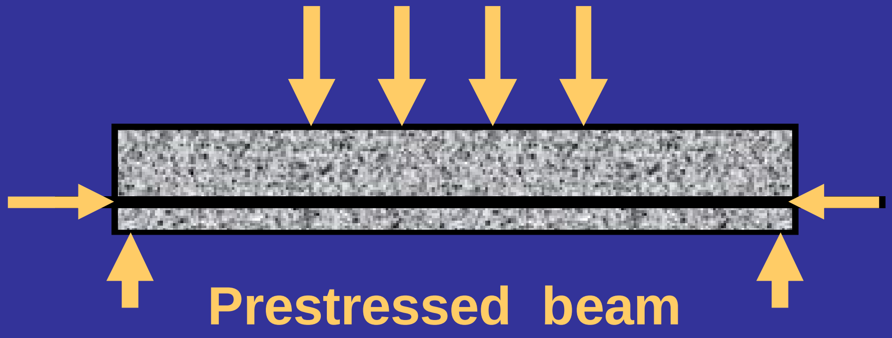
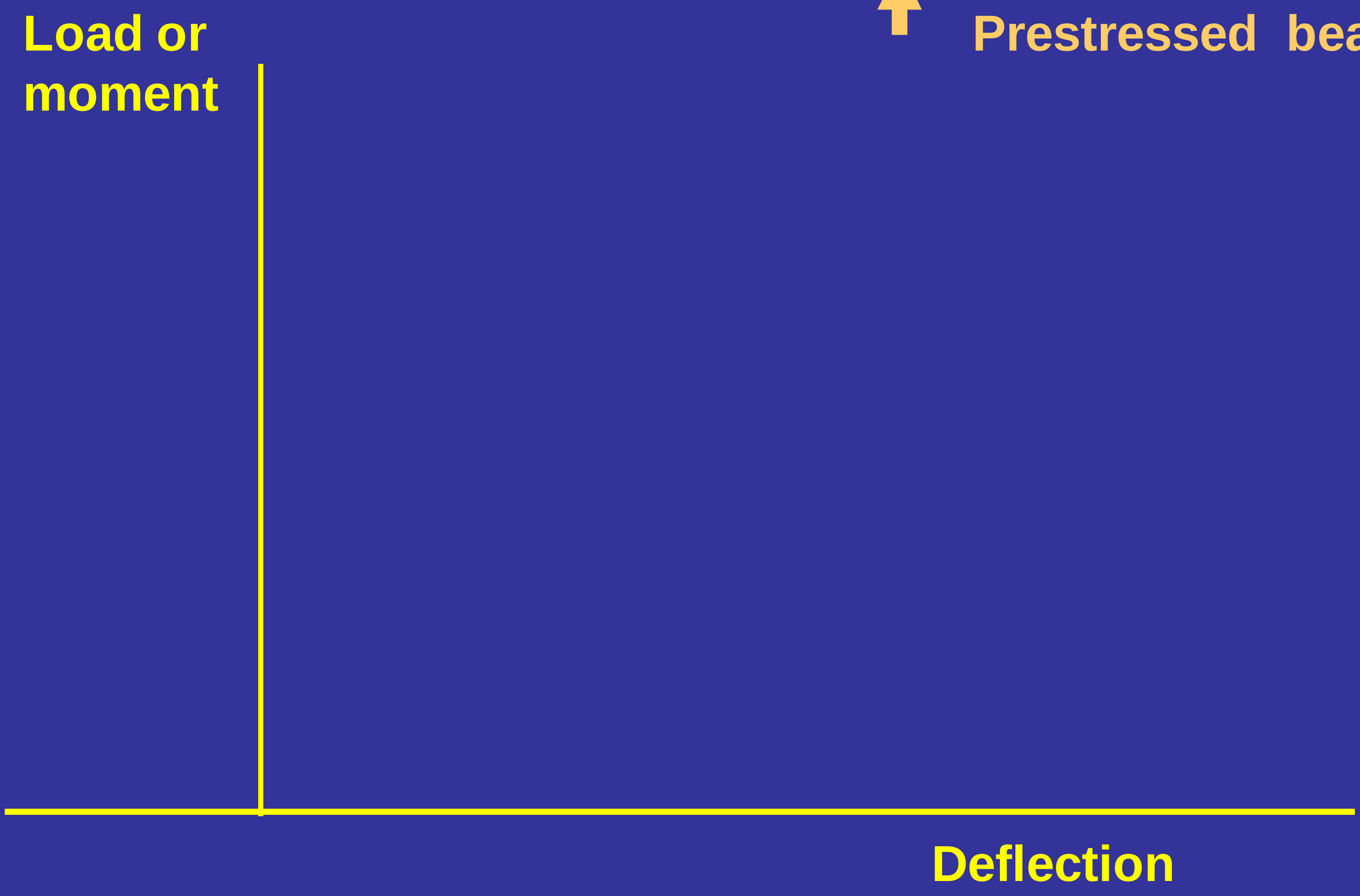
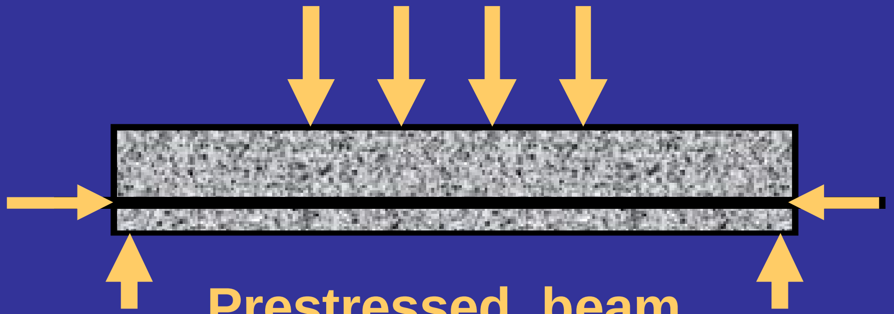
Compatibility
M_{Rd}

Serviceability stress check




Prestressed beam


Load or moment
Deflection



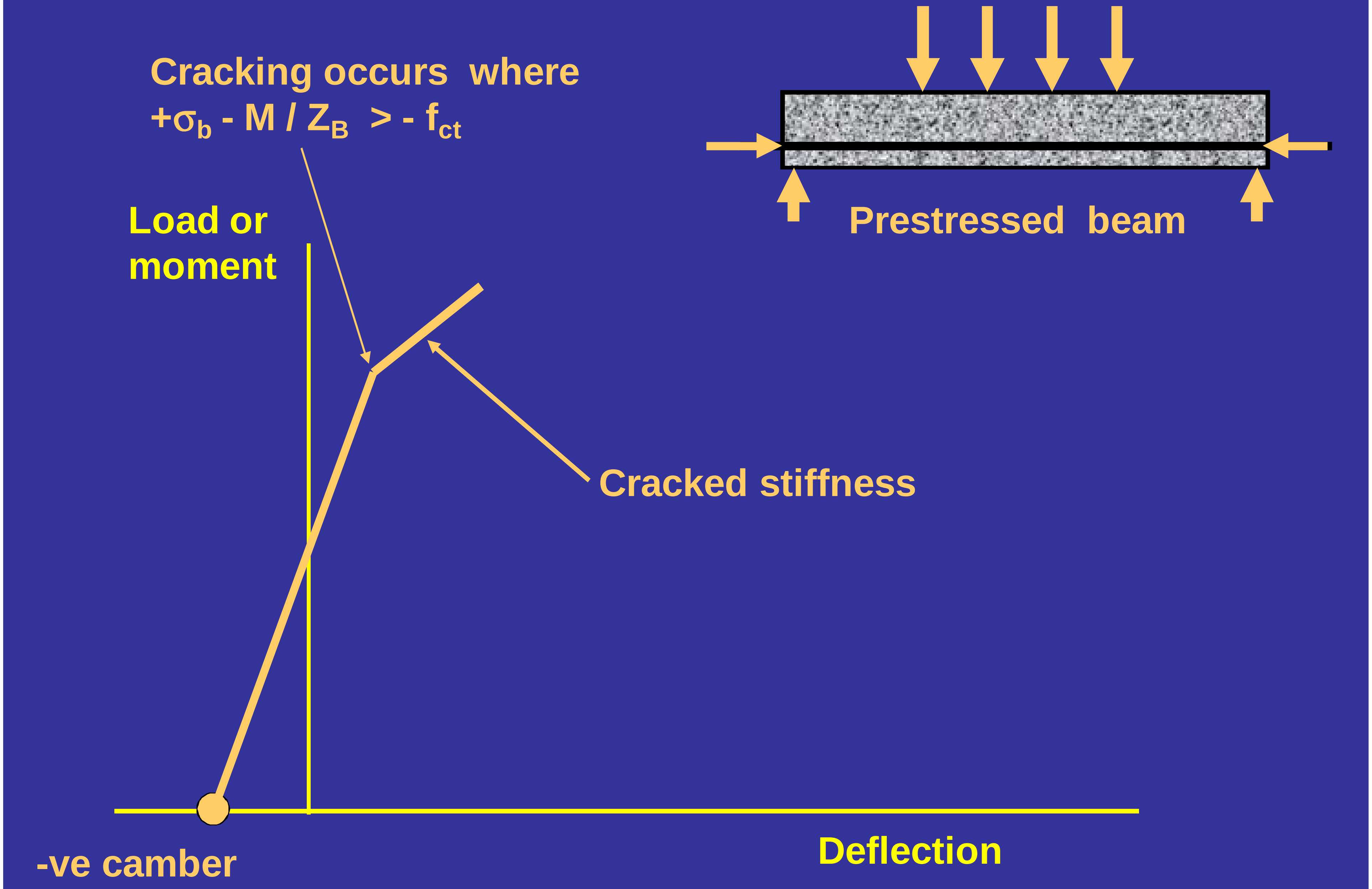
Prestressed beam

Load or moment

Uncracked stiffness

-ve camber

Deflection



Cracking occurs where $+\sigma_b - M / Z_B > - f_{ct}$

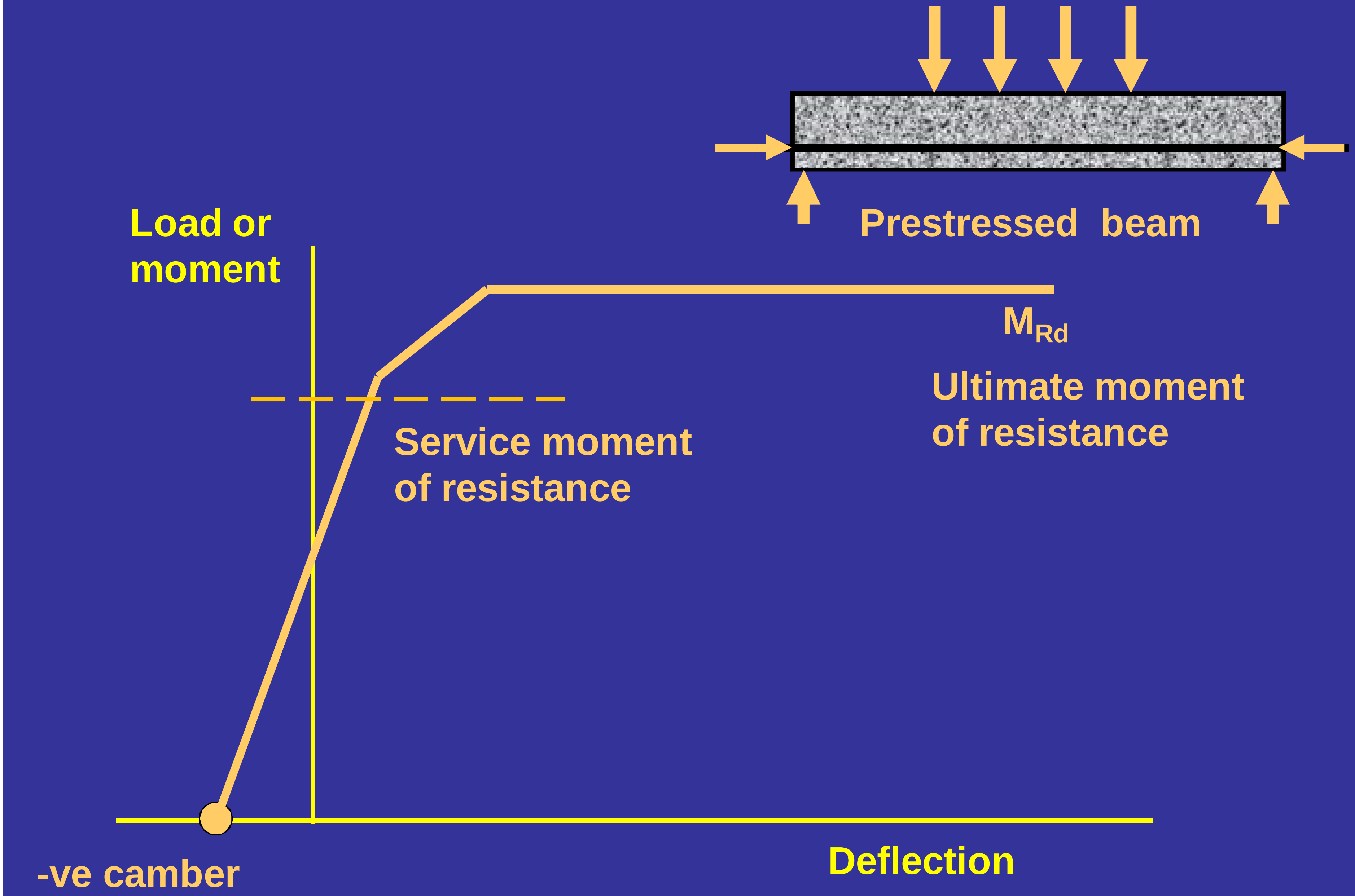
Load or moment

Prestressed beam

Cracked stiffness

-ve camber

Deflection



Prestressed beam

Load or moment

M_{Rd}

Ultimate moment of resistance

Service moment of resistance

-ve camber

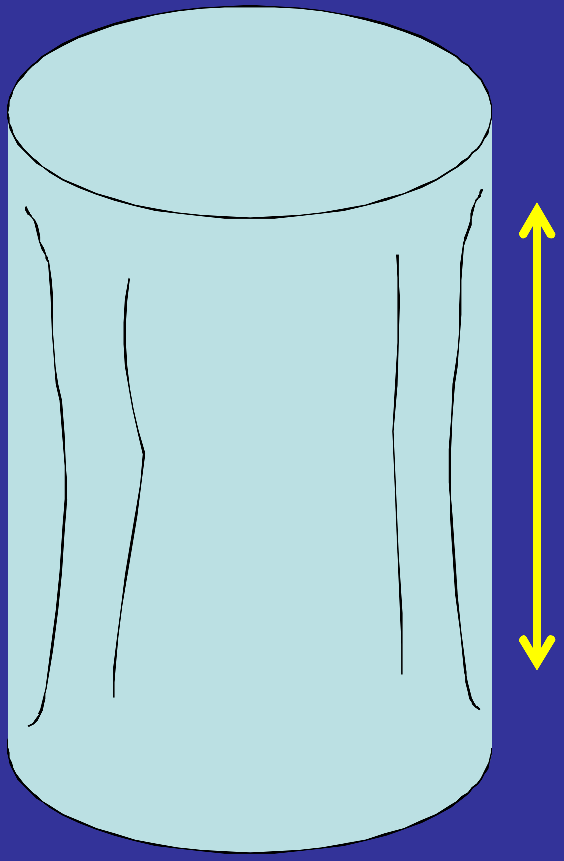
Deflection

Compressive cylinder strength f_{ck}

Design compressive strength

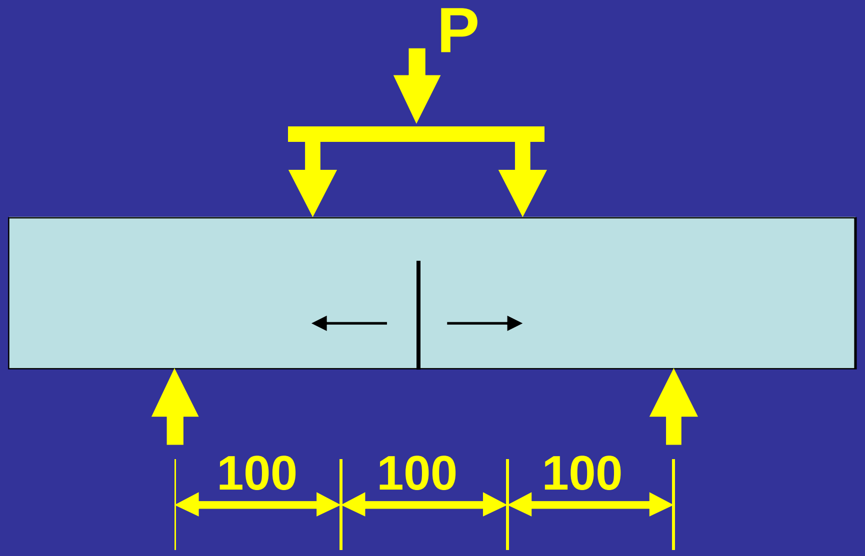
$$f_{cd} = 0.85\ f_{ck} / \gamma_c$$

for example 0.85 x 45 / 1.5 = 25.5 N/mm^2



Central region under uniform compressive stress

Tensile strength due to flexure f_{ctm}

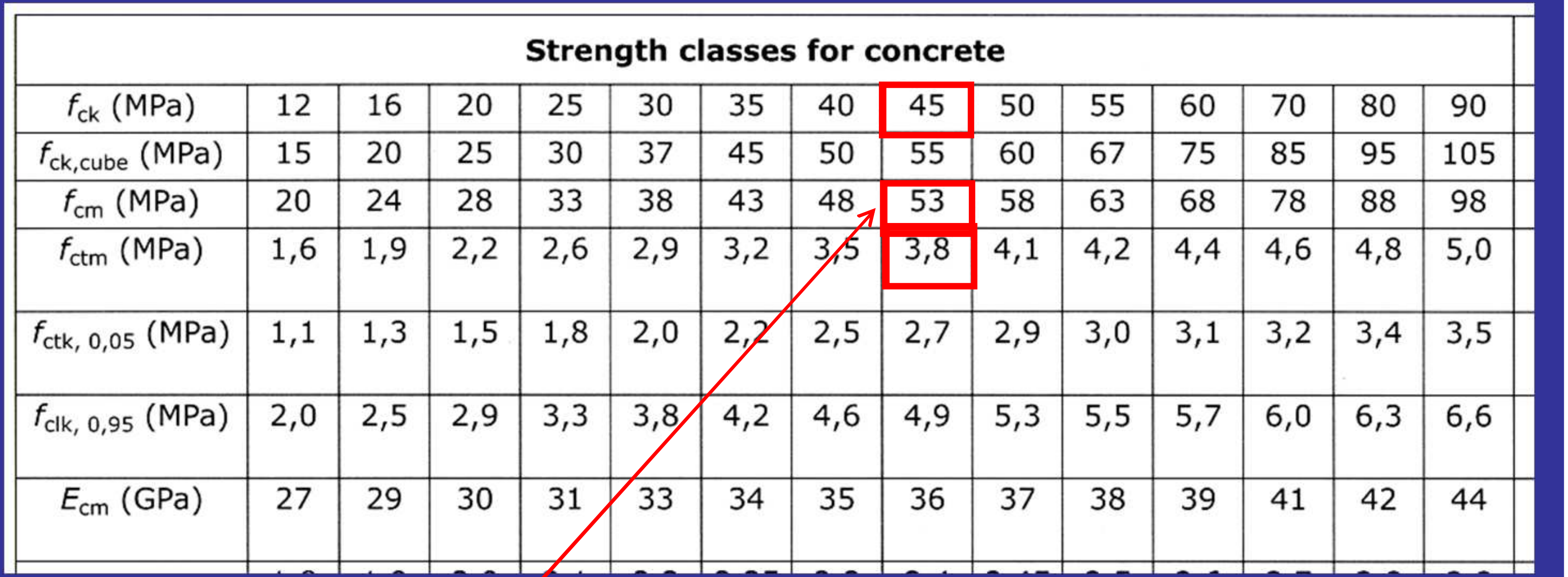


EC2-1-1, Table 3.1

$f_{ctm} = 0.3\ f_{ck}^{2/3}$

e.g. $= 0.3 \times 45^{2/3} = 3.80\ \text{N/mm}^2$

EC2-1-1 values listed in Table 3.1

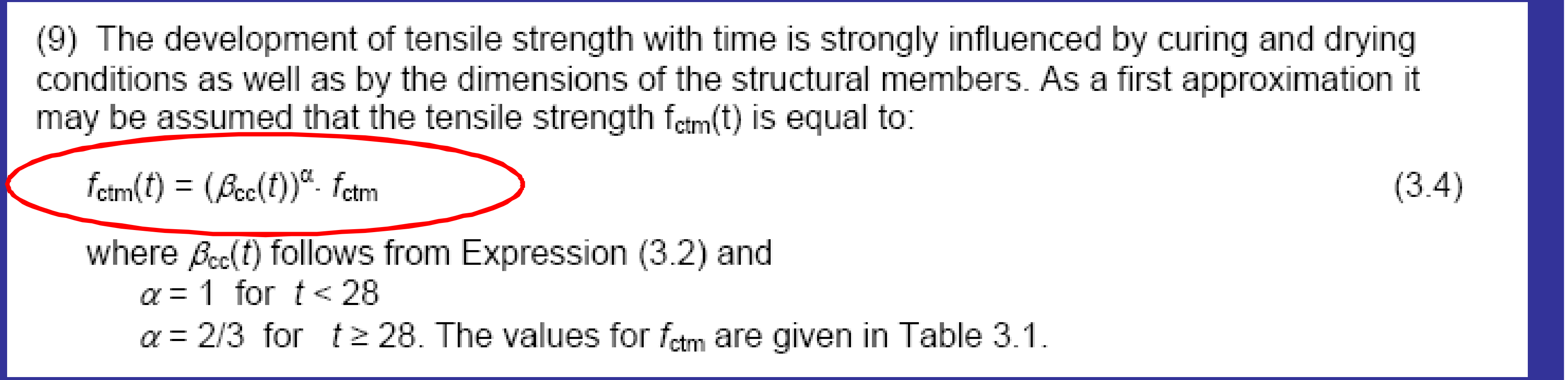


Strength classes for concrete														
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0
$f_{ctk,\ 0,05}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5
$f_{clk,\ 0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44

Mean $f_{cm} = f_{ck} + 8\ N/mm^2$

Early tensile stress at transfer

EC2-1-1, clause 3.1.2.



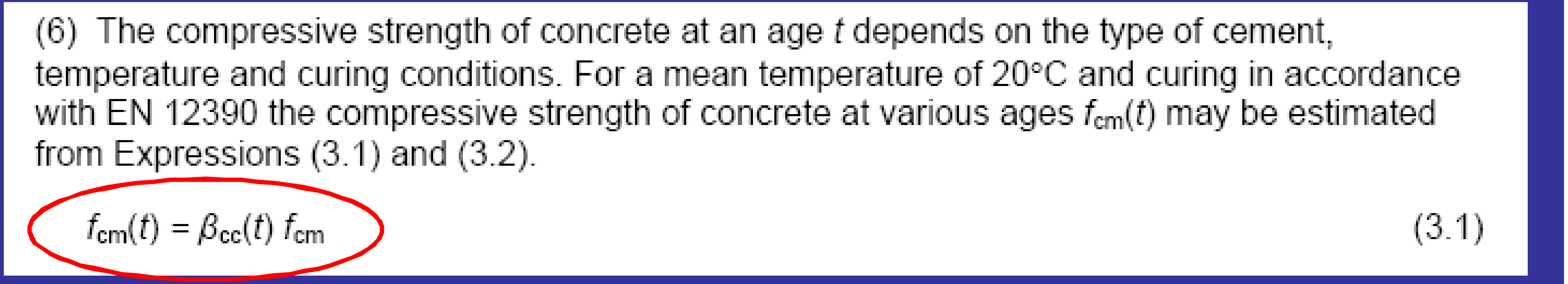
(9) The development of tensile strength with time is strongly influenced by curing and drying conditions as well as by the dimensions of the structural members. As a first approximation it may be assumed that the tensile strength $f_{ctm}(t)$ is equal to:

$$f_{ctm}(t) = (\beta_{cc}(t))^{\alpha} \cdot f_{ctm} \tag{3.4}$$

where $\beta_{cc}(t)$ follows from Expression (3.2) and

$\alpha = 1$ for $t < 28$

$\alpha = 2/3$ for $t \geq 28$. The values for f_{ctm} are given in Table 3.1.



(6) The compressive strength of concrete at an age t depends on the type of cement, temperature and curing conditions. For a mean temperature of 20°C and curing in accordance with EN 12390 the compressive strength of concrete at various ages $f_{cm}(t)$ may be estimated from Expressions (3.1) and (3.2).

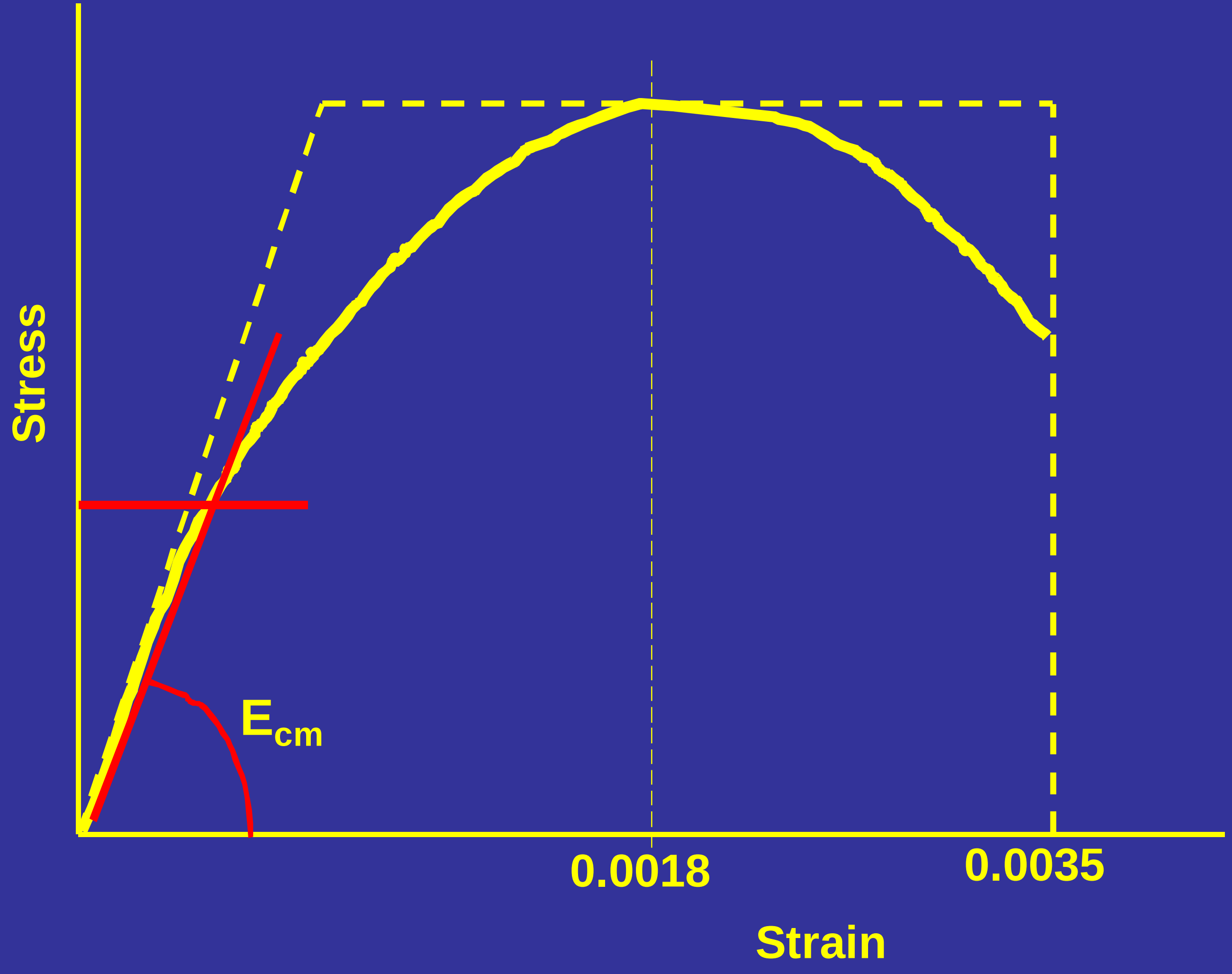
$$f_{cm}(t) = \beta_{cc}(t)\ f_{cm} \tag{3.1}$$

$\therefore f_{ctm}(t) = f_{ctm} \times f_{cm}(t)/ f_{cm}$

e.g. $f_{ctm}(t) = 3.80 \times 38 / 53 = 2.72$ N/mm^2

E_{cm} = 22 (f_{cm} / 10)$^{0.3}$ kN/mm^2 for gravel and granite.
For limestone x 0.9.

e.g. = 22 x (53/10)$^{0.3}$ = 36.3 kN/mm^2



Prestressing tendons stress v strain

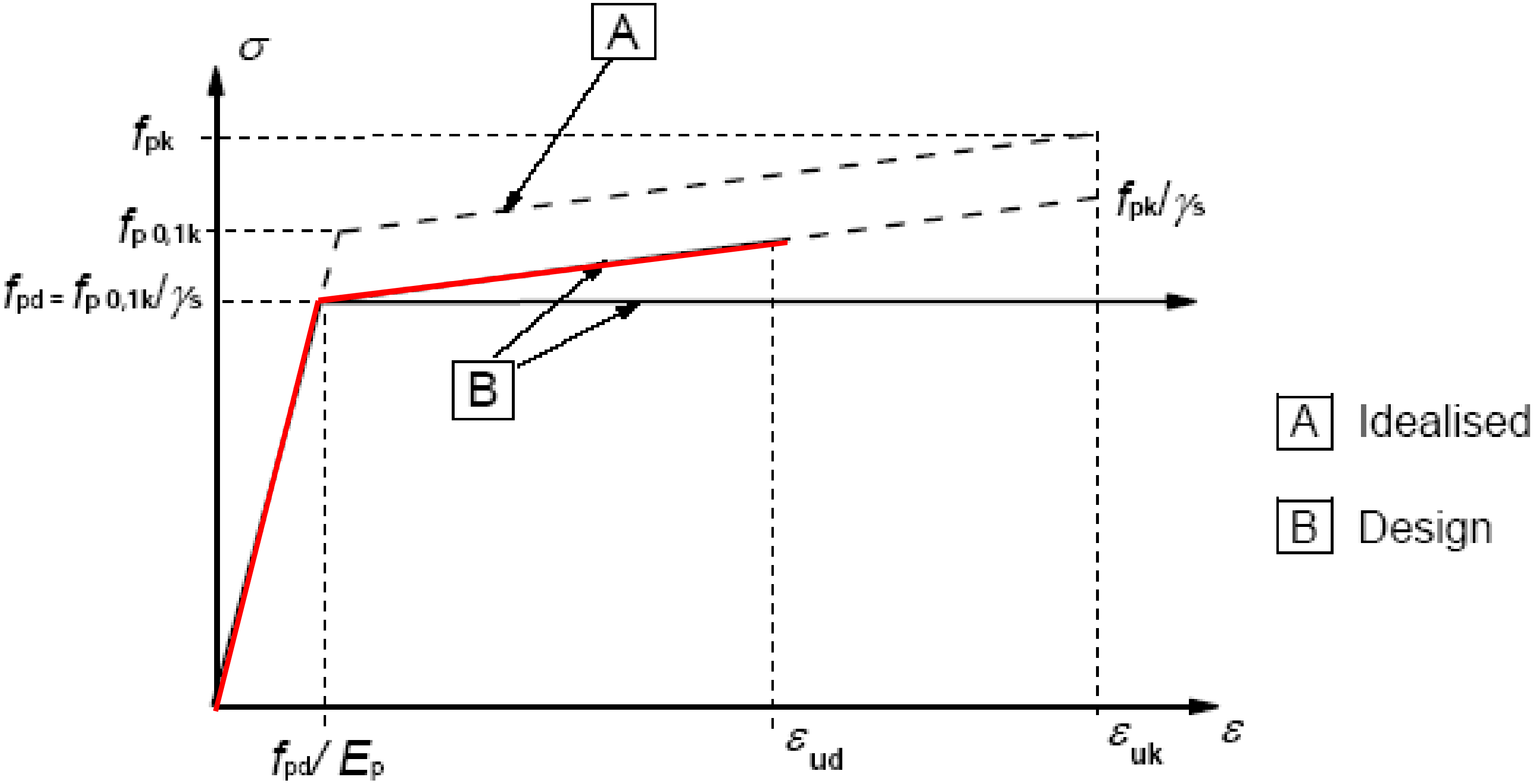


Figure 3.10: Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)

e.g. f_{pk} = 1770 N/mm^2

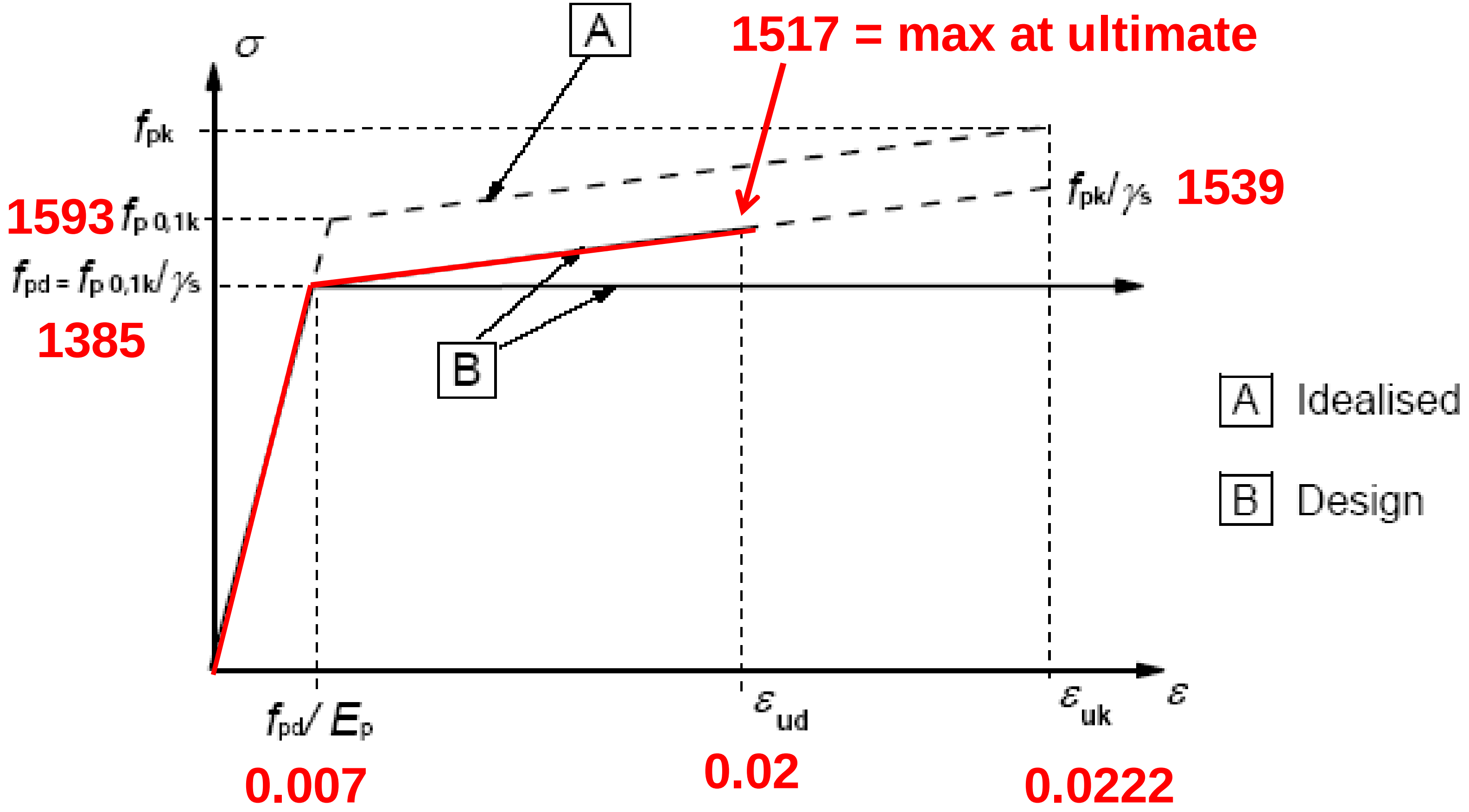


Figure 3.10: Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)

Durability = nominal cover c = $c_{min,dur} + \Delta c_{dev}$

For precast slabs > C30/37, use S2

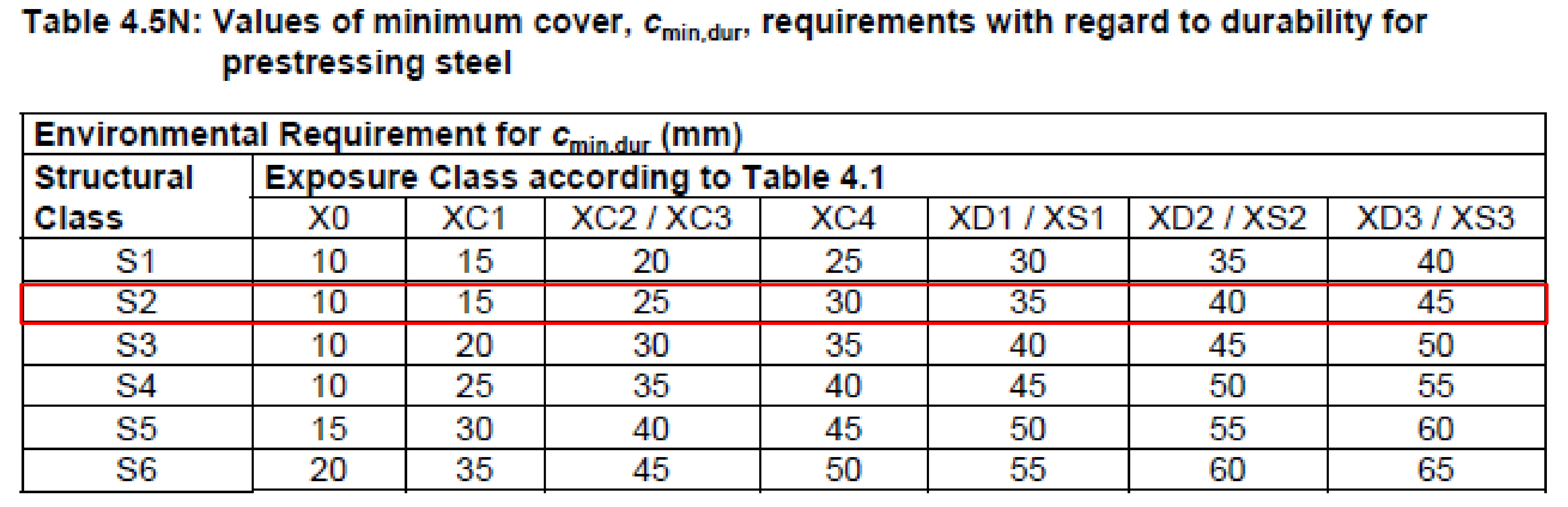
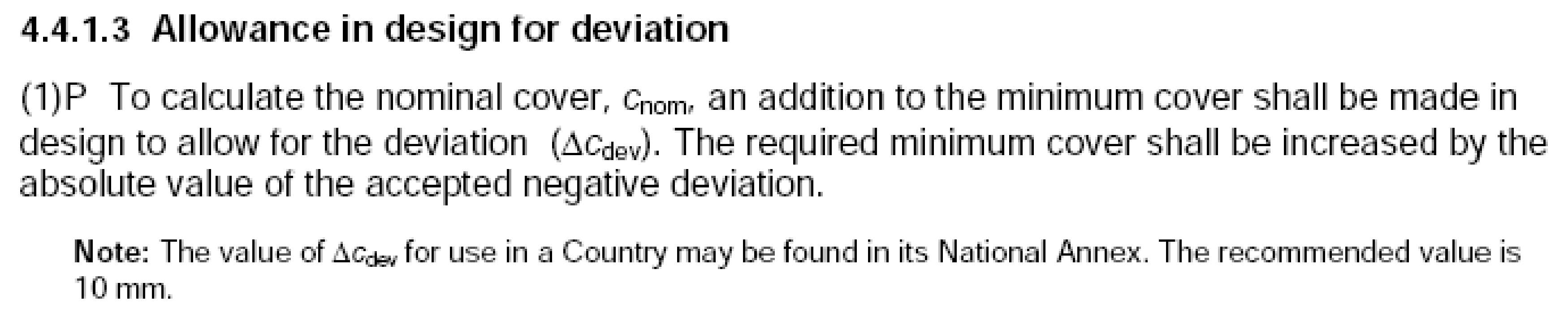


Table 4.5N: Values of minimum cover, $c_{min,dur}$, requirements with regard to durability for prestressing steel

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	15	20	25	30	35	40
S2	10	15	25	30	35	40	45
S3	10	20	30	35	40	45	50
S4	10	25	35	40	45	50	55
S5	15	30	40	45	50	55	60
S6	20	35	45	50	55	60	65

Much better and recent information in BS 8500-1: 2015

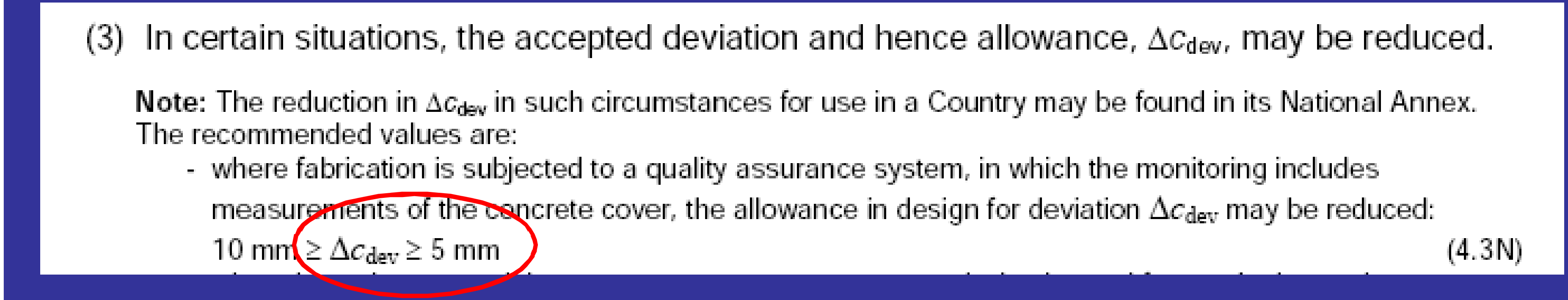
Construction deviation Δc_{dev} = 5 mm if tendon positions are controlled



4.4.1.3 Allowance in design for deviation

(1)P To calculate the nominal cover, c_{nom}, an addition to the minimum cover shall be made in design to allow for the deviation (Δc_{dev}). The required minimum cover shall be increased by the absolute value of the accepted negative deviation.

Note: The value of Δc_{dev} for use in a Country may be found in its National Annex. The recommended value is 10 mm.



(3) In certain situations, the accepted deviation and hence allowance, Δc_{dev}, may be reduced.

Note: The reduction in Δc_{dev} in such circumstances for use in a Country may be found in its National Annex. The recommended values are:

- where fabrication is subjected to a quality assurance system, in which the monitoring includes measurements of the concrete cover, the allowance in design for deviation Δc_{dev} may be reduced:

$$10 \text{ mm} \geq \Delta c_{dev} \geq 5 \text{ mm} \qquad (4.3N)$$

Syllabus

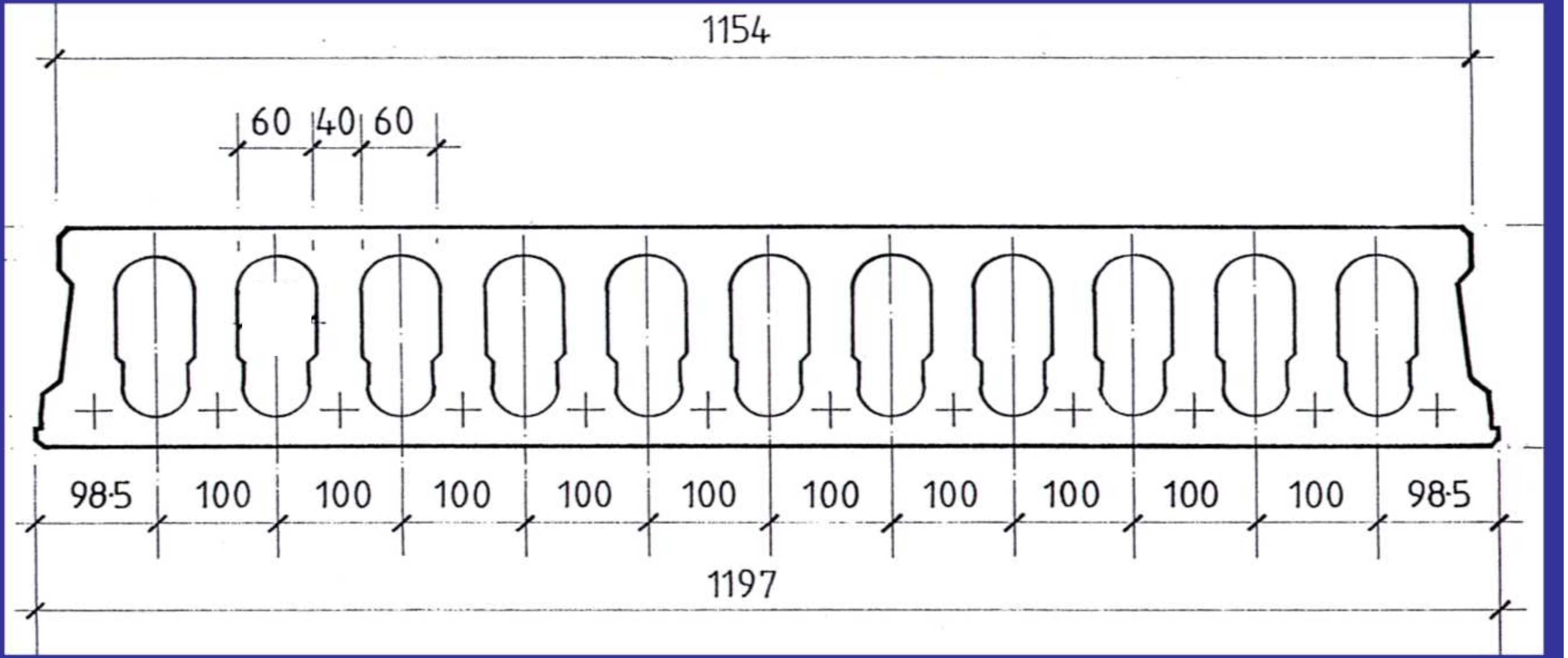
Prestress.

Losses.

Limit f_{ctm}

Moment of resistance.

Prestressed concrete hollow core floor units



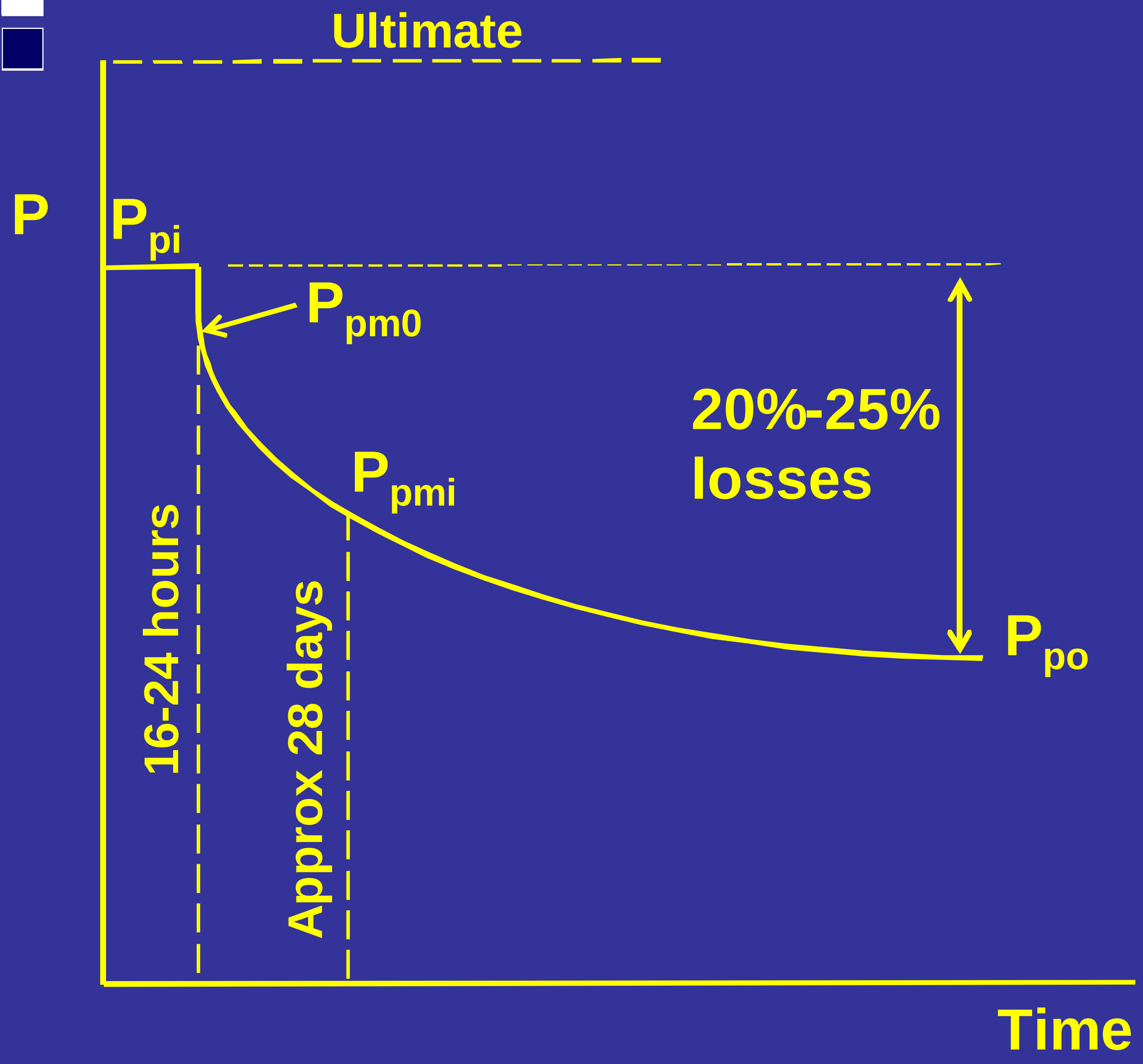
Calculate: Area A_c

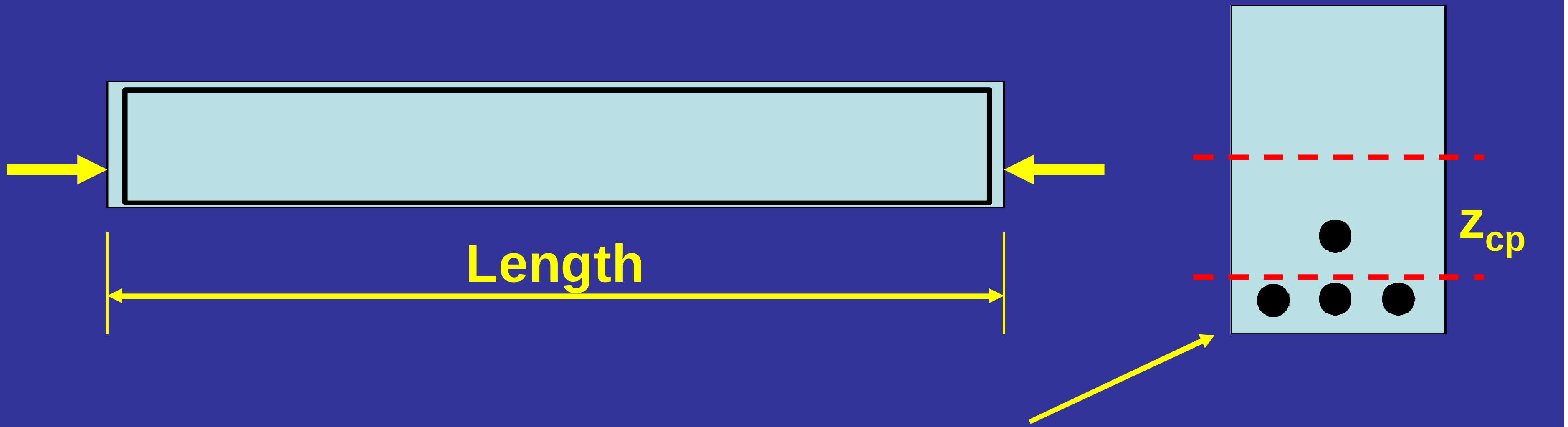
Centroid height y_b (and $y_t = h - y_b$)

2^{nd} moment of area I_{x-x}

1^{st} moment of area S_{x-x} (for shear only)

Section modulus $Z_b = I_{x-x} / y_b$ and $Z_T = I_{x-x} / y_t$


Ultimate
P
P_{pi}
P_{pm0}
P_{pmi}
20%-25%
losses
P_{po}
16-24 hours
Approx 28 days
Time



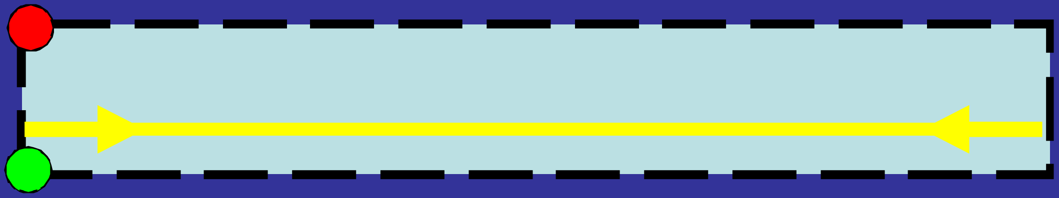
Initial prestressing force $P_{pi} = \eta\ f_{pk}\ A_{ps}$

Eccentricity z_{cp}

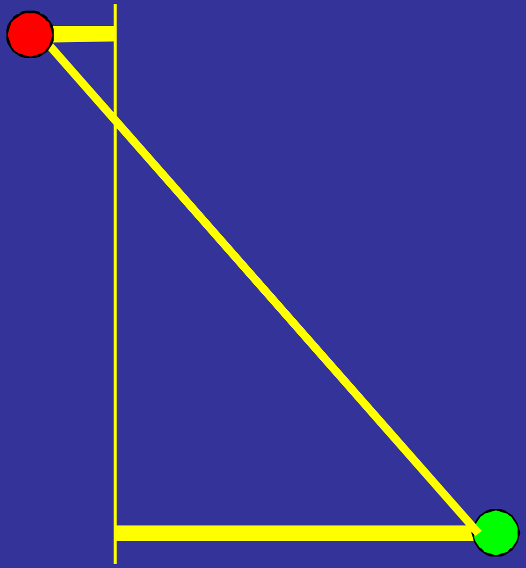
Typically 0.7 x 1770

Immediate strand relaxation and elastic shortening at transfer …

Transfer stress limits, e.g. based on $f_{ck}(t) = 30\ N/mm^2$

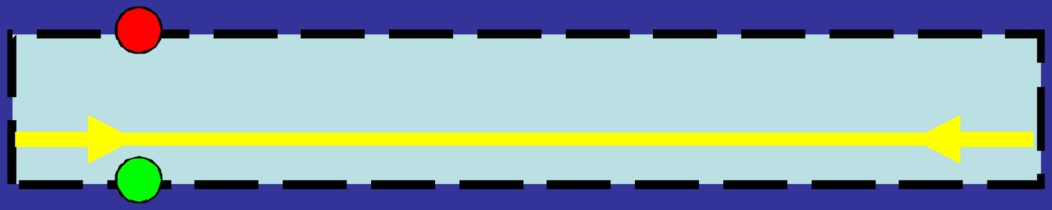


$\sigma_{t(t)} \geq\ - f_{ctm}(t) = -\ 2.72\ N/mm^2$

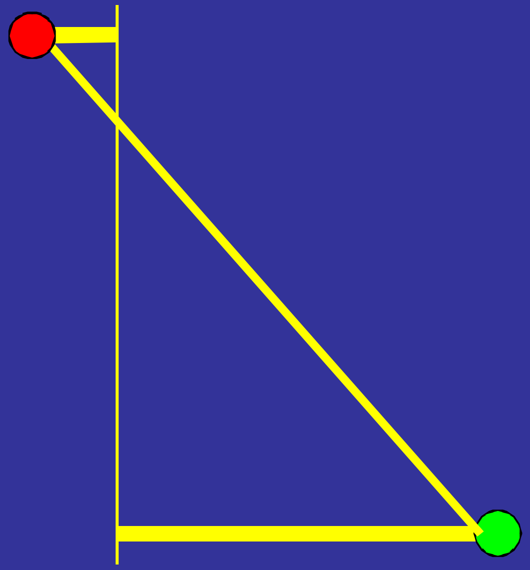


$\sigma_{b(t)} \leq 0.6\ f_{ck}(t) = +18.00\ N/mm^2$

Can be checked at the end of the transmission length l_{pt}



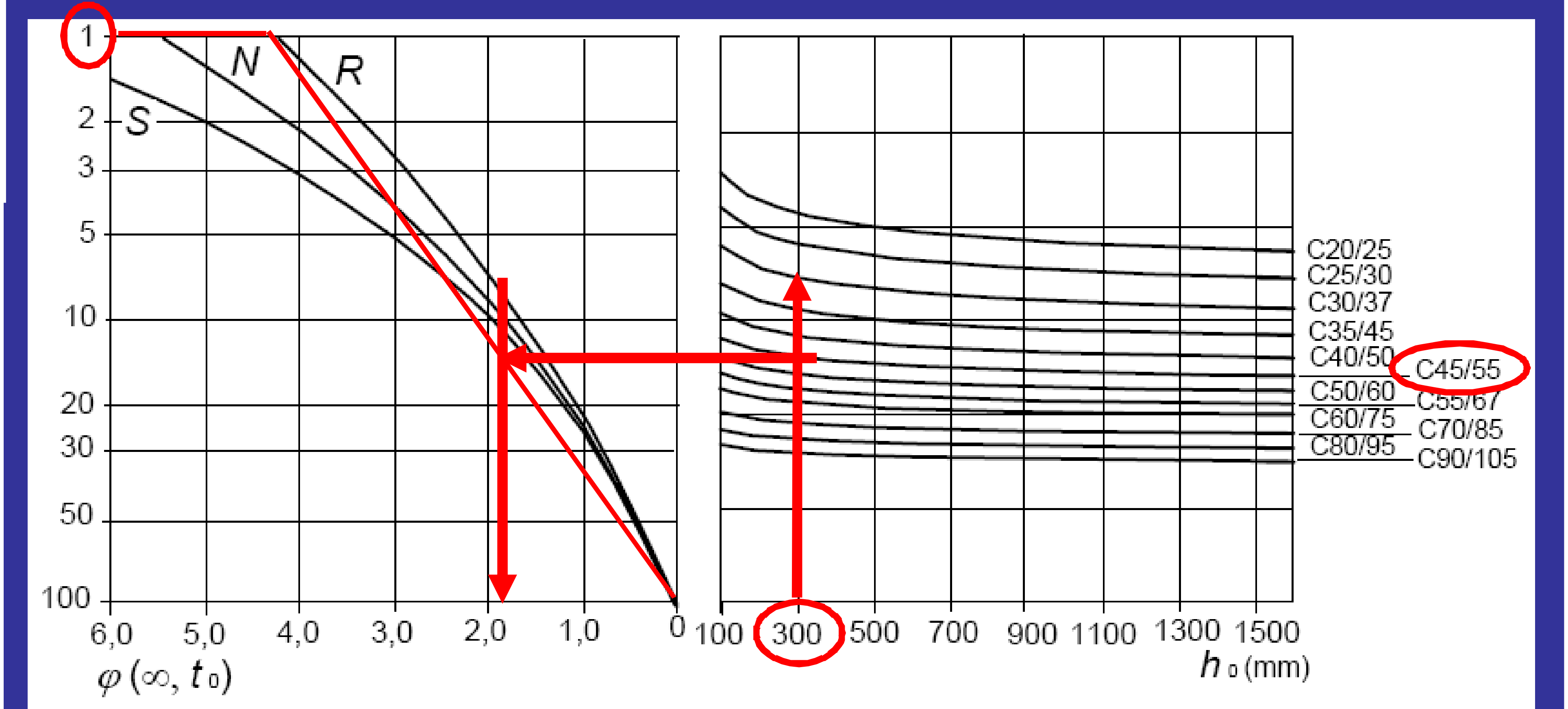
$$\sigma_{t(t)} + M_{self} / Z_t \geq - f_{ctm}(t) = - 2.72 \text{ N/mm}^2$$



$$\sigma_{b(t)} - M_{self} / Z_b \leq 0.6 f_{ck}(t) = +18.00 \text{ N/mm}^2$$

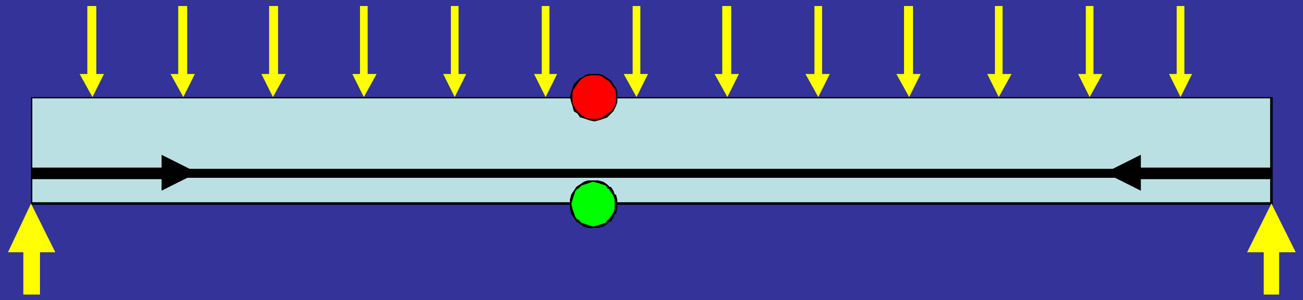
Followed by long term losses due to creep, shrinkage and further strand relaxation

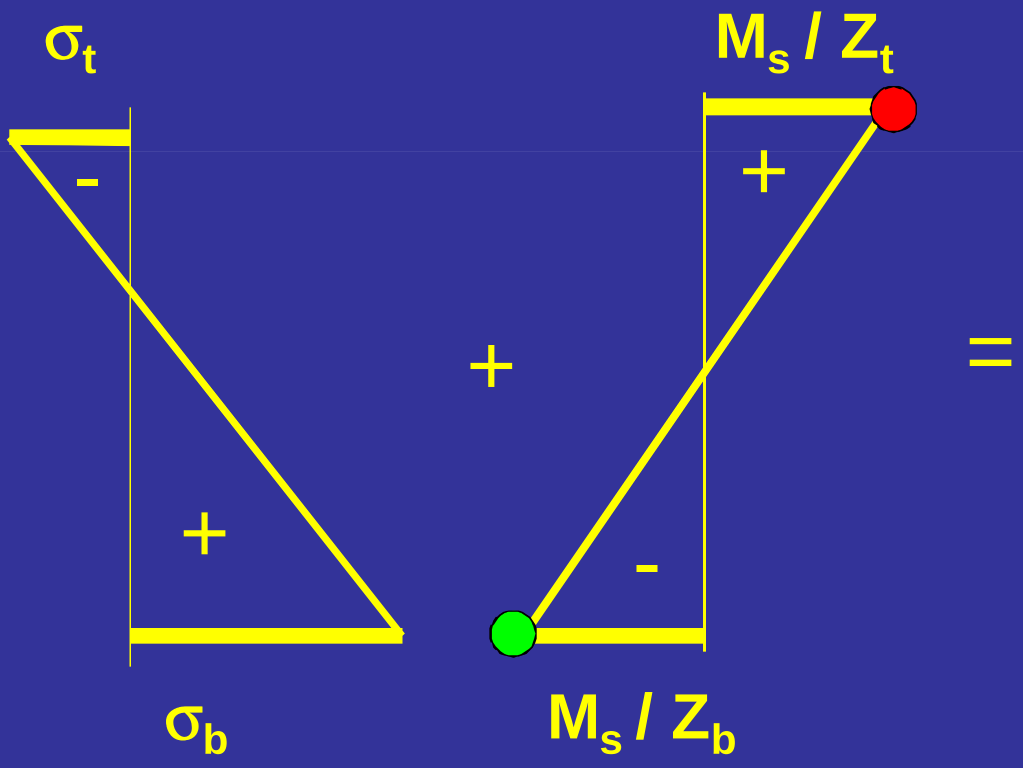
Results in a lower prestress σ_b and σ_t



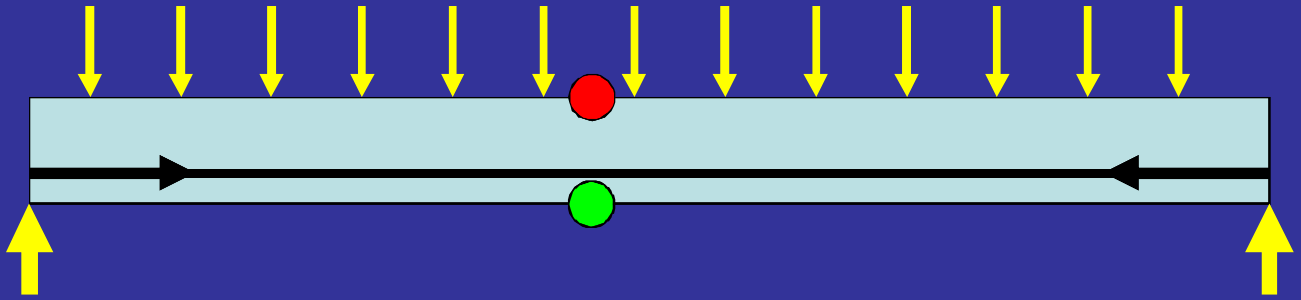
b) outside conditions - RH = 80%

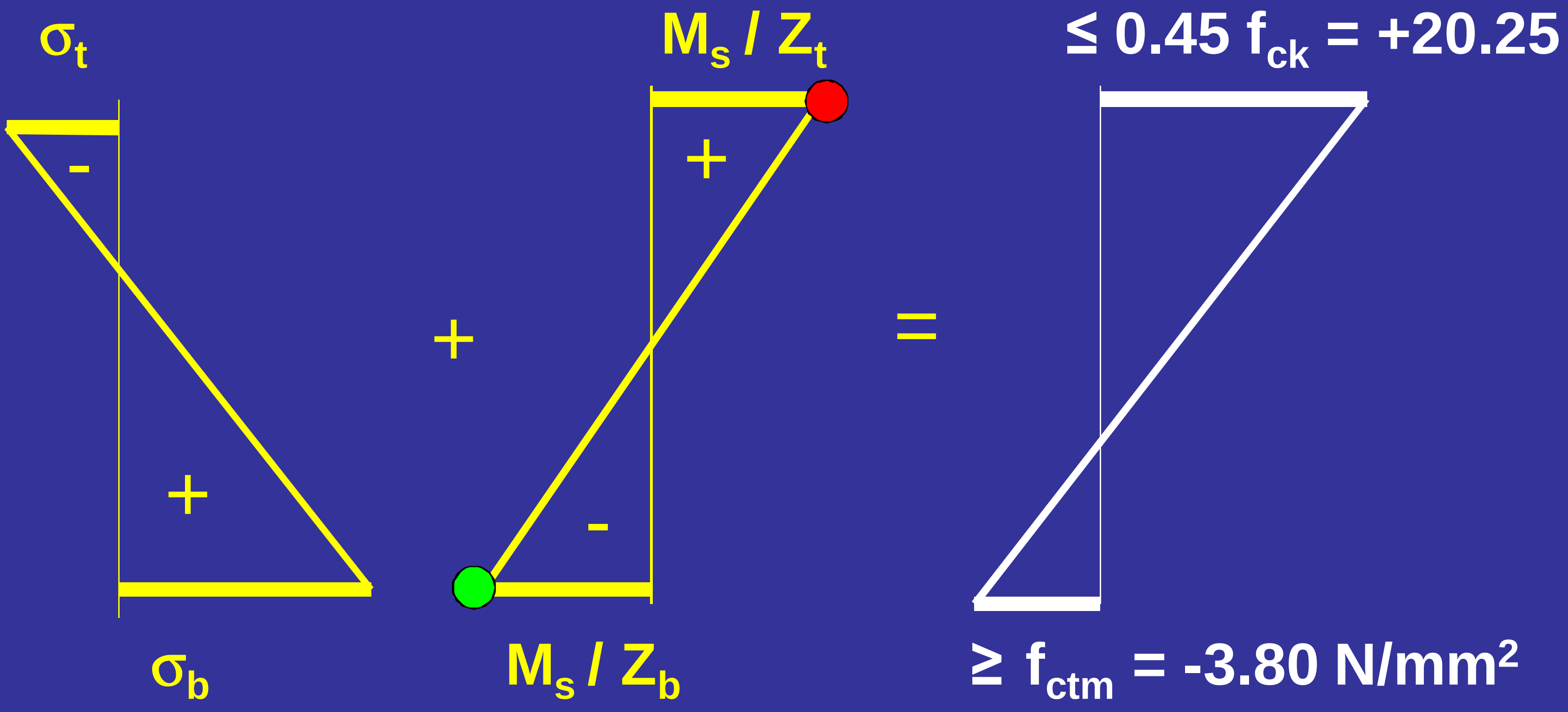
Adding prestress to imposed service stress =





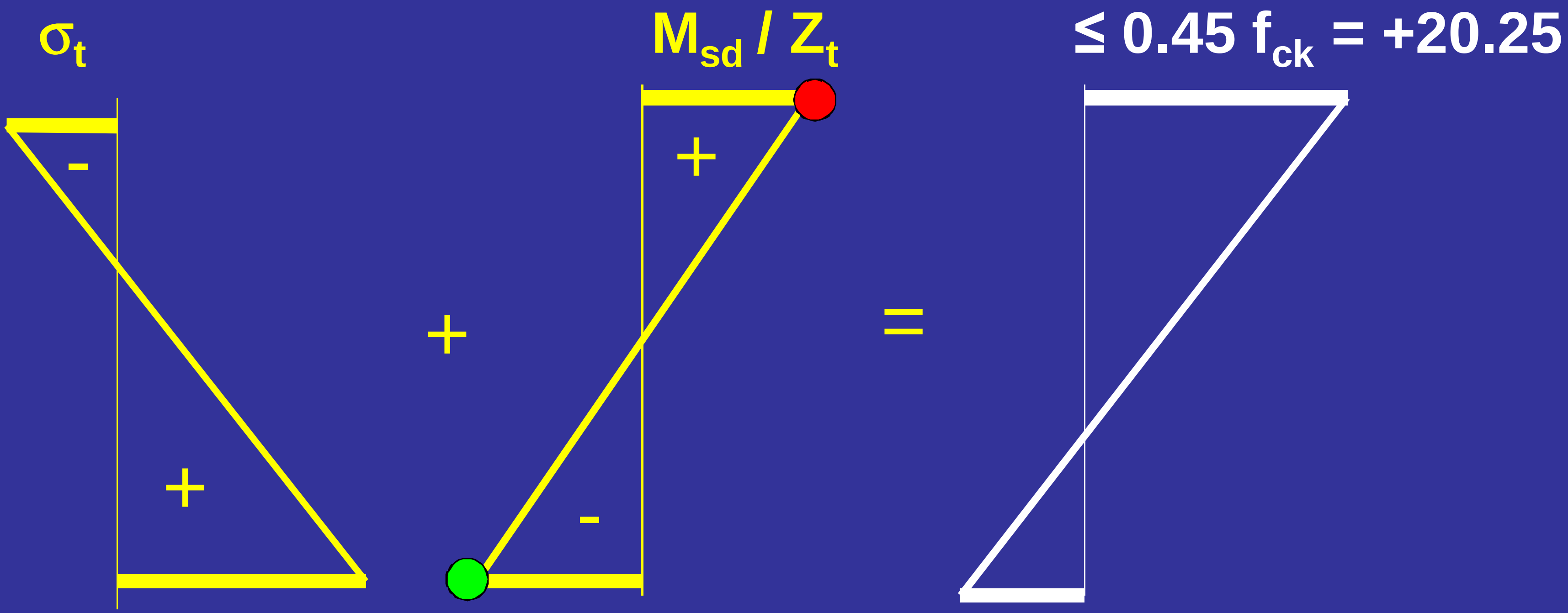
Adding prestress to imposed service stress =





Service moment of resistance is lesser of

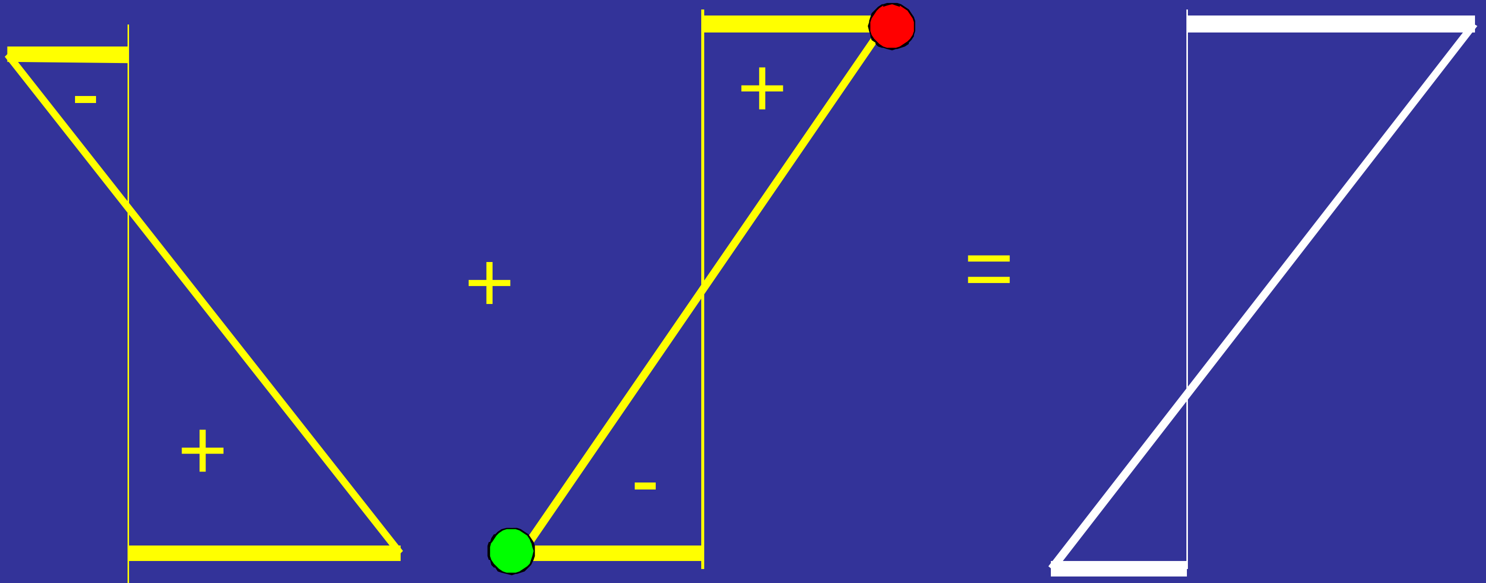
$M_{Sd} = (\sigma_t + 0.45f_{ck}) Z_t$



Service moment of resistance is lesser of

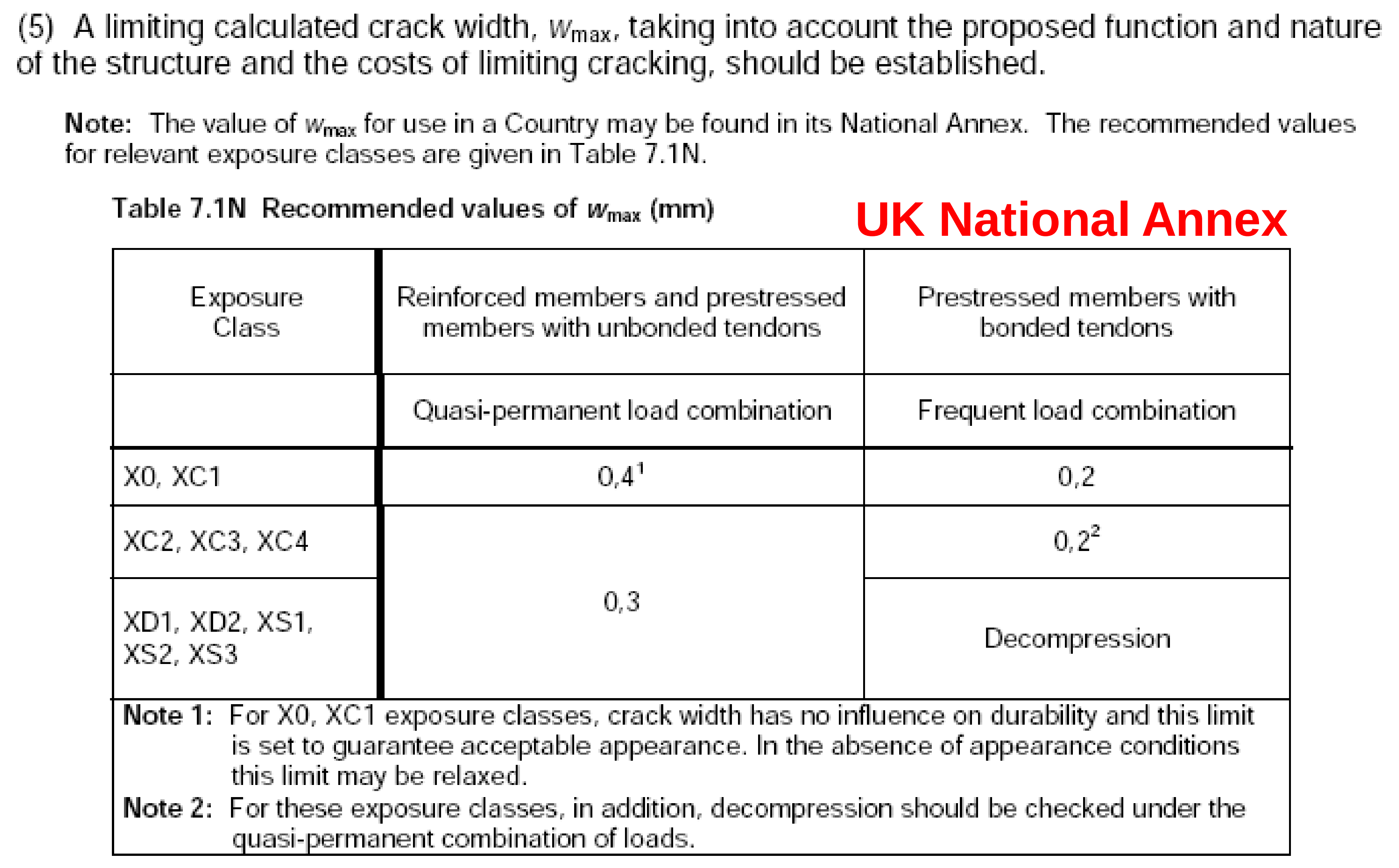
$M_{Sd} = (\sigma_t + 0.45f_{ck})\ Z_t$

$M_{Sd} = (\sigma_b + f_{ctm})\ Z_b$ Mostly critical



σ_b $M_{sd}\ /\ Z_b$ $\geq f_{ctm} = -3.80\ N/mm^2$

But for exposure > XC1, permissible tension f_{ctm} is reduced (depending on each country)



(5) A limiting calculated crack width, w_{max}, taking into account the proposed function and nature of the structure and the costs of limiting cracking, should be established.

Note: The value of w_{max} for use in a Country may be found in its National Annex. The recommended values for relevant exposure classes are given in Table 7.1N.

Table 7.1N Recommended values of w_{max} (mm)

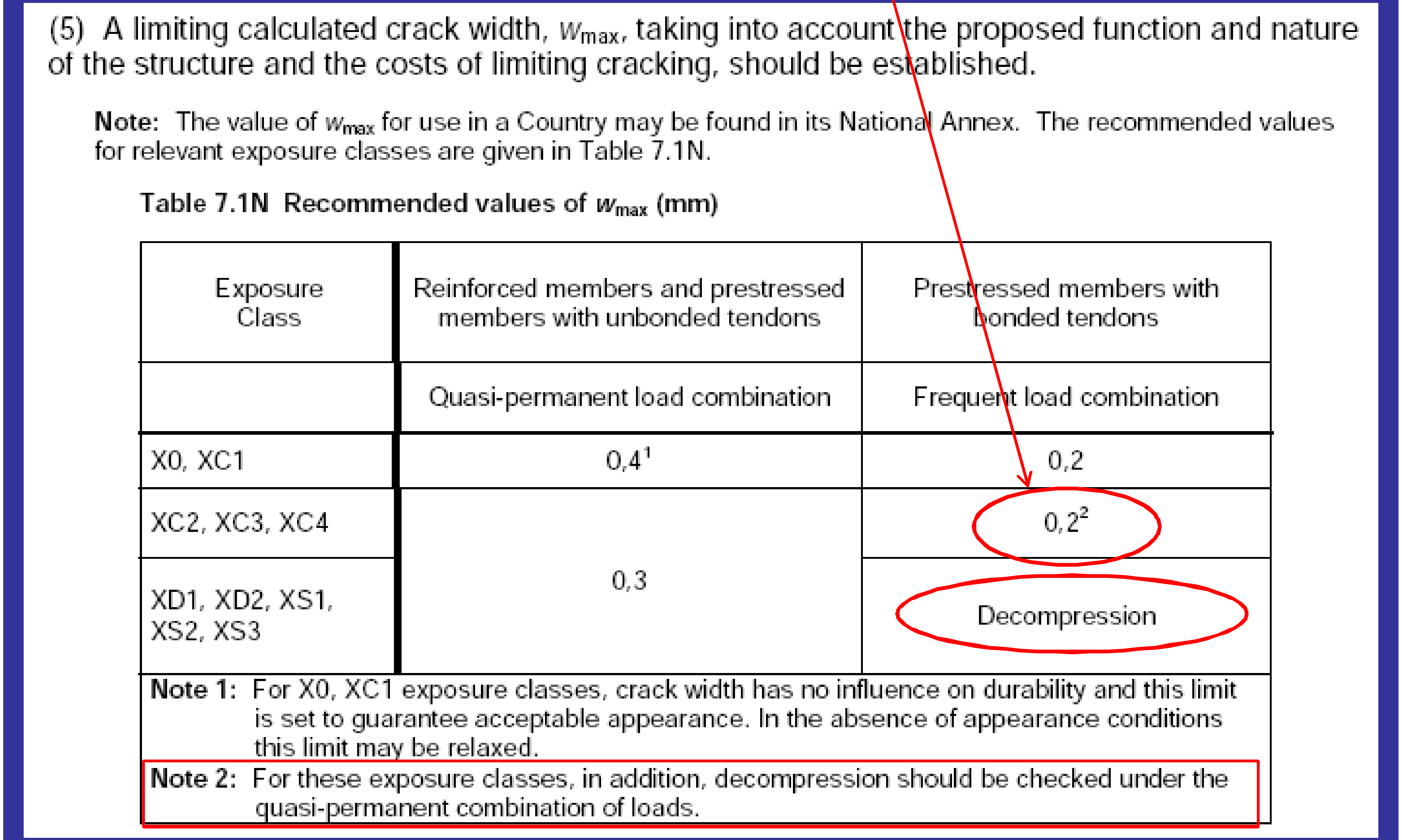
UK National Annex

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	$0{,}4^1$	0,2
XC2, XC3, XC4	0,3	$0{,}2^2$
XD1, XD2, XS1, XS2, XS3		Decompression

Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.

Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

For Class XC2-XC4 (external) no tension allowed. But can use quasi-permanent live load x ψ_2



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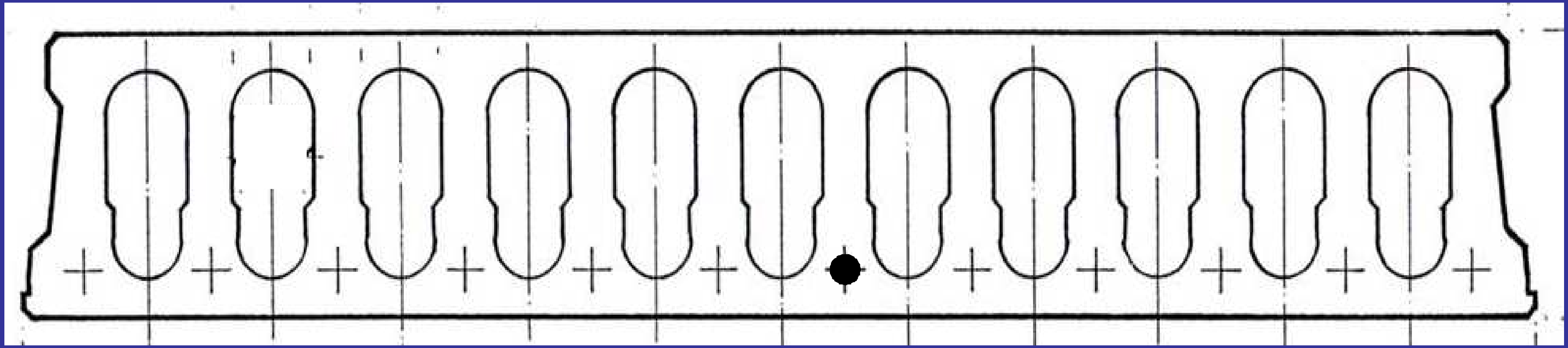
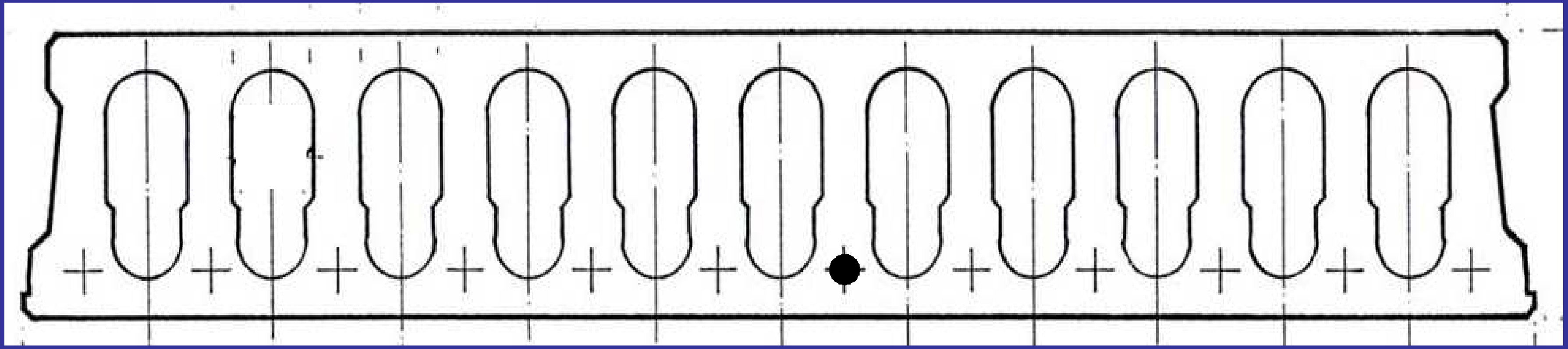
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4[1]	0,2
XC2, XC3, XC4	0,3	0,2[2]
XD1, XD2, XS1, XS2, XS3		Decompression

Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.

Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.



Worked example:



Worked example: XC1 exposure

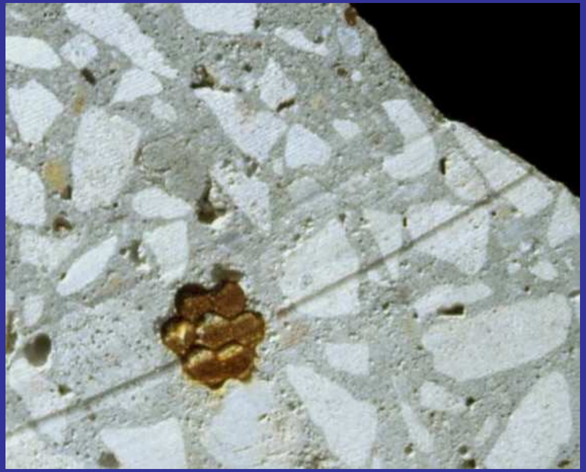
1200 wide x 250 depth

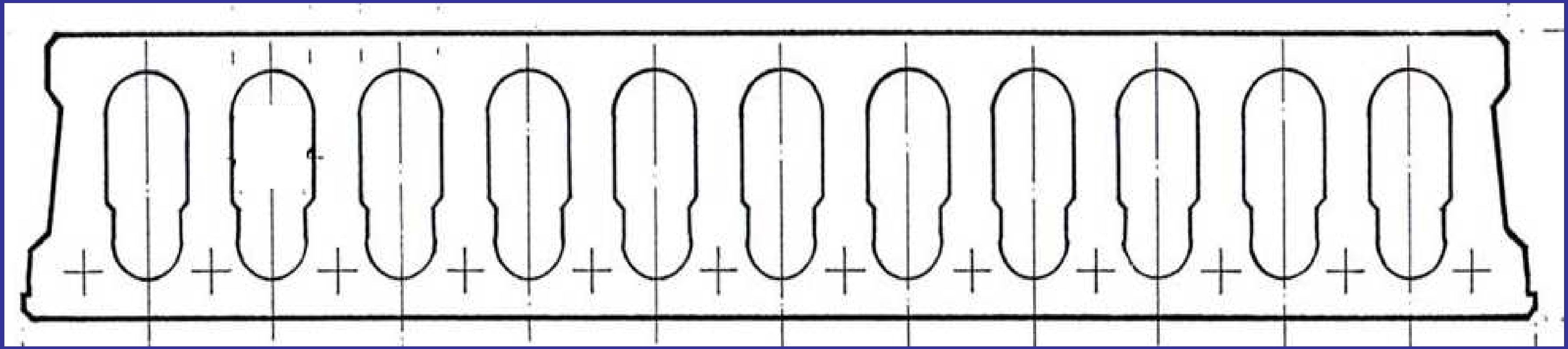
12 no. 9.3 mm strands at 35 mm cover

Axis a = 39.6 mm

f_{pk} = 1770 N/mm^2

Initial stressing to 70% = 1239 N/mm^2





A_c = 182791 mm^2

y_b = 122.4 mm

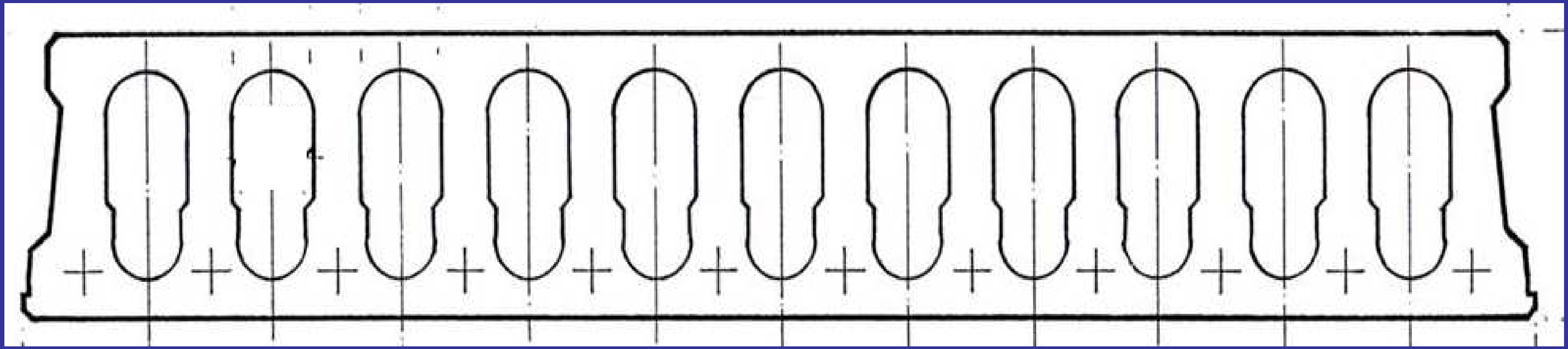
$I_{x\text{-}x}$ = 1270.3 x 10^6 mm^4

Z_b = 1270.3 / 122.4 = 10.378 x 10^6 mm^3

Z_T = 9.955 x 10^6 mm^3

A_{ps} = 12 x 52 = 624 mm^2

z_{cp} = 122.4 – 39.6 = 82.8 mm



Initial prestress P_{pi} = 624 x 1239 x 10^{-3} = 773.1 kN

Relaxation Class 2. 2.5% at 1000 hours

Immediate relaxation loss = 4.95 N/mm^2 (0.40%)

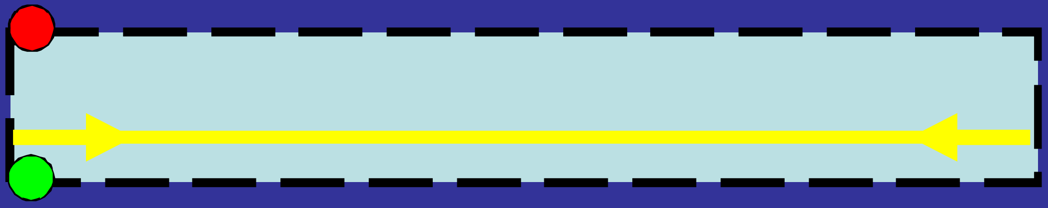
E_p (strand) 195 kN/mm^2

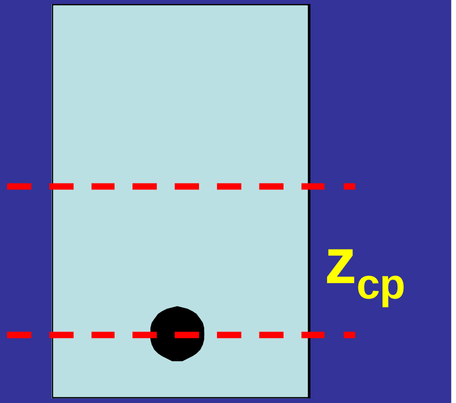
$E_{cm}(t)$ (gravel aggregate) = 32.8 kN/mm^2

Elastic shortening loss = 49.67 N/mm^2 (4.01%)

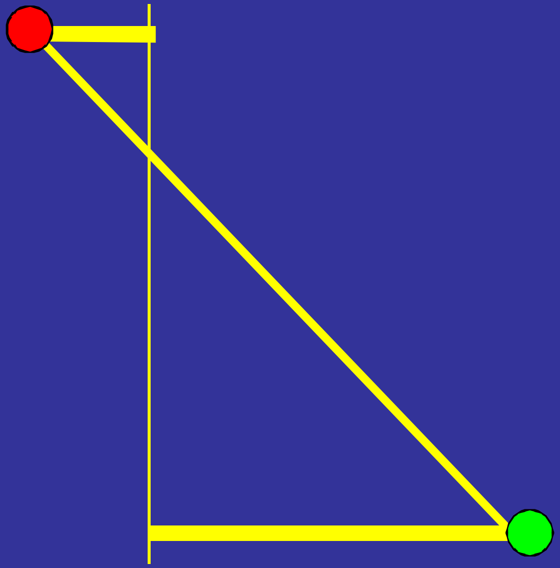
Remaining force at transfer

P_{pm0} = 739.0 kN (4.41% loss)





$\sigma_{t(t)}$ = - 2.10 ≥ – 2.72 N/mm^2



$\sigma_{b(t)}$ = 9.94 ≤ 18.0 N/mm^2 OK

Further loss of prestress at installation at (say) 28 days:

RH at transfer = 70%

Creep coefficient at 28 days = 0.84

Creep loss of stress at 28 days = 34.0 N/mm^2 (2.74%)

P_{pmi} = 717.8 kN (this is used later to determine camber after installation)

Final loss of prestress at 500,000 hours (57 years):

RH at service (indoor exposure) = 50%

Creep coefficient from installation to life = 1.60

Creep loss of stress = 61.8 N/mm^2 (4.99%)

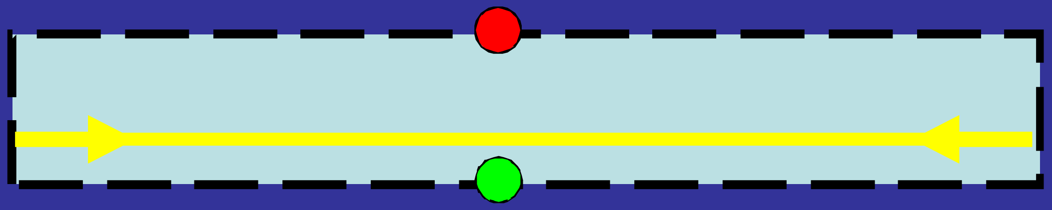
Shrinkage ε_{sh} = 420 x 10^{-6}

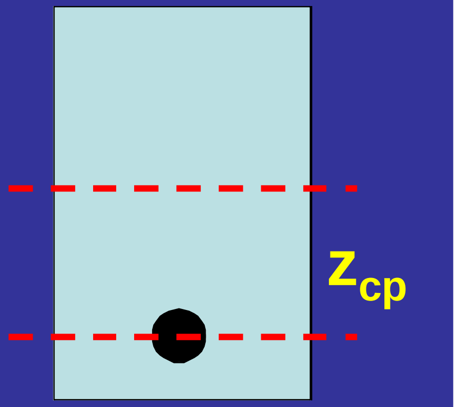
Shrinkage loss = 74.7 N/mm^2 (6.03%)

Long term relaxation loss = 40.3 N/mm^2 (3.25%)

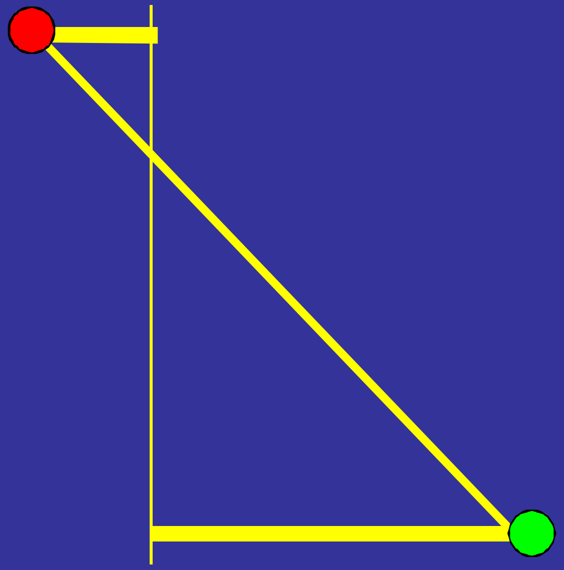
After all losses

P_{p0} = 614.3 kN (20.5% loss)



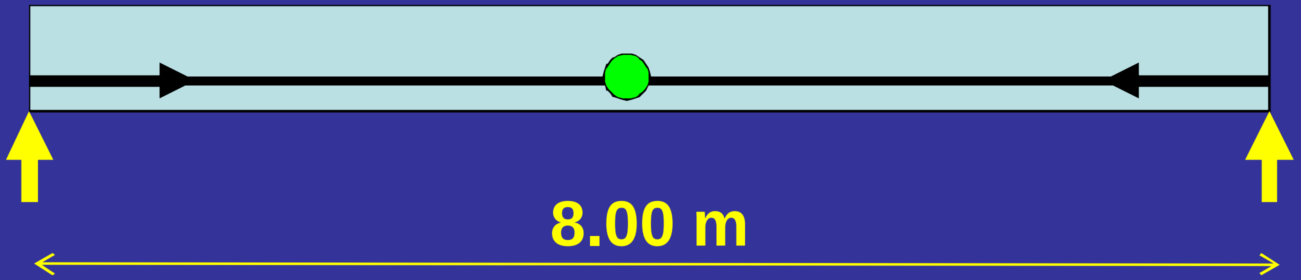


σ_t = - 1.746 N/mm^2

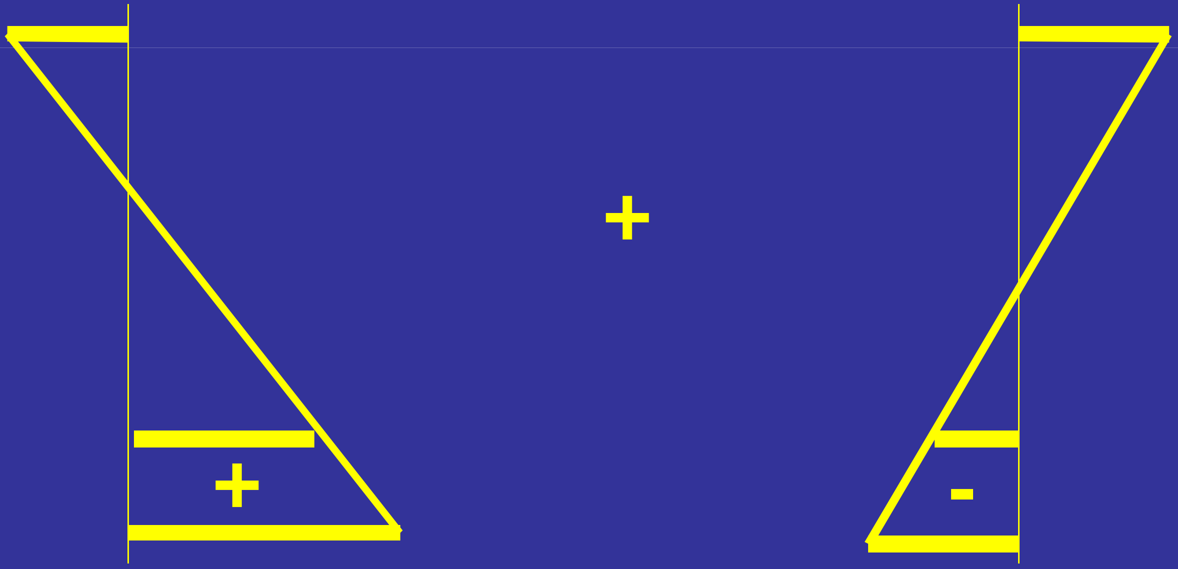


σ_b = 8.26 N/mm^2

But ! creep losses may be reduced by reversal of prestress due to self weight and dead loads



Self weight M = 26.87 kNm



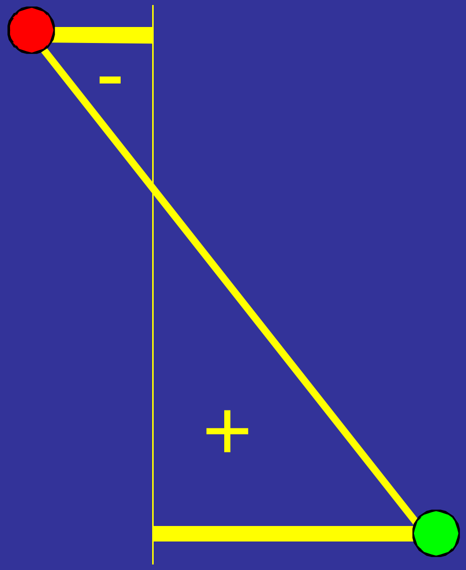
Creep loss at support
= 95.8 N/mm^2

Creep loss at mid-span
= 63.4 N/mm^2

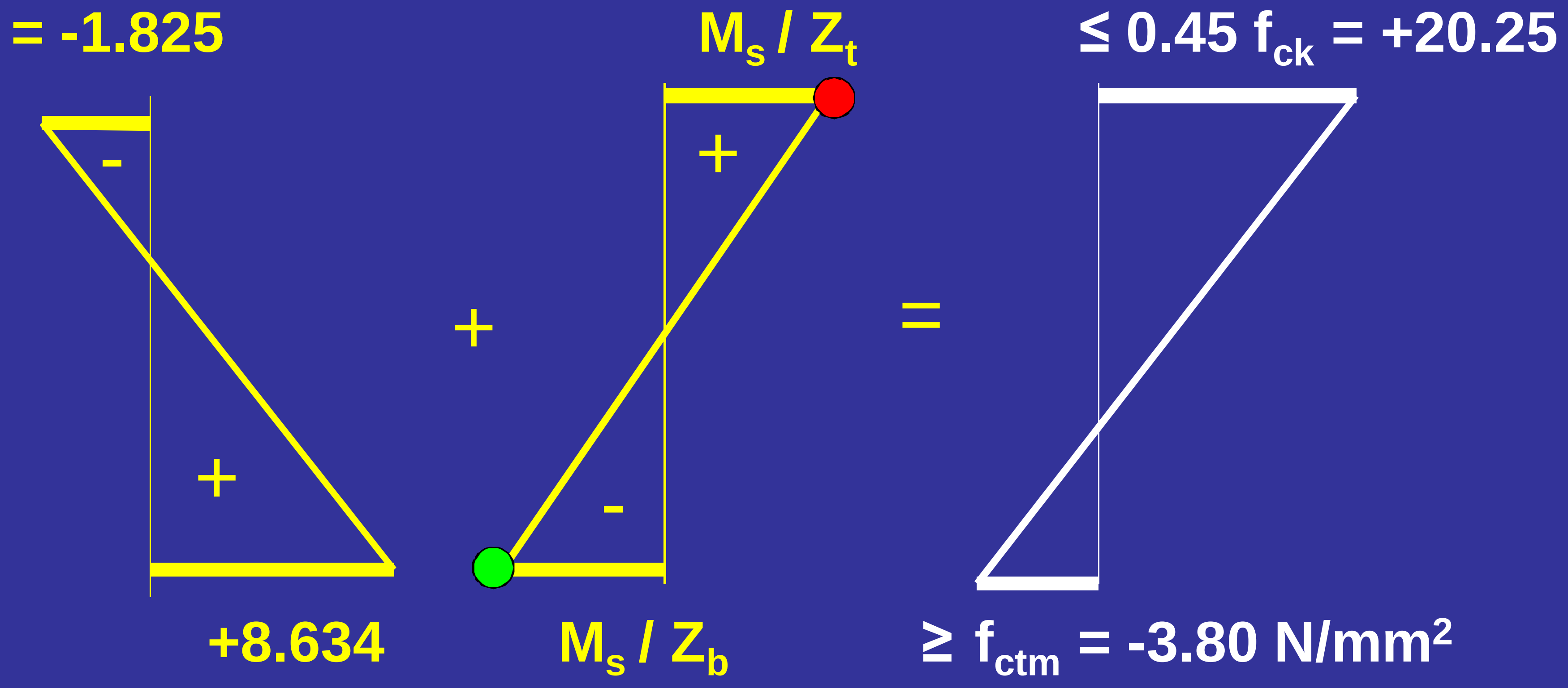
Final σ_b = 8.634 N/mm^2 (increase of 4.5%)

σ_t = -1.825 N/mm^2

= -1.825



+8.634



$= -1.825$

M_s / Z_t

$\leq 0.45\ f_{ck} = +20.25$

-

+

+

+

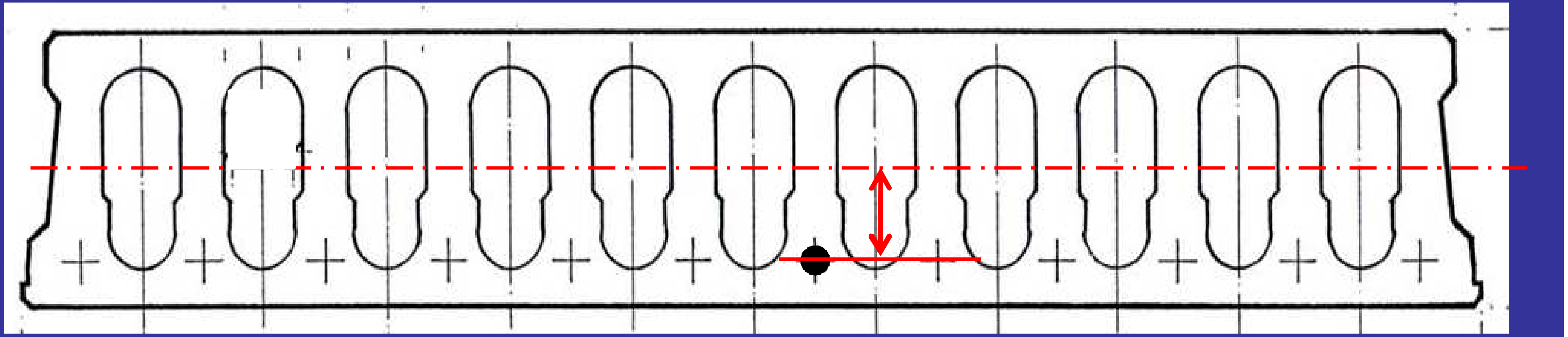
-

=

$+8.634$

M_s / Z_b

$\geq f_{ctm} = -3.80\ N/mm^2$



But ! compound values may be used for I_{xx}, Z_b and Z_t based on the *transformed area* of the strands:

$m = E_p / E_{cm} = (195 / 36.3) -1 = 4.37$

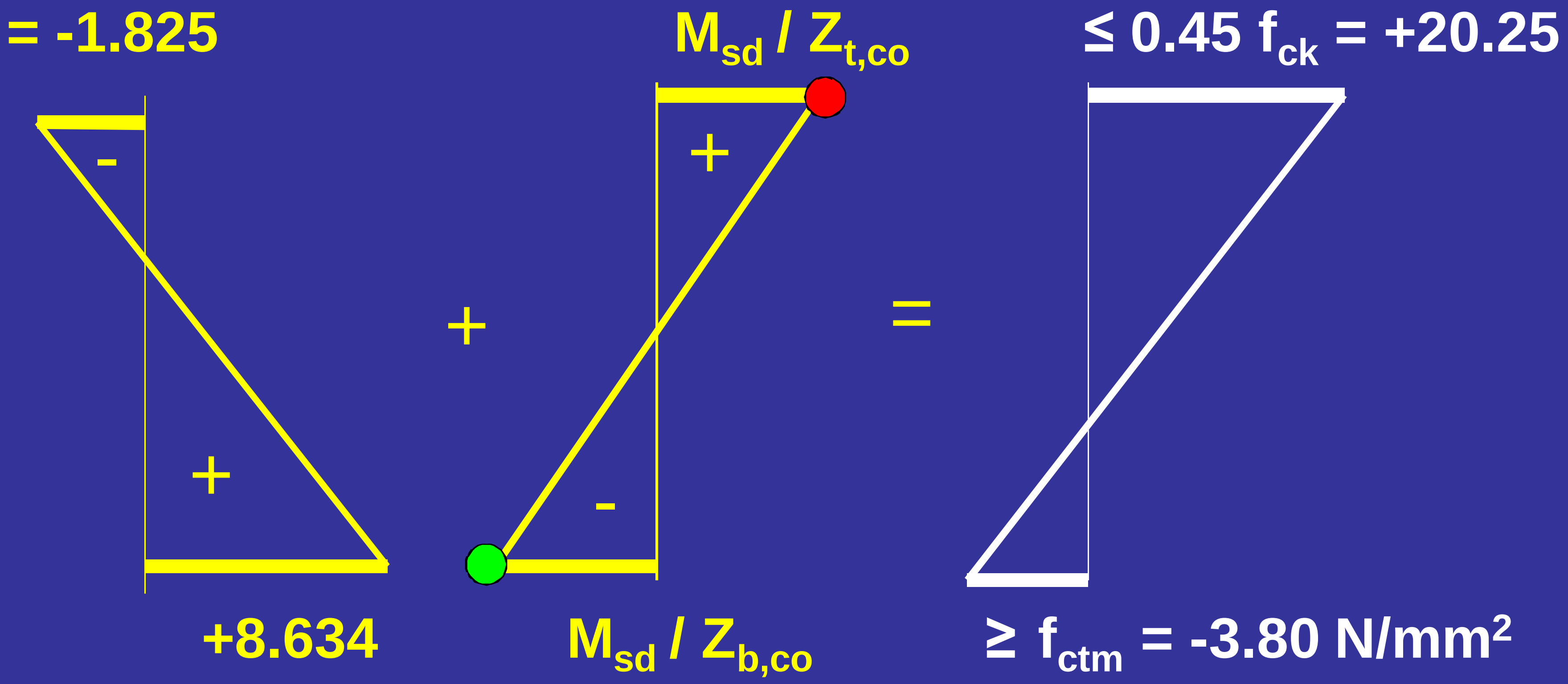
Then $I_{xx} = 1289 \times 10^6$ mm^4

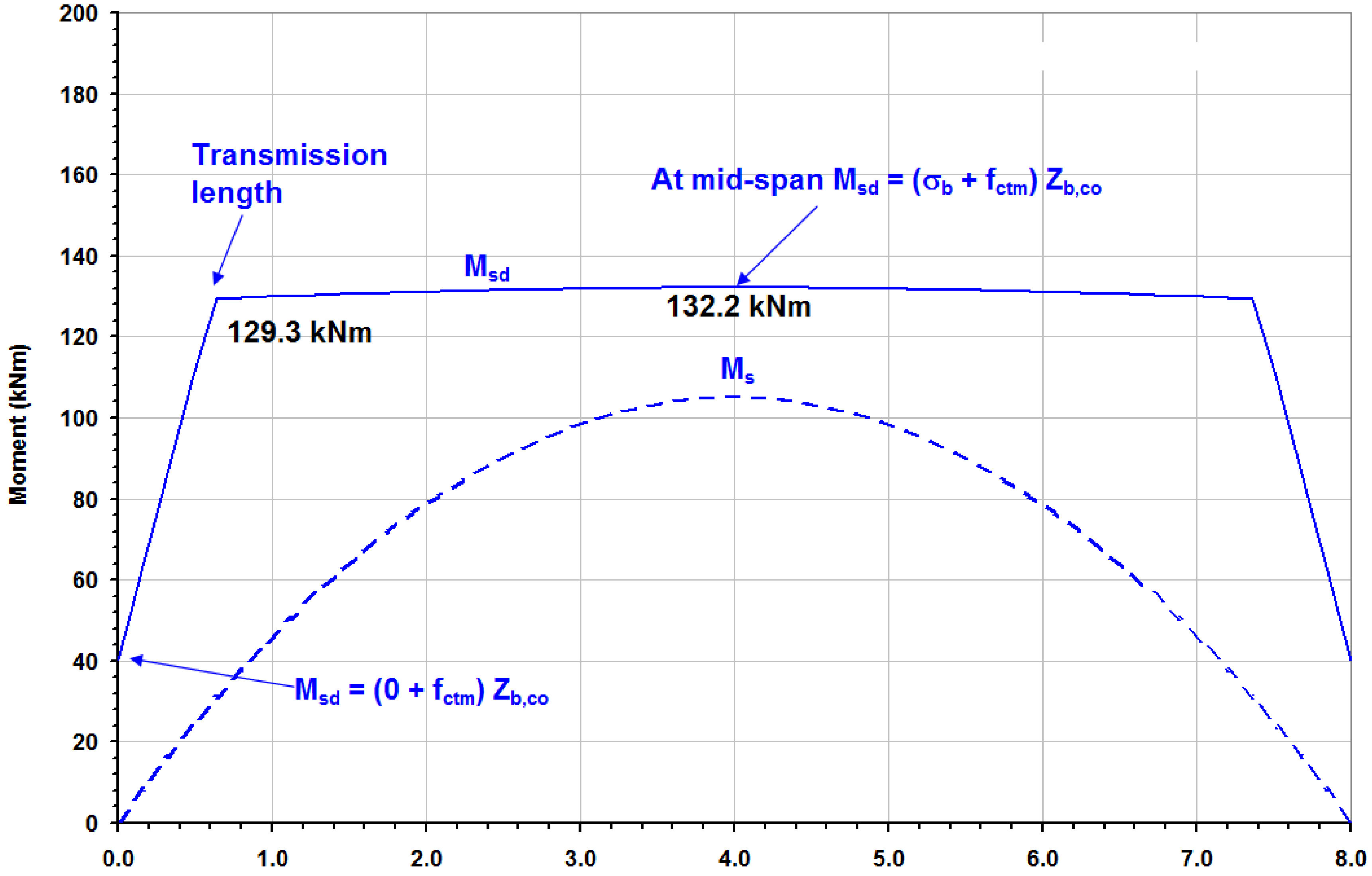
$Z_{b,co} = 10.634 \times 10^6$ mm^3 (increase of 2.5%)

Service moment of resistance is lesser of

M_{Sd} = (20.25 + 1.825) x 10.004 = 220.8 kNm

M_{Sd} = (8.634 + 3.80) x 10.634 = 132.2 kNm





Transmission length
At mid-span $M_{sd} = (\sigma_b + f_{ctm}) Z_{b,co}$
M_{sd}
129.3 kNm
132.2 kNm
M_s
$M_{sd} = (0 + f_{ctm}) Z_{b,co}$
Moment (kNm)
200
180
160
140
120
100
80
60
40
20
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

Syllabus

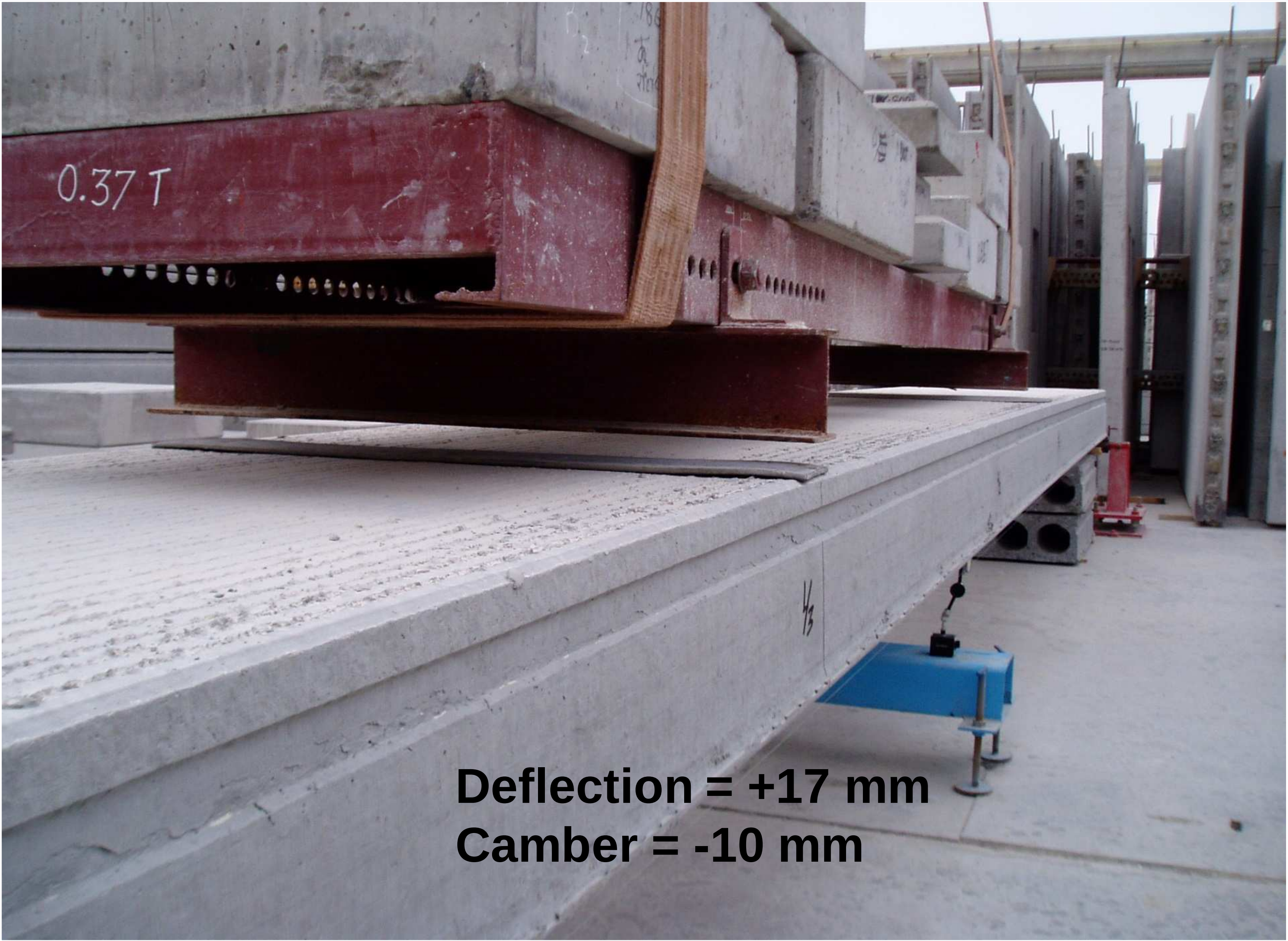
Ultimate strength.
Equilibrium.
Compatibility.
M_{Rd}.

4 point bending test of prestressed hollow core slab.

Initial camber = -27 mm

1200 x 320 deep x 11.0 m span

Self + imposed M_{Ed} = service moment of resistance



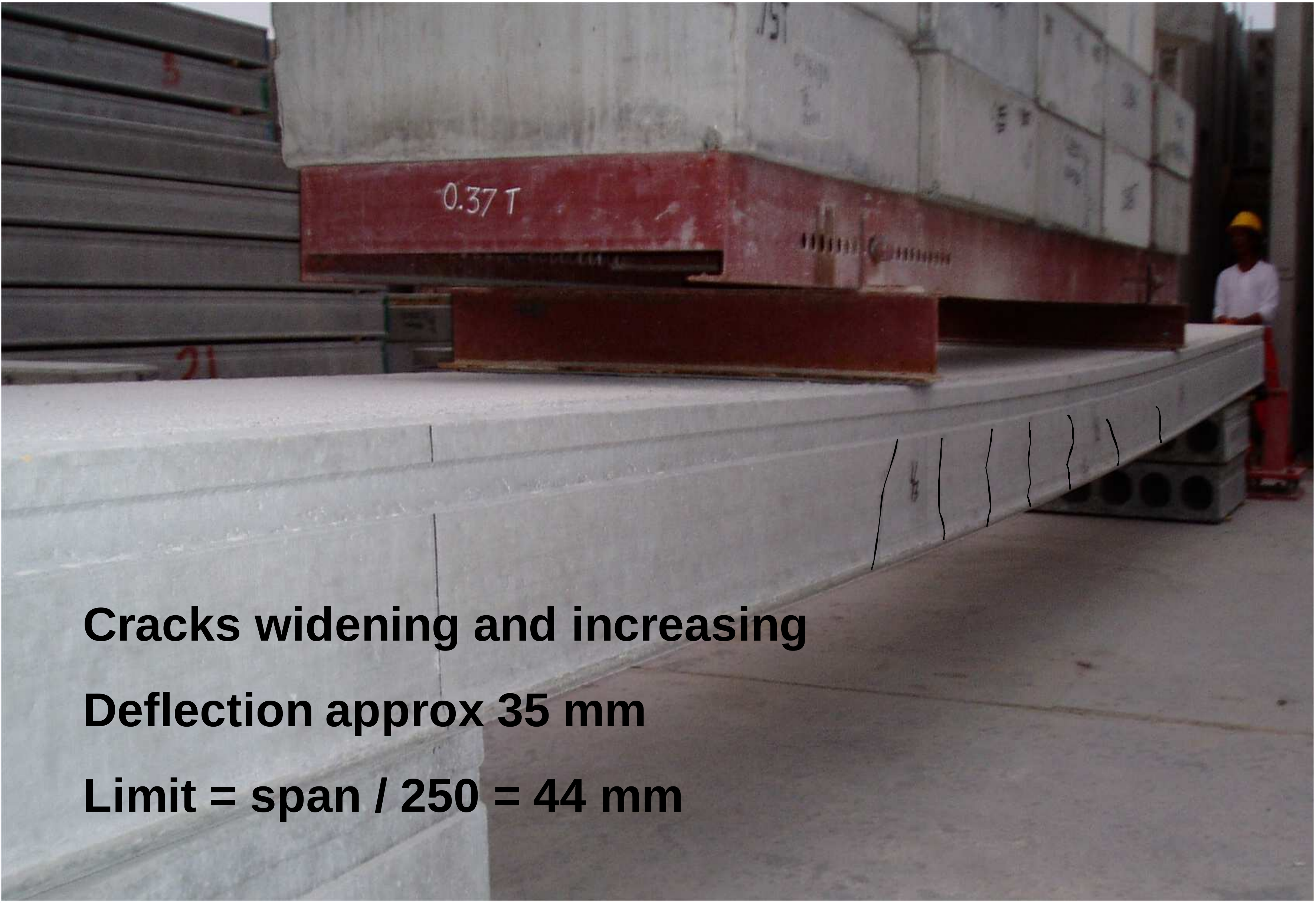
Deflection = +17 mm
Camber = -10 mm

Self + imposed $M_{Ed} = M_{Rd}$ (ultimate resistance)

First cracking

Deflection approx 25 mm

Self + imposed M_{Ed} = 1.25 M_{Rd} (ultimate load + 25%)

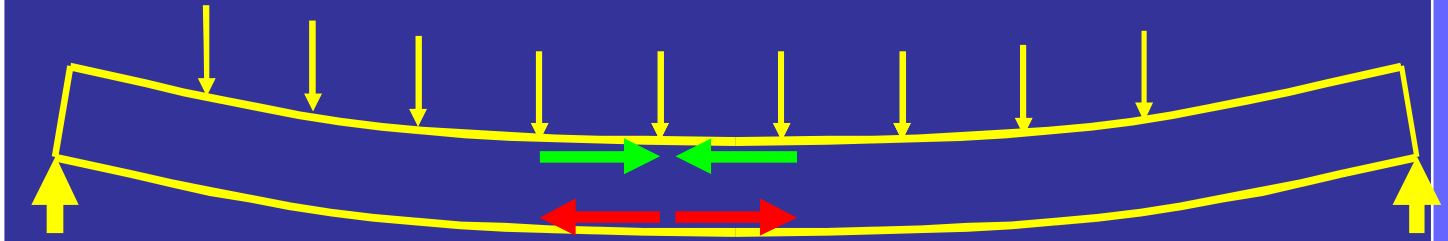


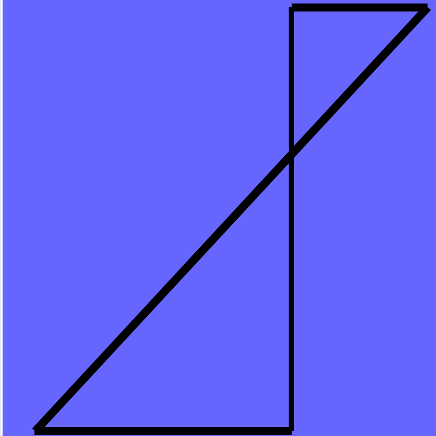
Cracks widening and increasing

Deflection approx 35 mm

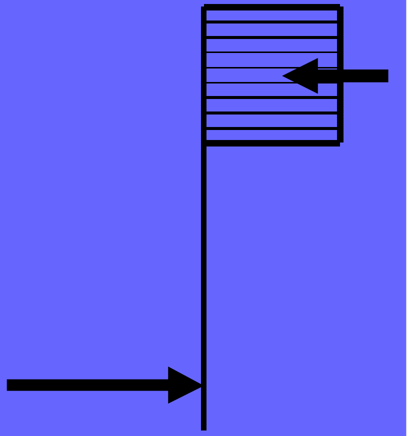
Limit = span / 250 = 44 mm

Ultimate moment of resistance



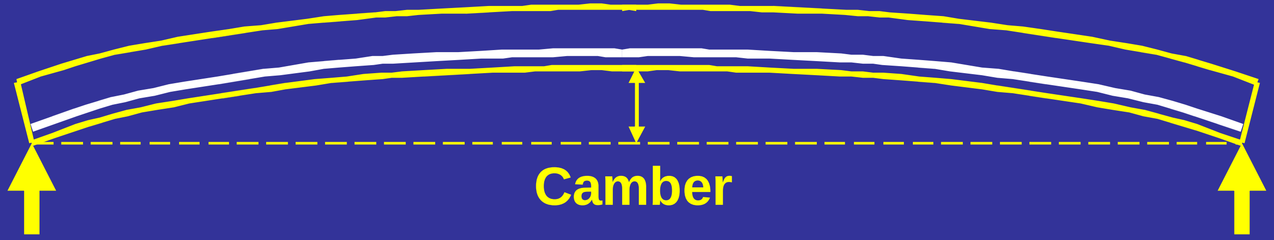


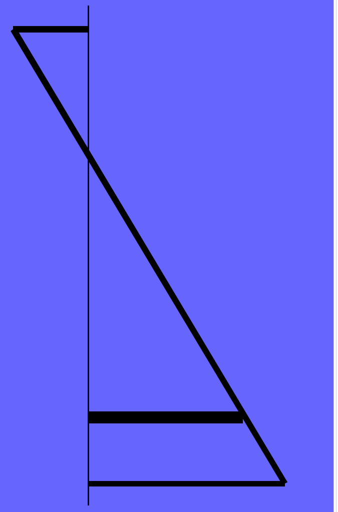
Strains



Stress

Strain development from initial prestress to ultimate

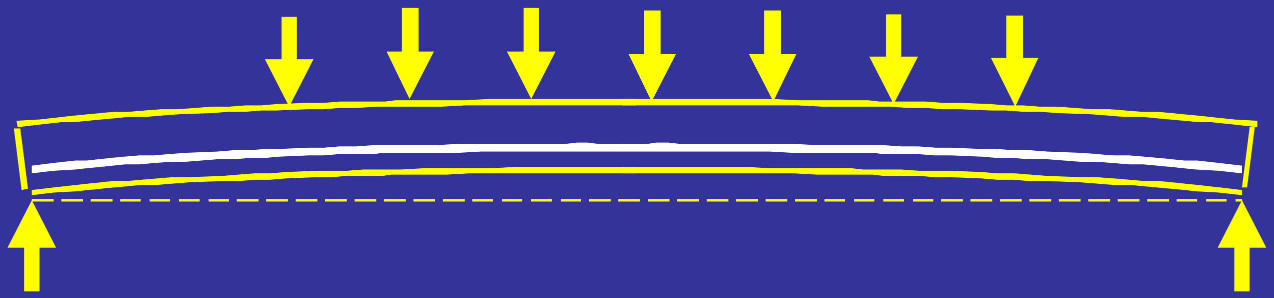


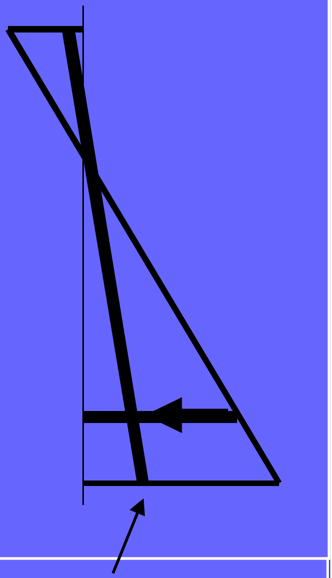


First is the pre-strain due to final prestress after all losses =

$$\varepsilon_{po} = \sigma_{po} / E_p$$

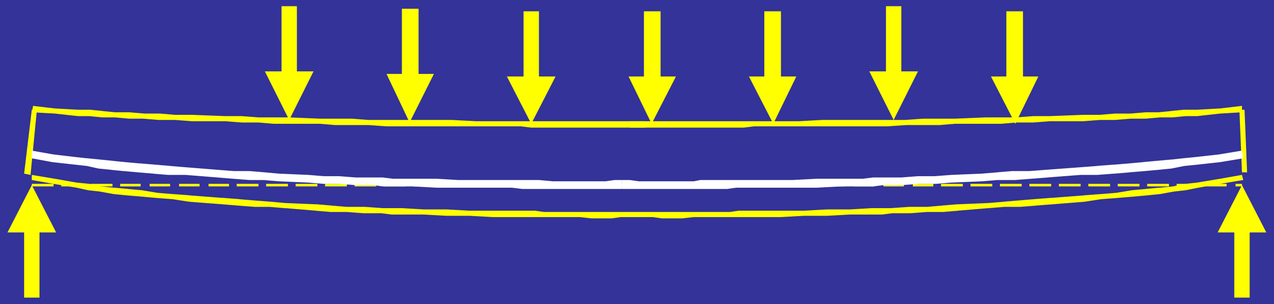
Strain development from initial prestress to ultimate

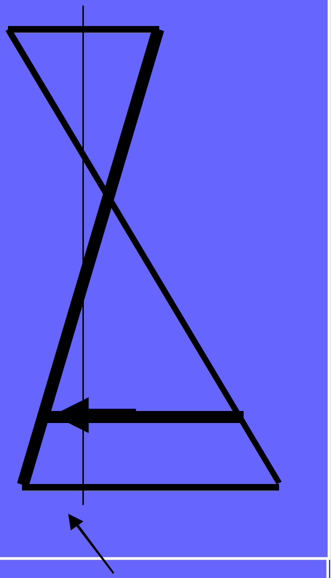




Bending strain added to pre-strain

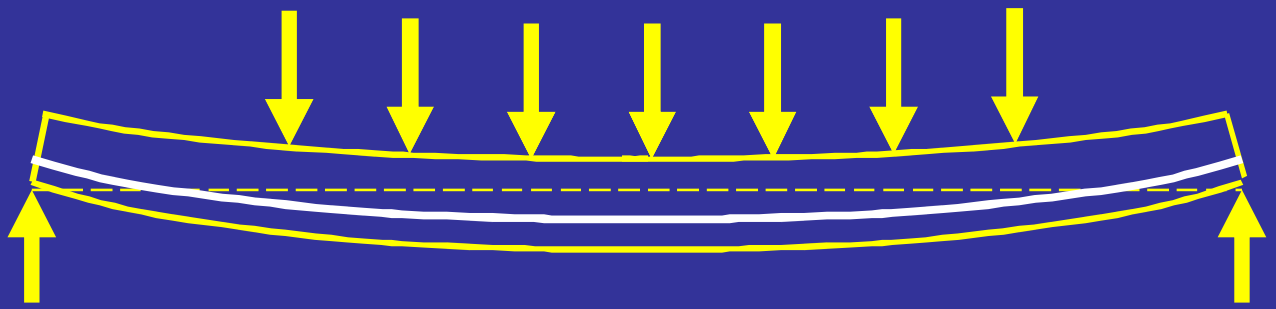
Strain development from initial prestress to ultimate

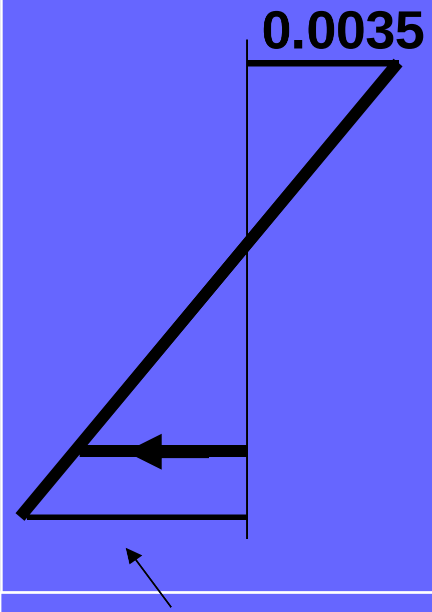




Bending strain now overtake the pre-strain

Strain development from initial prestress to ultimate



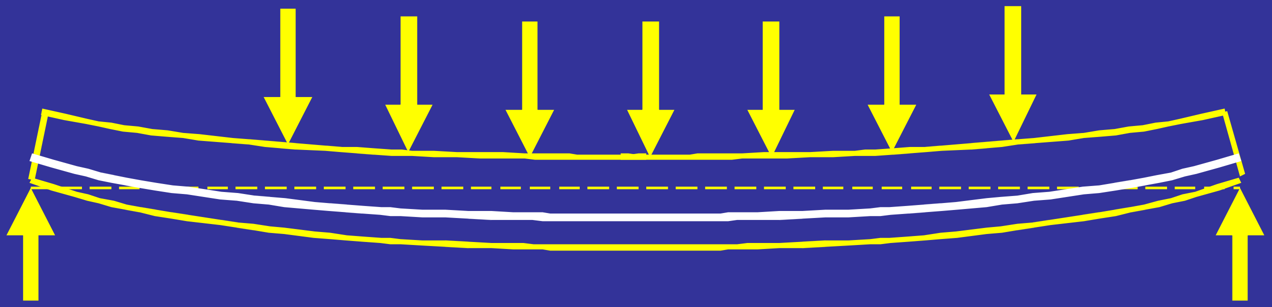


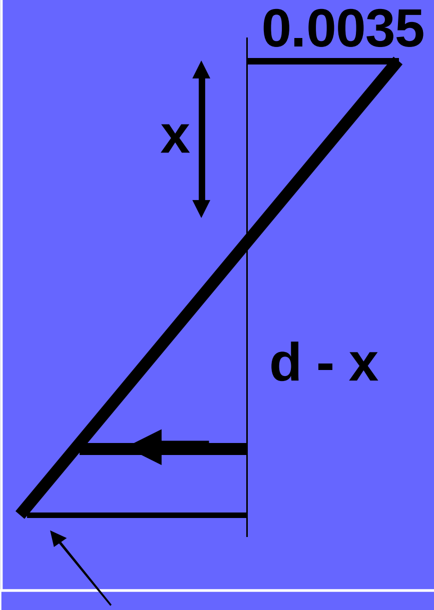
Final ultimate strains

ε_{cu} = 0.0035

$\varepsilon_{p} < \varepsilon_{ud}$ code value 0.02

Strain development from initial prestress to ultimate





Total strain

$\varepsilon_p = \varepsilon_{po} + 0.0035 (d - x) / x$

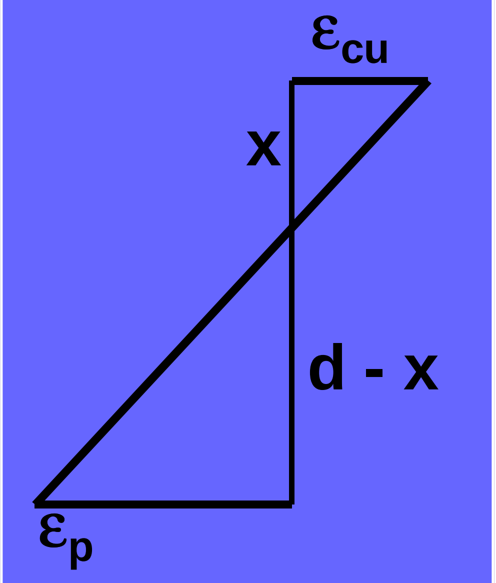
Now find x and the stresses

Final ultimate strains

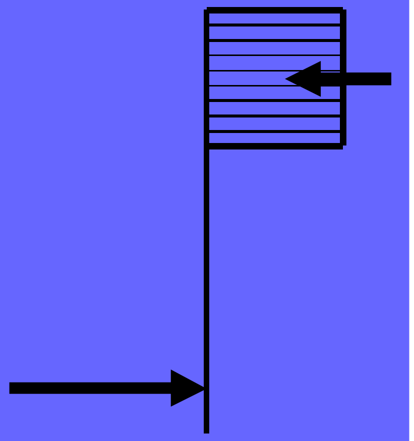
$\varepsilon_{cu} = 0.0035$

$\varepsilon_p < \varepsilon_{ud}$ code value 0.02

Constitutive relationship stress v strain



Strains



Stress

Constitutive relationship stress v strain

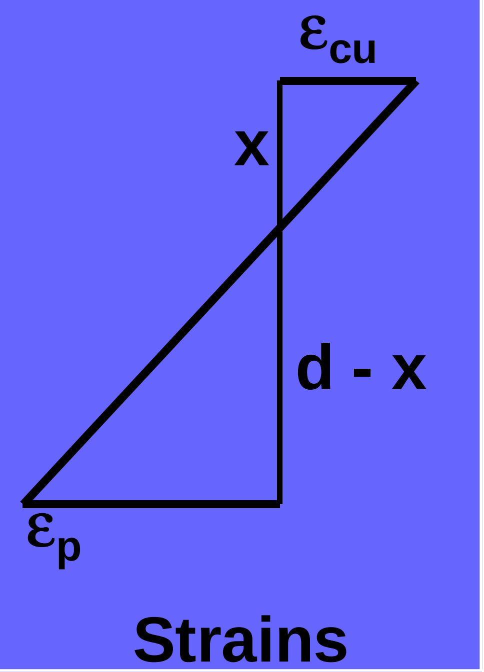
Equilibrium

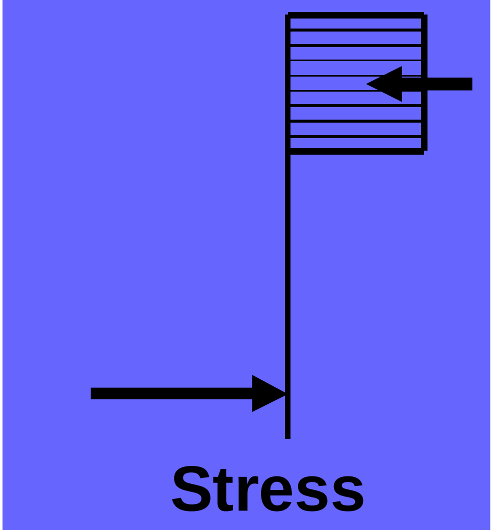
$F_c = F_s$

$0.567\ f_{ck}\ b\ 0.8\ x = f_p\ A_p$

and compatibility

$$\frac{x}{(d - x)} = \frac{\varepsilon_{cu}}{\varepsilon_p}$$

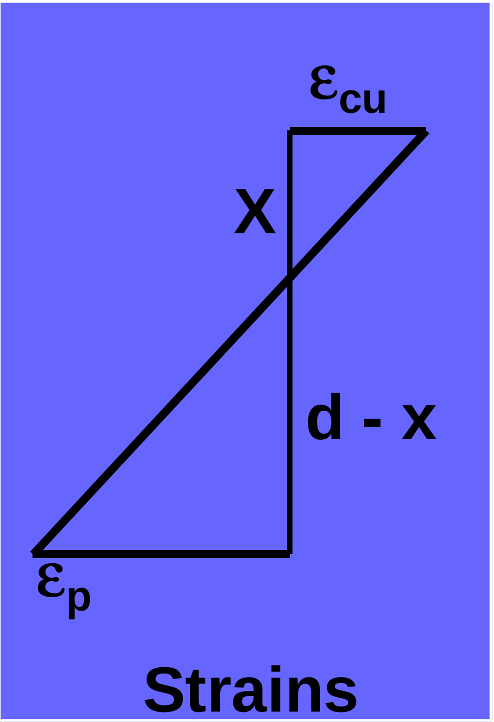


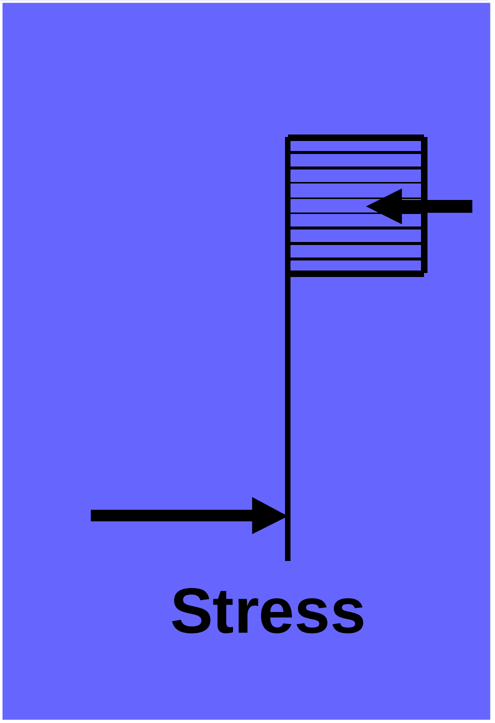


Combining ;

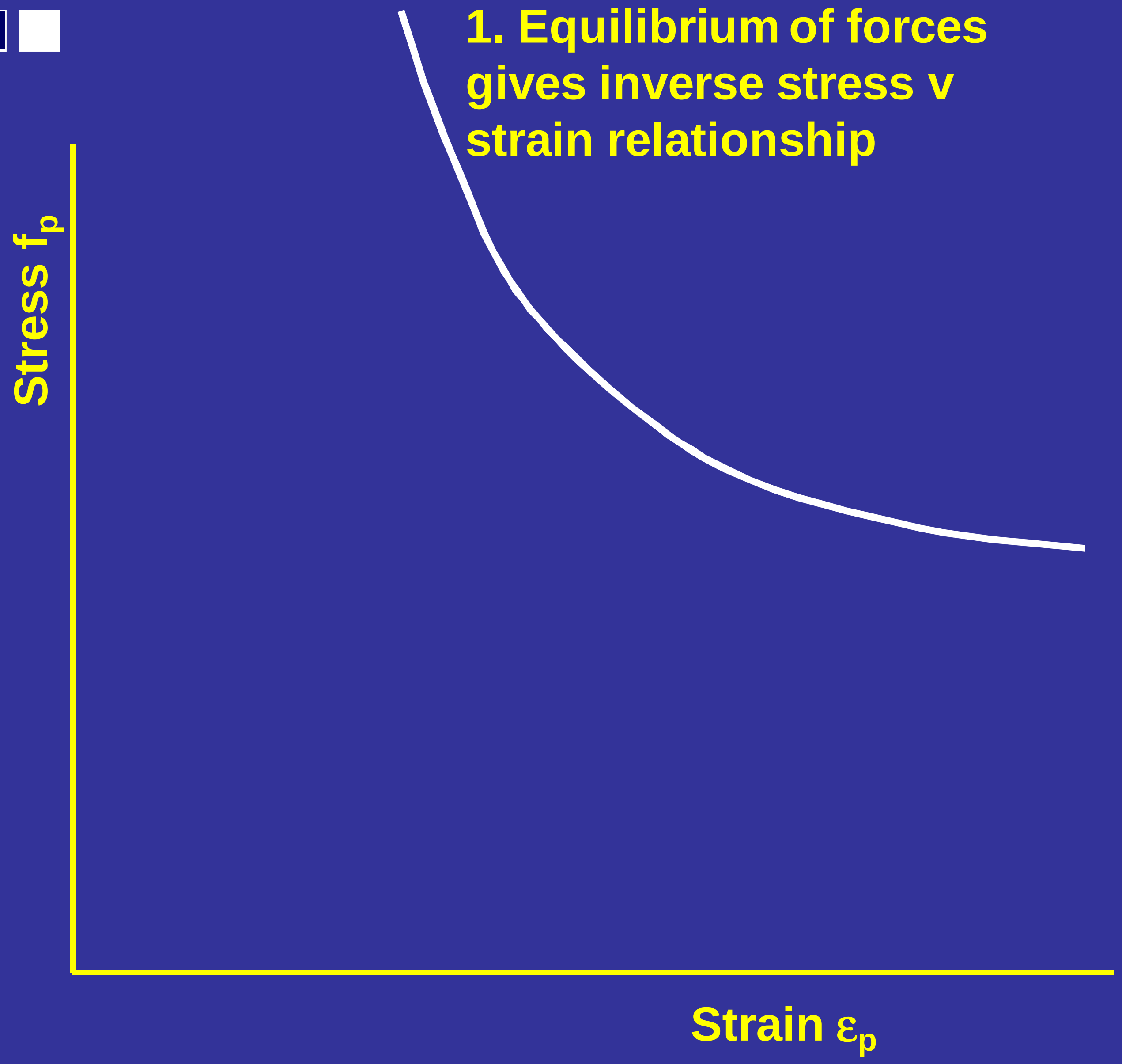
$f_p \;=\; 0.567\, f_{ck}\, b\, 0.8\, (d\text{-}x)\, \varepsilon_{cu} \,/\, A_p\, \varepsilon_p$

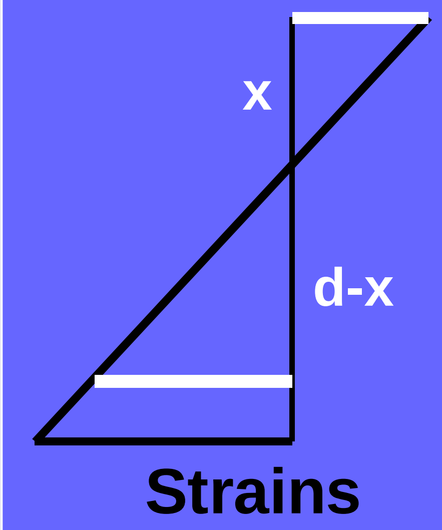
or stress = inverse of strain

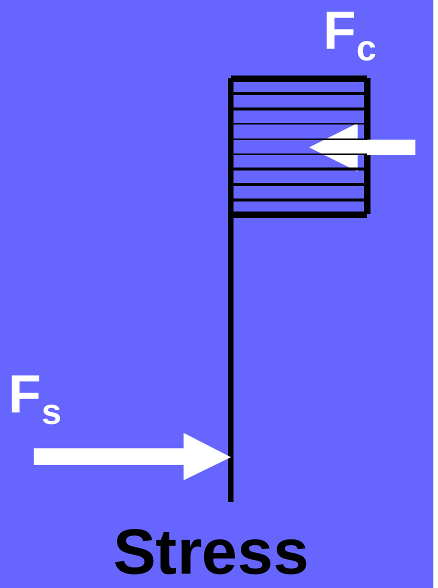




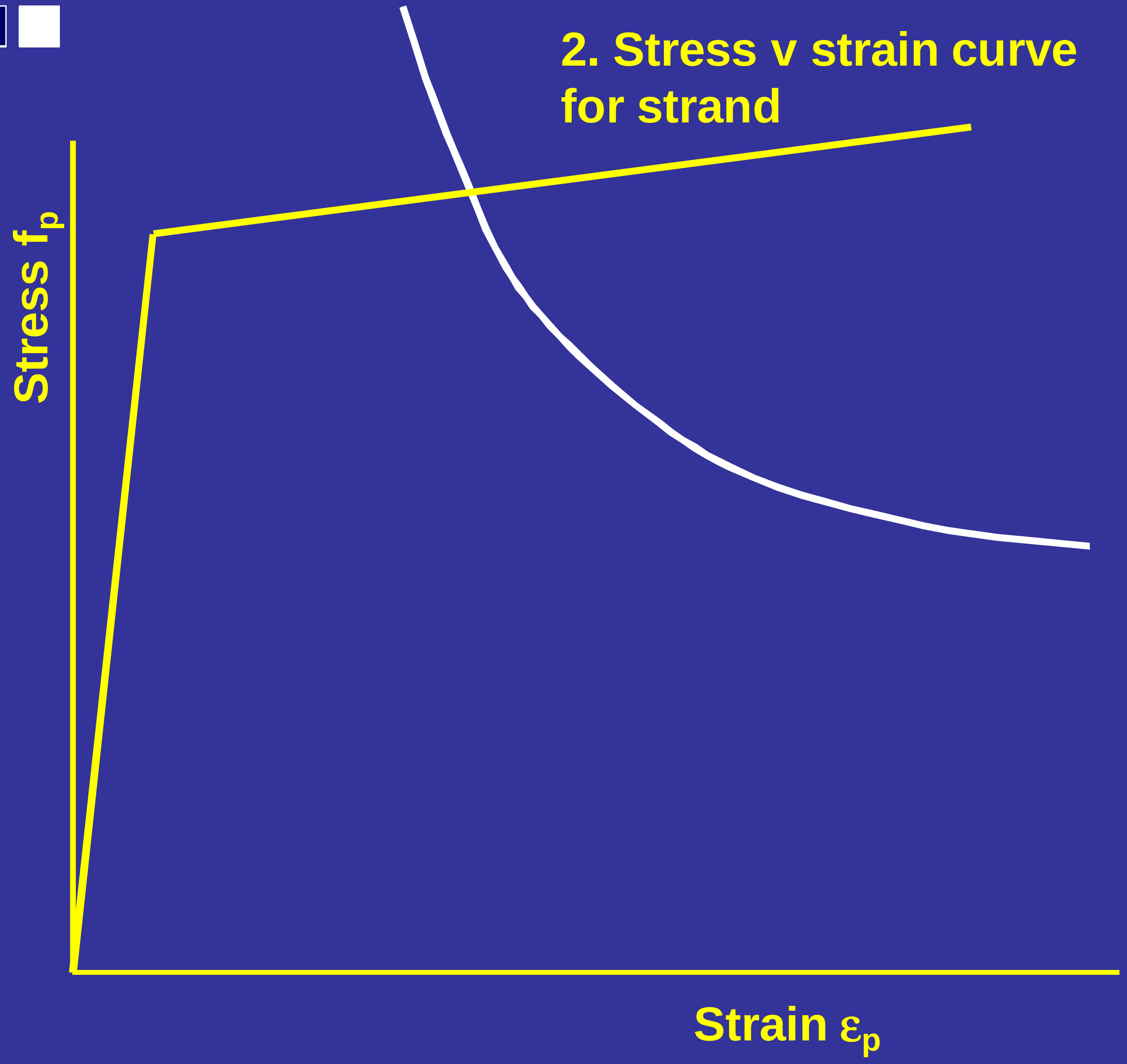
1. Equilibrium of forces gives inverse stress v strain relationship

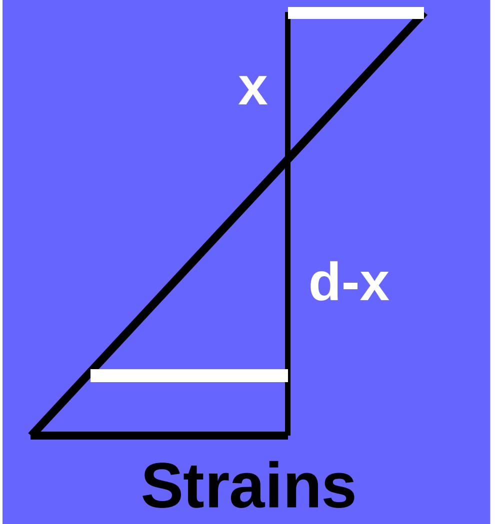


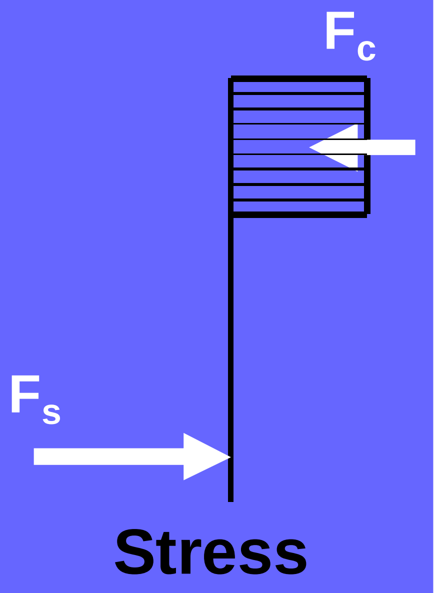




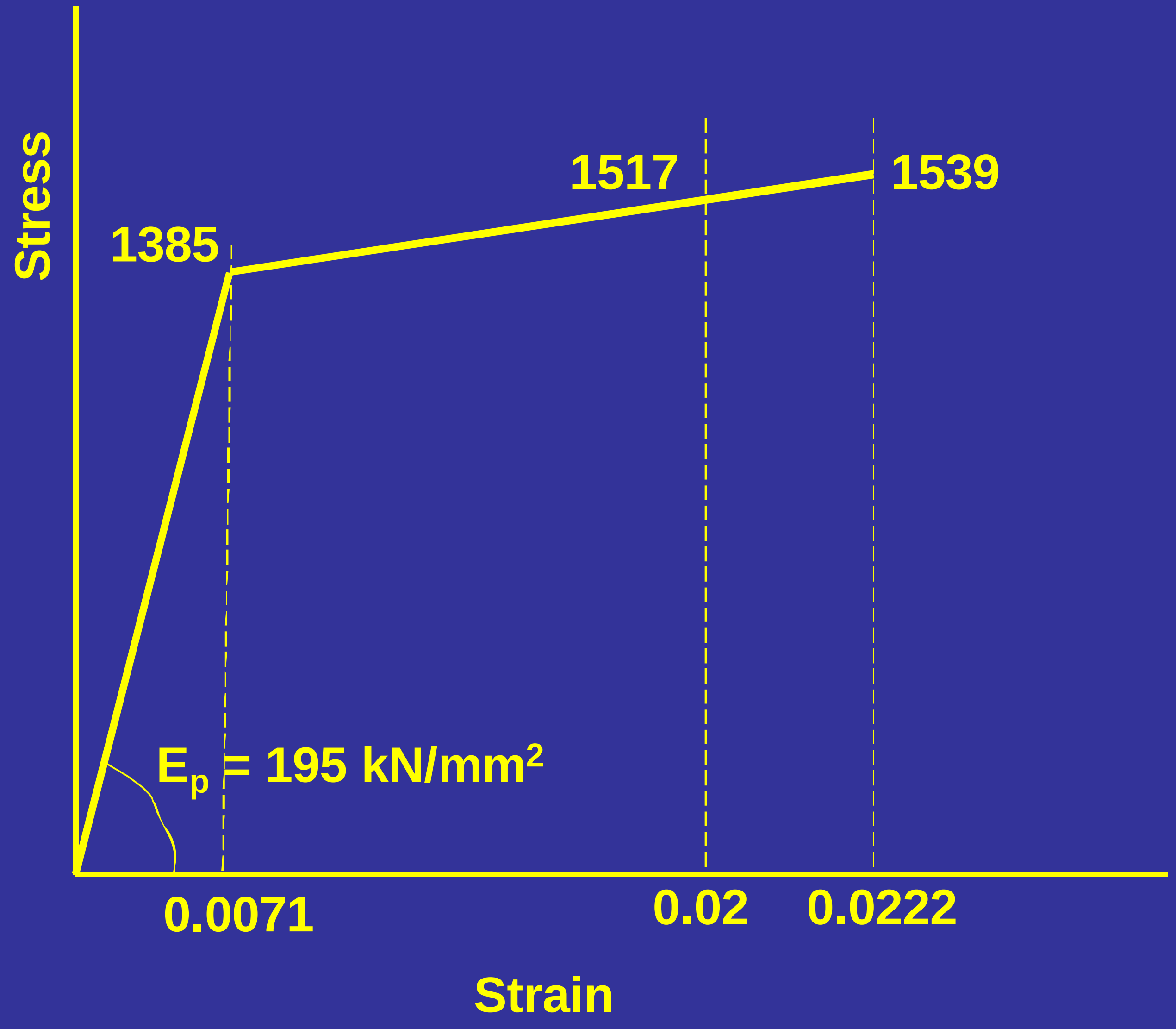
2. Stress v strain curve for strand



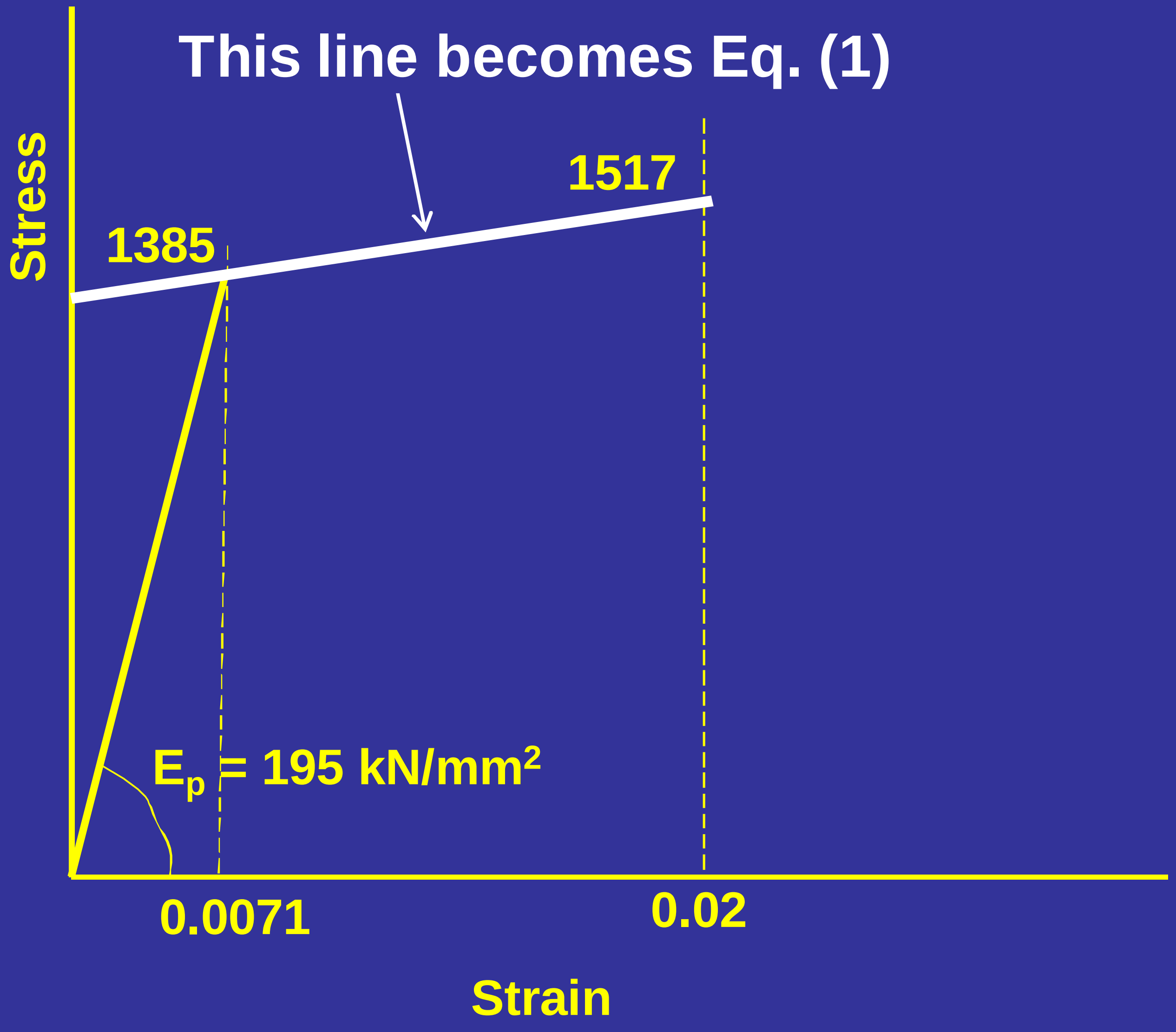




Stress v strain diagram, e.g. for strand with f_{pk} = 1770 N/mm^2



Stress v strain diagram, e.g. for strand with $f_{pk} = 1770$ N/mm^2



Total strain = pre-strain + compatibility concrete strain ε_p

$= \varepsilon_{po} + \varepsilon_{cu} (d / x - 1)$ …(2)

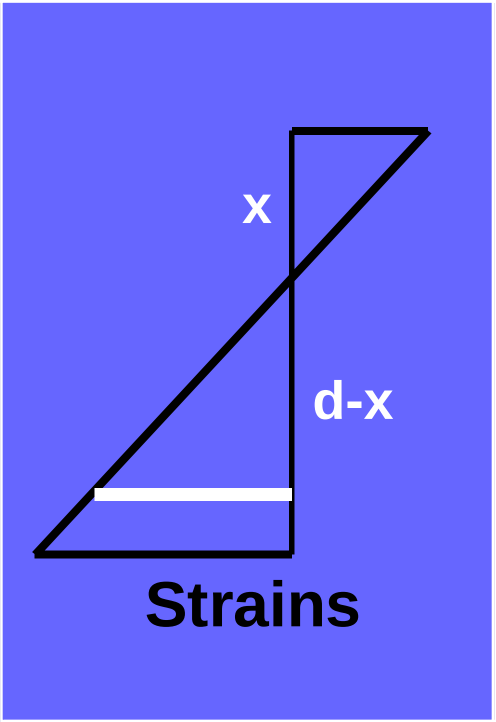
where, pre-strain after losses

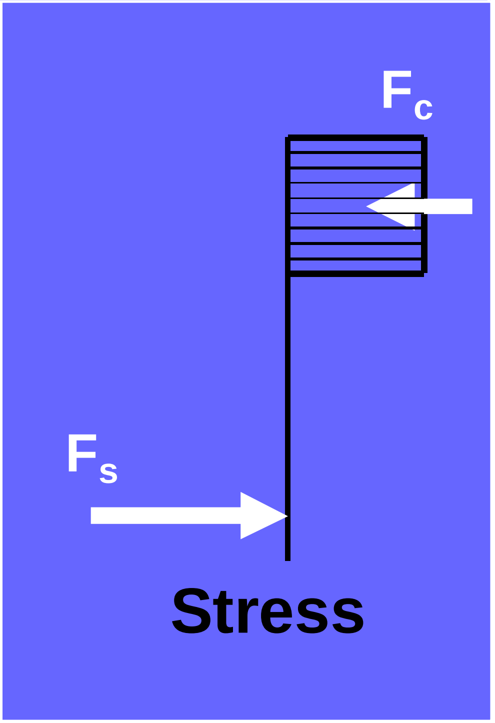
$\varepsilon_{po} = \sigma_{po} / E_p$

Force equilibrium

$F_s = F_c$

$f_p A_p = 0.567 f_{ck} b 0.8 x$ …(3)





Combining 3 equations gives the quadratic solution:

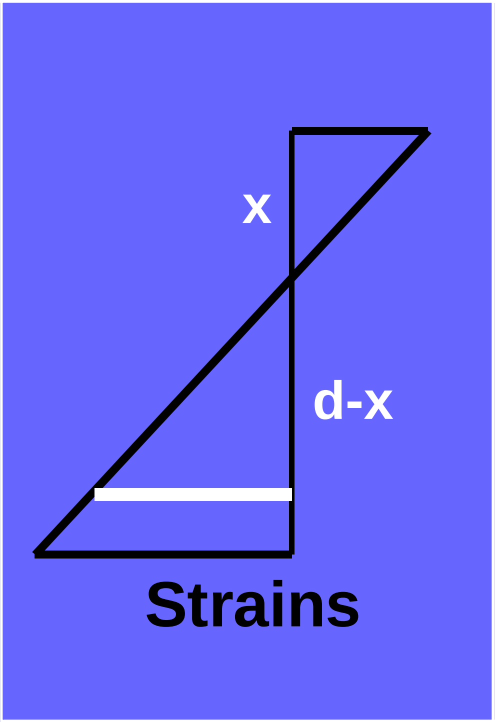
$$0.567\, f_{ck}\ 0.8\ b\ (\varepsilon_{uk} - \varepsilon_{LOP})\ x^2$$

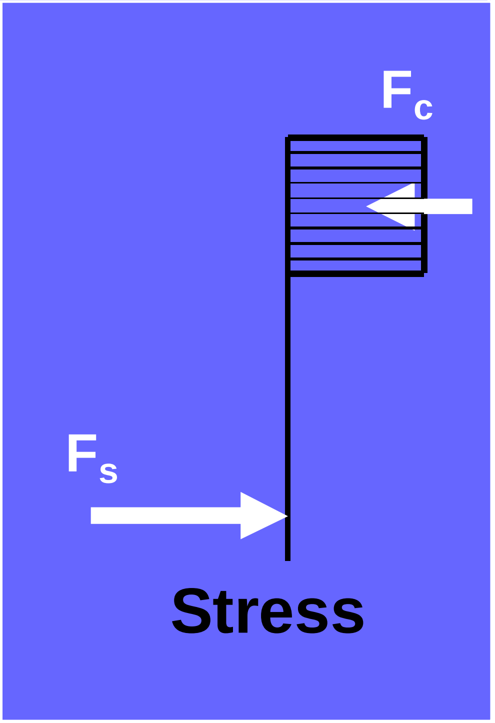
$$-[0.9(\varepsilon_{uk} - \varepsilon_{LOP}) + 0.1(\varepsilon_{po} - \varepsilon_{cu} - \varepsilon_{LOP})]\ A_p\ f_{pd}\ x$$

$$-0.1\varepsilon_{cu}\ d\ A_p\ f_{pd} = 0$$

Solving yields x

Then ε_p and f_p are found



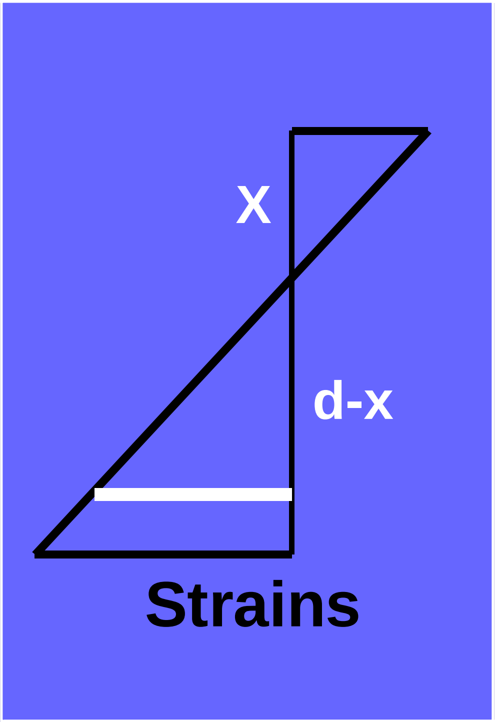


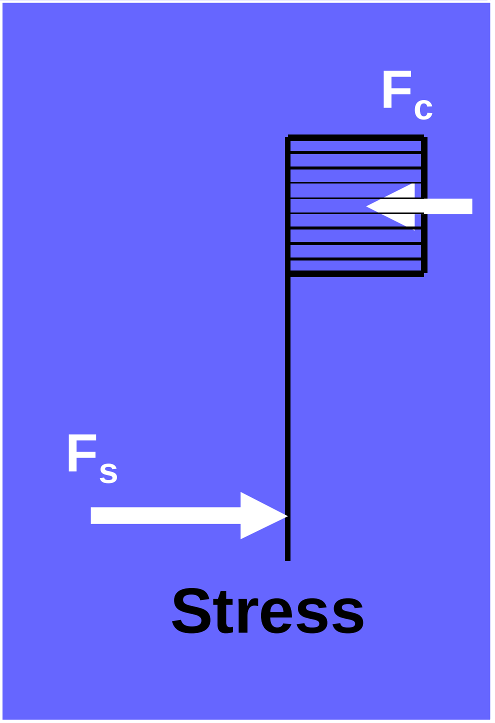
Check that f_p is not greater than the maximum allowed, e.g. $f_{pk,max} = 1517 \text{ N/mm}^2$

Check 0.8x < depth of top flange

Determine the centroid of the compression block, $d_n = 0.4x$

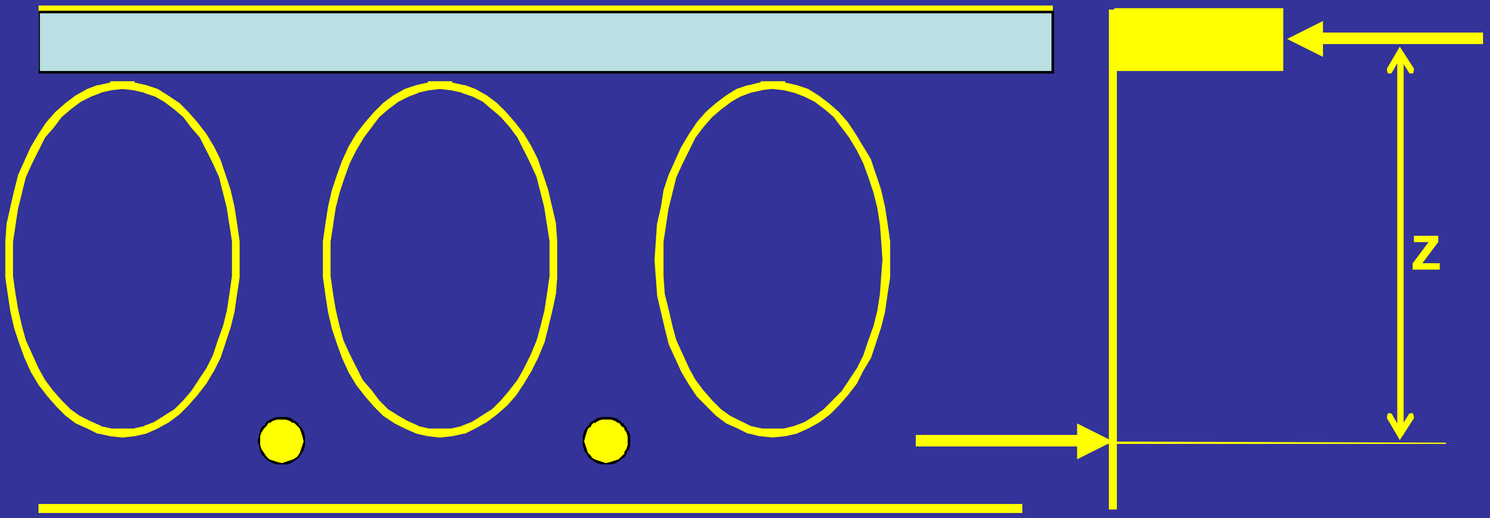
Lever arm $z = d - d_n$



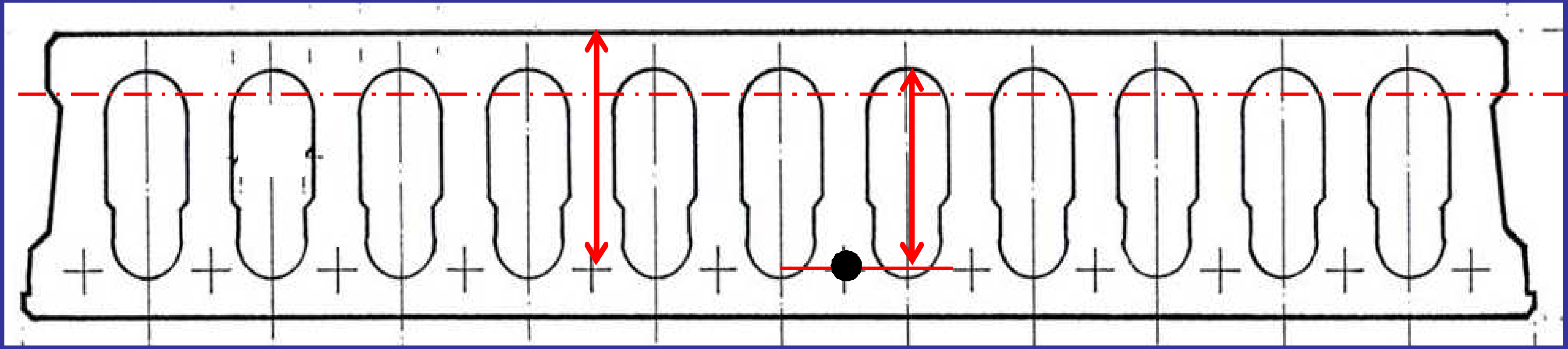


Ultimate moment of resistance

$M_{Rd} = f_p\ A_p\ z$



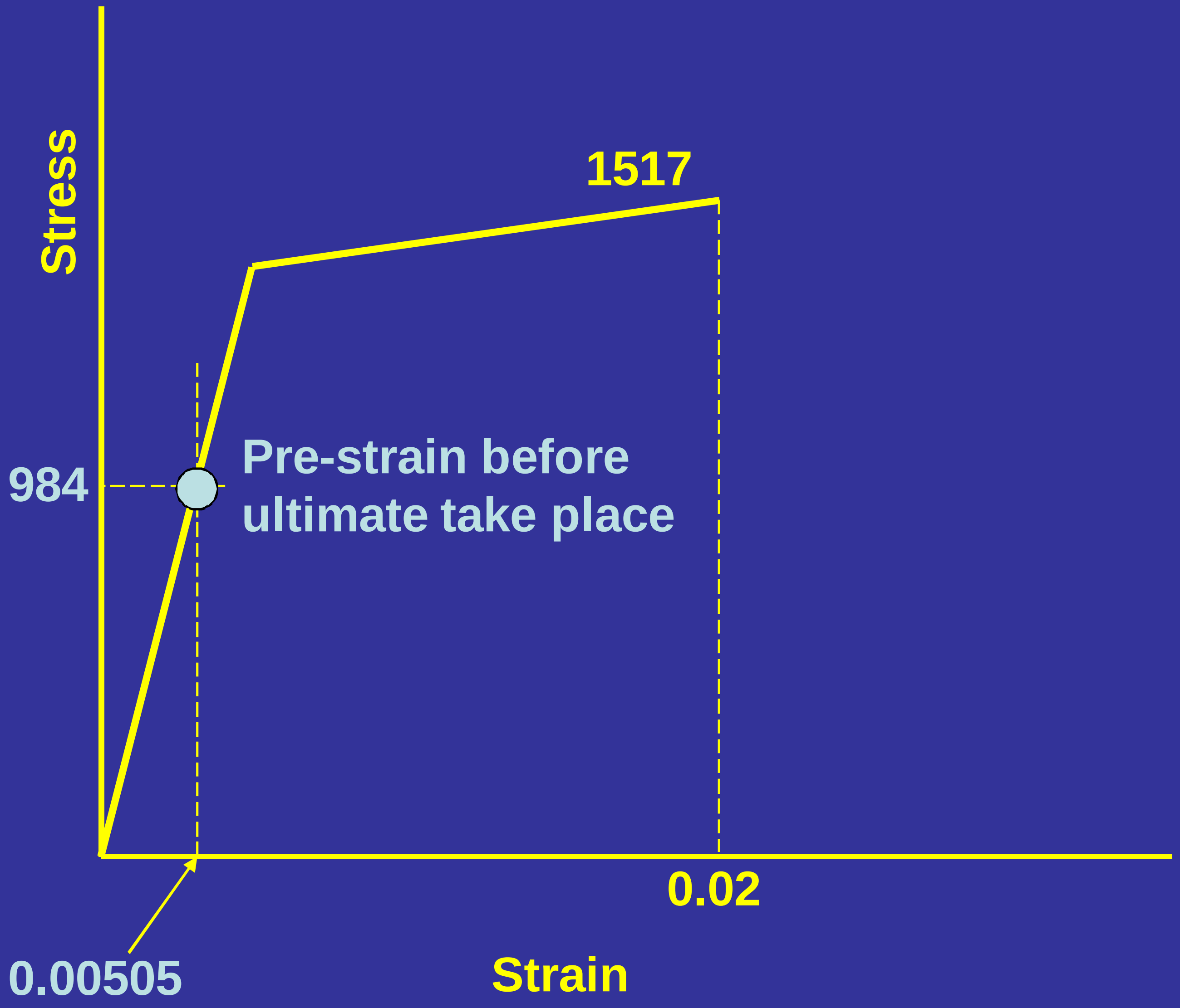
Worked example (continued)

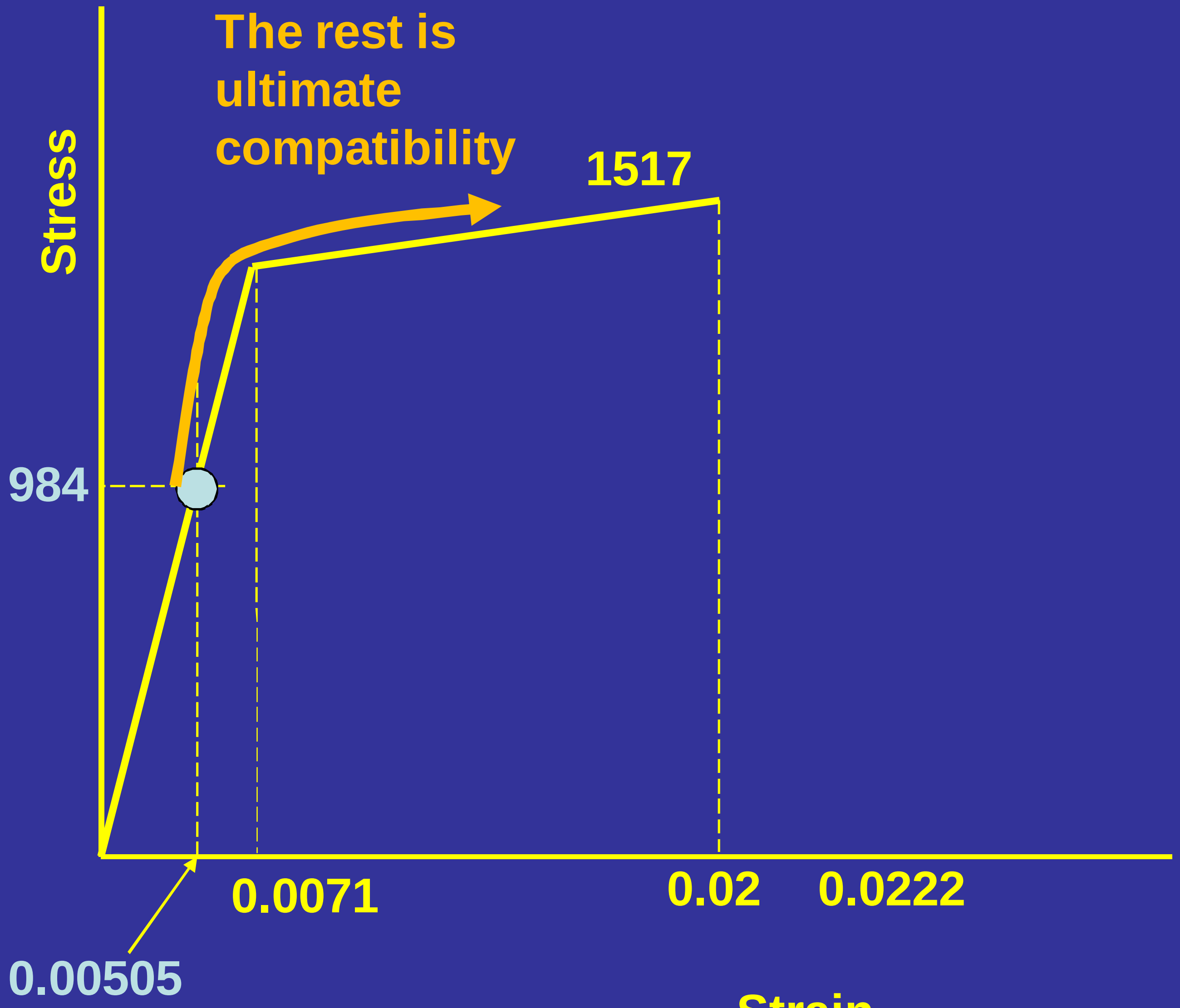


$A_p = 624 \text{ mm}^2$

$d = 250 – 39.6 = 210.4 \text{ mm}$

$\varepsilon_{po} = 984.5 / 195000 = 0.00505$


Stress
1517
984
Pre-strain before
ultimate take place
0.02
0.00505
Strain


The rest is ultimate compatibility
1517
Stress
984
0.0071
0.02
0.0222
0.00505
Strain

Worked example

The quadratic terms are:

$369.6\ x^2 – 12515\ x – 70708 = 0$

$x = 38.8$ mm

Compression depth = 0.8 x 38.8 = 31.0 mm

< top flange depth = 35 mm

$d_n = 0.4 \times 38.8 = 15.5$ mm

$z = 210.4 – 15.5 = 194.9$ mm

Worked example

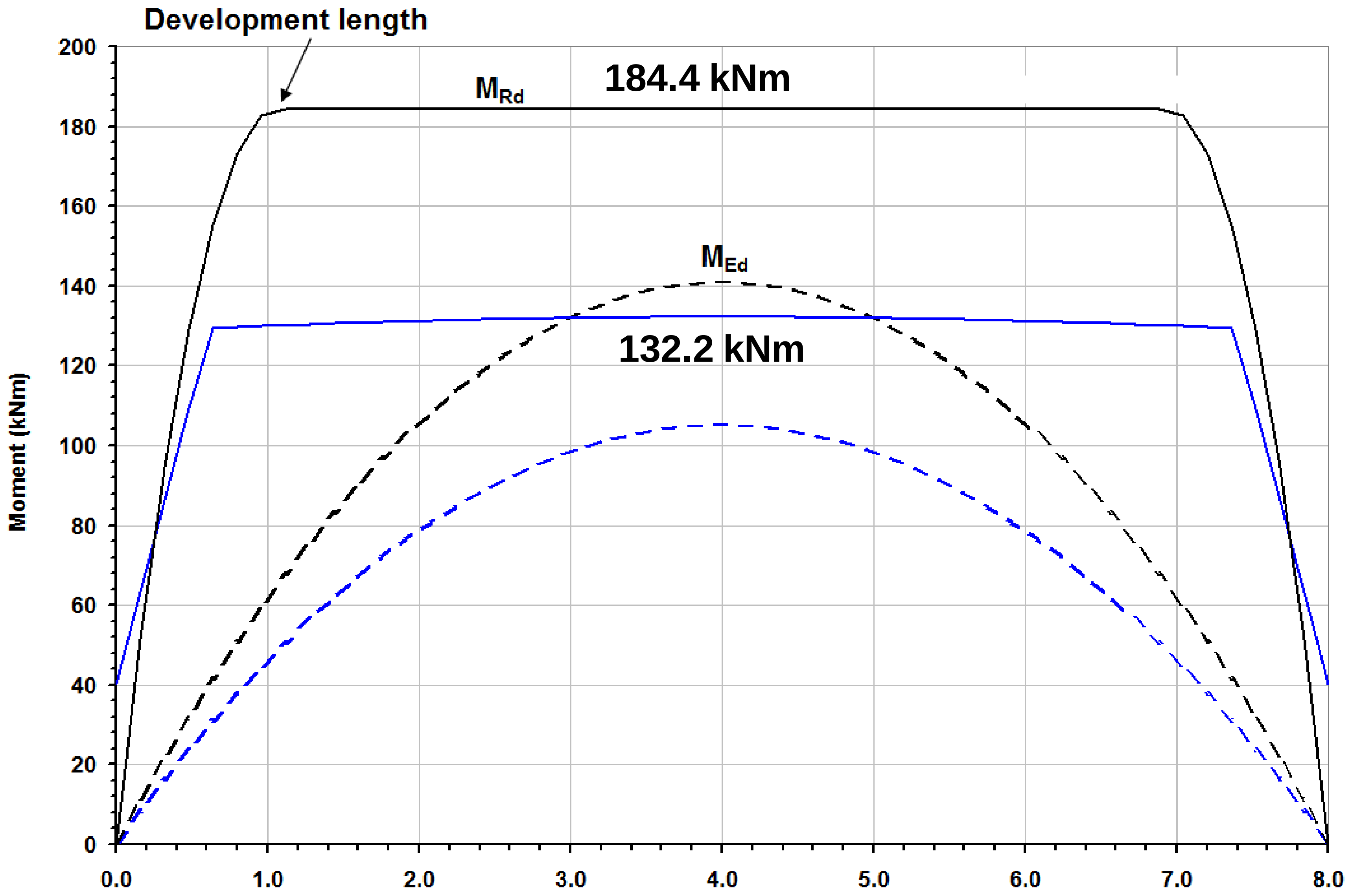
Then $\varepsilon_p = 0.202025 > 0.02$

$\therefore\ f_p = 1517\ \text{N/mm}^2$

$M_{Rd} = 624 \times 1517 \times 194.9 \times 10^{-6} = \underline{184.4\ \text{kNm}}$

Remember $M_{sd} = 132.2$ kNm $\therefore M_{Rd} / M_s = 1.39$

A good margin for most dead and live load combinations


Development length
M_Rd
184.4 kNm
M_Ed
132.2 kNm
Moment (kNm)
200
180
160
140
120
100
80
60
40
20
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

Syllabus

Camber.
Creep.
Deflections.
Limits.

Pre-camber, here 3 days after transfer

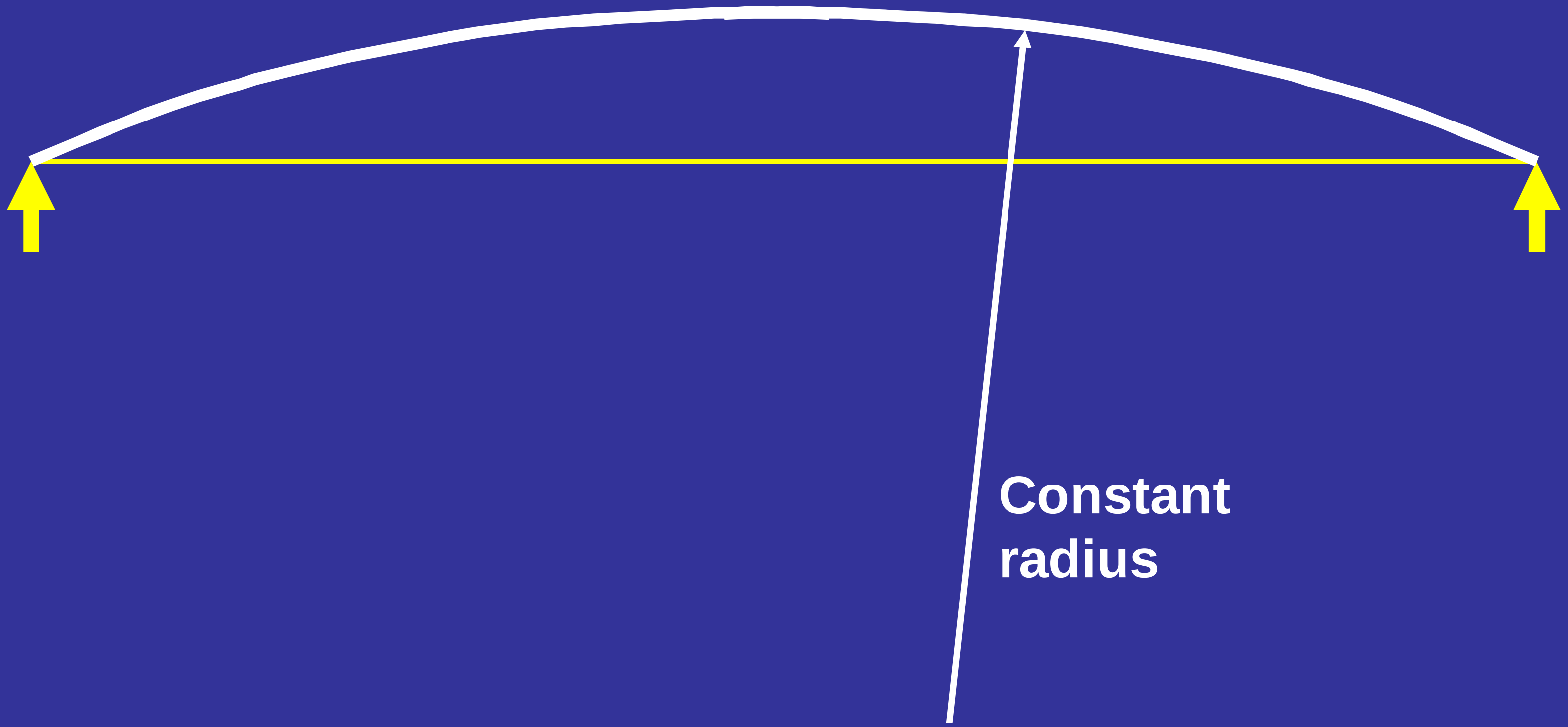
Camber & Deflections

1. Pre-camber at transfer < L/300 ±50%
2. Deflection due to self weight at installation < L/250
3. Long-term total deflection < L/250*
4. Active deflection (after installation) < L/500* (or L/350 if non-brittle finishes)

* EC2-1-1 limits

Upward camber due to transfer force

$\delta_1 = - P_{pm0}\ z_{cp}\ L^2 / 8\ E_{cm}(t)\ I_{xx}$



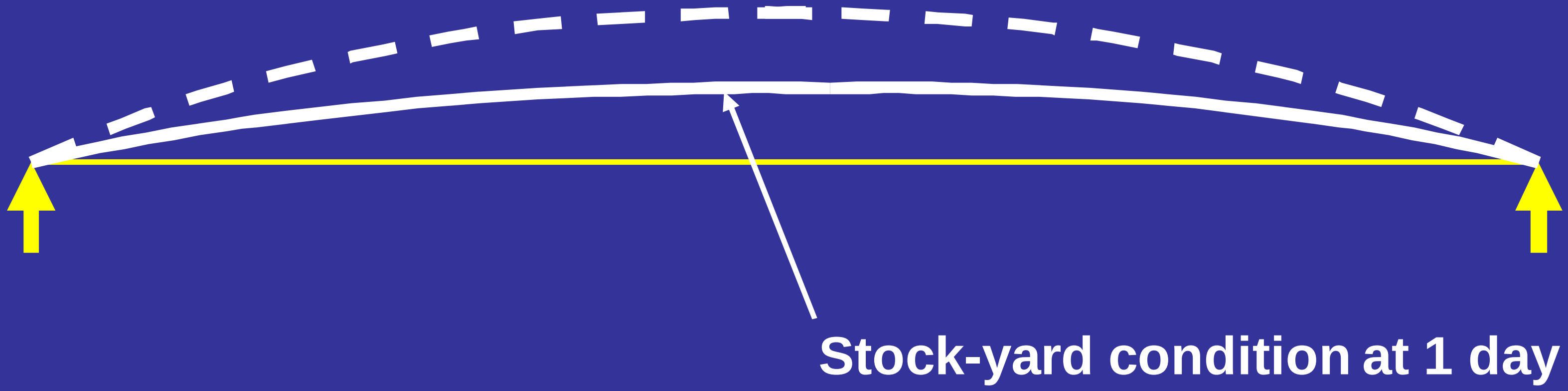
Constant radius

Upward camber due to transfer force

$\delta_1 = - P_{pm0}\ z_{cp}\ L^2 / 8\ E_{cm}(t)\ I_{xx}$

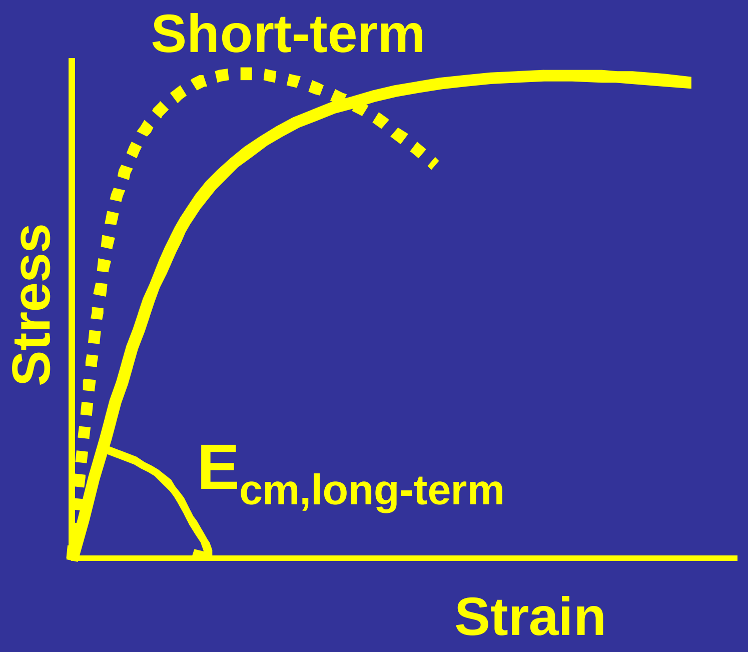
plus downward due to self weight

$\delta_2 = +5\ w_o\ L^4 / 384\ E_{cm}(t)\ I_{xx}$



Camber & Deflections

Creep of concrete causes a reduction in Young’s modulus, but at the same time the concrete is gaining strength and stiffness to 28 days.



Camber & Deflections

Creep of concrete causes a reduction in Young's modulus, but at the same time the concrete is gaining strength and stiffness to 28 days.

Creep coefficient φ_∞ = 2.5

Coefficient of development at:

transfer = 0.1

15 days = 0.3

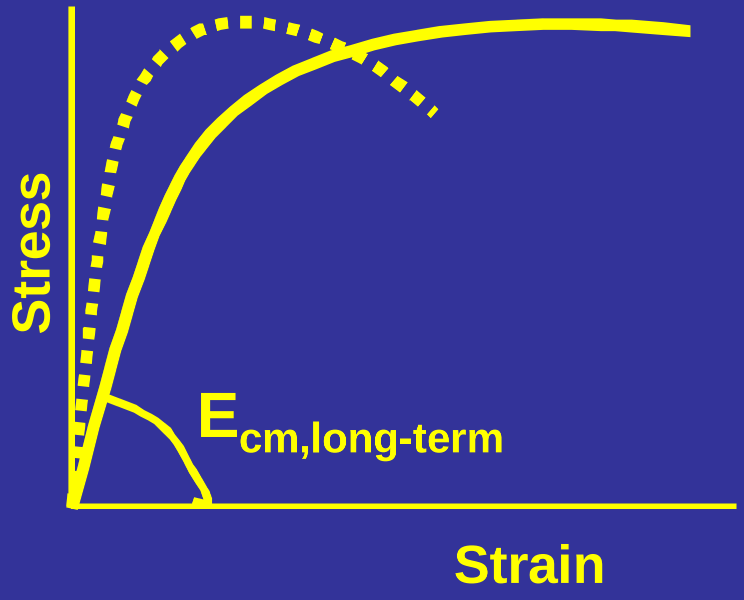
28 days = 0.4

2 months = 0.5

3 months = 0.6

∞ = 1.0

Values from ASSAP, Italy



Camber & Deflections

Creep coefficient $\varphi_\infty = 2.5$

Coefficient of development at:
transfer = 0.1
28 days = 0.4

So the net effect is to average the 1 and 28 day values

$$\varphi_1 = E_{cm}(t) / 0.5 \text{ x } [E_{cm} + E_{cm}(t)]$$

Camber & Deflections

Creep coefficient $\varphi_\infty = 2.5$

Coefficient of development at:
transfer = 0.1
28 days = 0.4

$$\varphi_1 = E_{cm}(t) \; / \; 0.5 \times [E_{cm} + E_{cm}(t)] \times 2.5 \times (0.4 - 0.1)$$

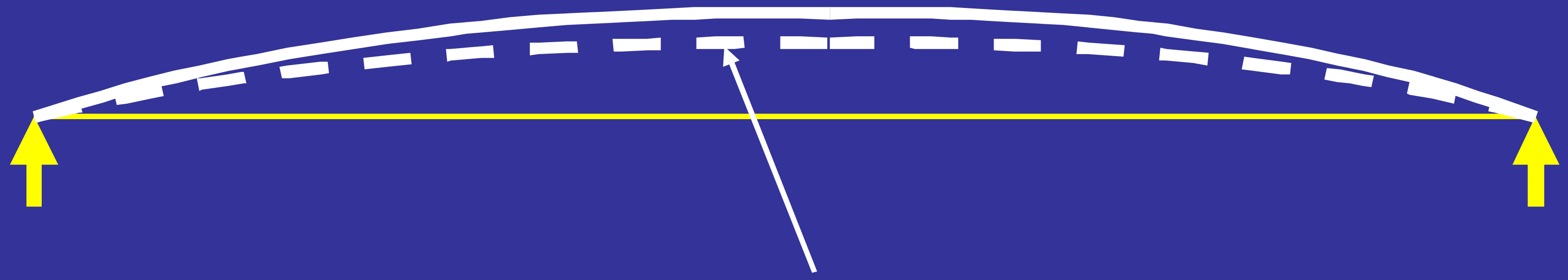
$$= 0.75 \times E_{cm}(t) \; / \; 0.5 \times [E_{cm} + E_{cm}(t)]$$

At 28 days, - creep camber + a bit for the small change in prestress force + creep deflection =

$\delta_3 = - (1+\varphi_1)\ \delta_1 + (P_{pm0} - P_{pmi})\ z_{cp}\ L^2 / 8\ E_{cm}\ I_{xx}$

plus downward due to self weight

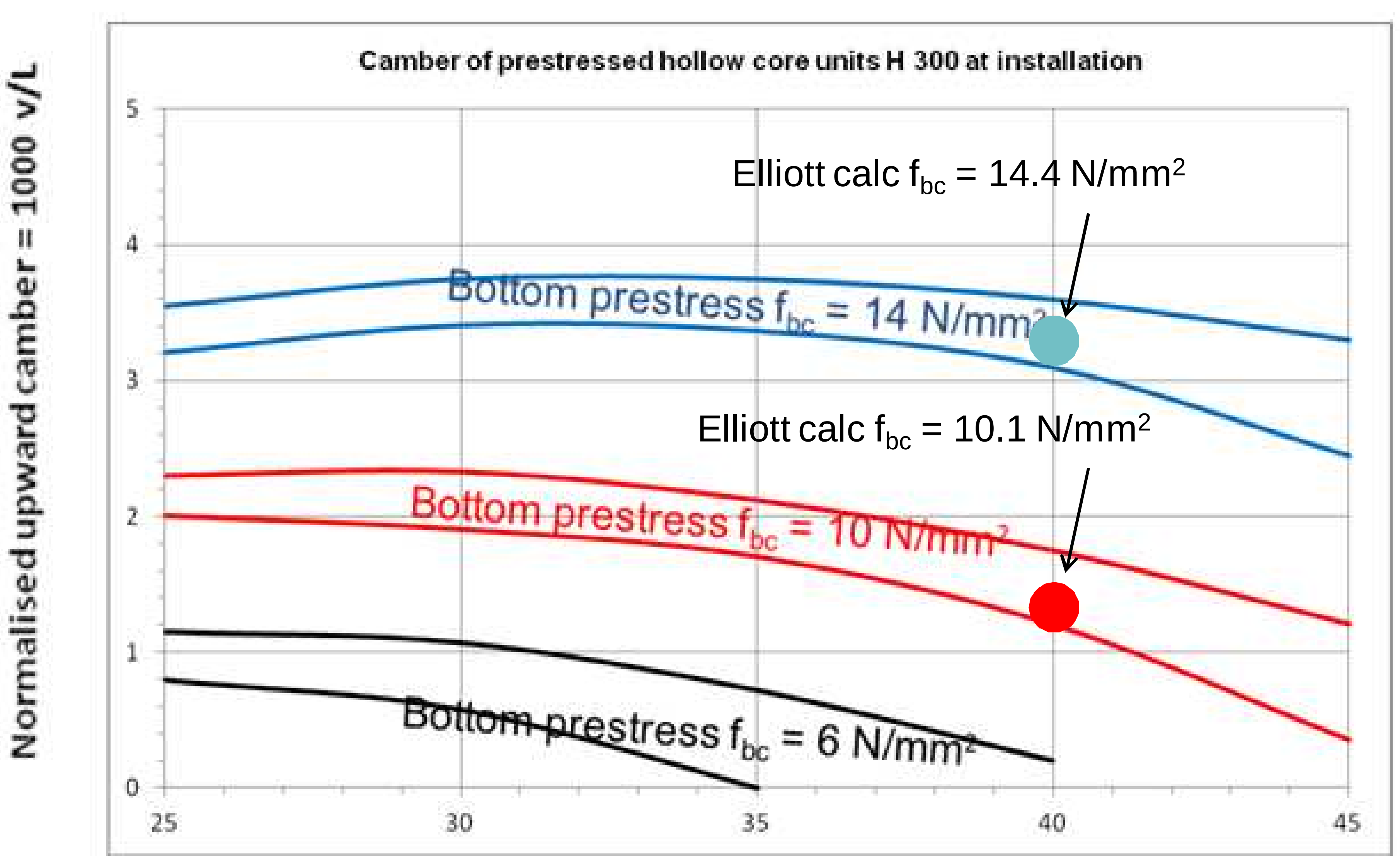
$\delta_4 = + (1+\varphi_1)\ \delta_2$



Site installation at 28 day

fib Manual

Camber at installation for 300 mm deep hcu



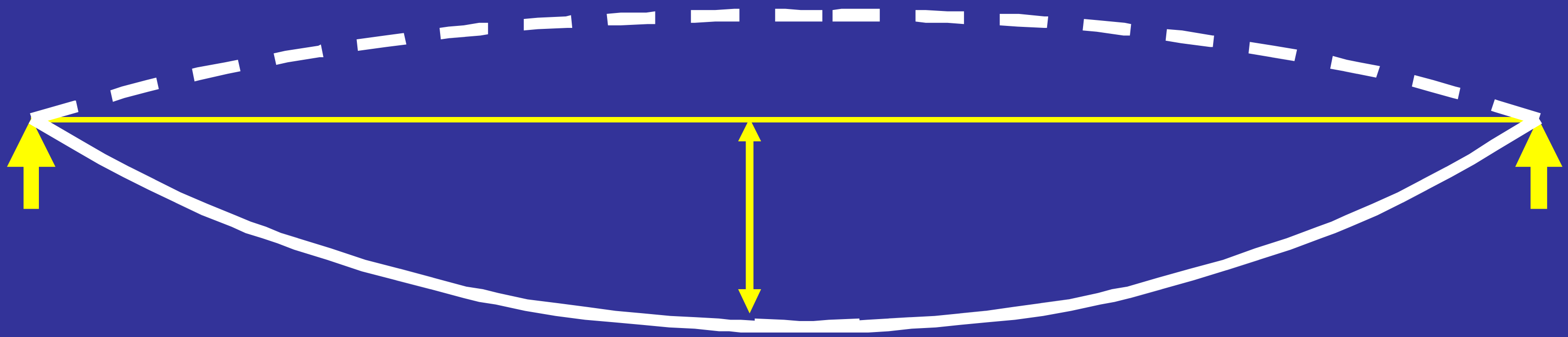
Long-term changes from E_{cm} to $E_{cm} / (1+\varphi_{\infty})$

$0.8\varphi_{\infty} = 0.8 \times 2.5 = 2.0$

0.8 is a long-term concrete aging coefficient

For loads after installation

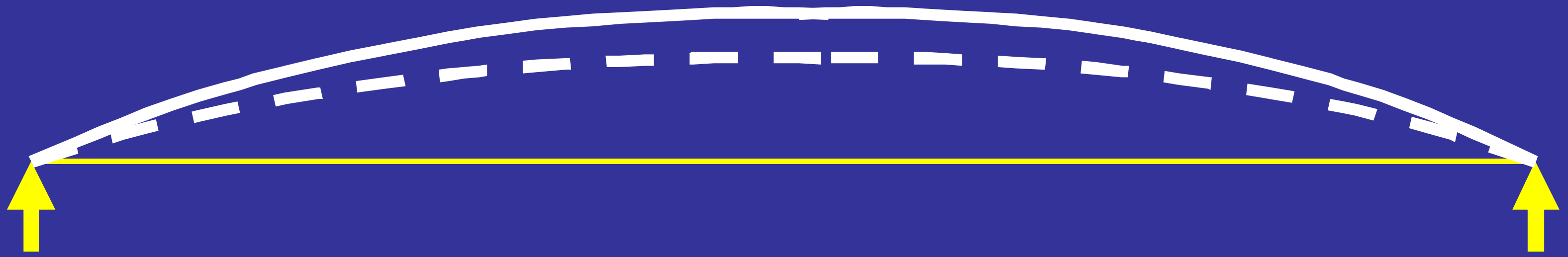
$\varphi_{28} = 2.0 \times (1.0 - 0.4) = 1.20$



Final long-term deflection
from many sources

First, camber increases upwards, less a bit for the change in prestress

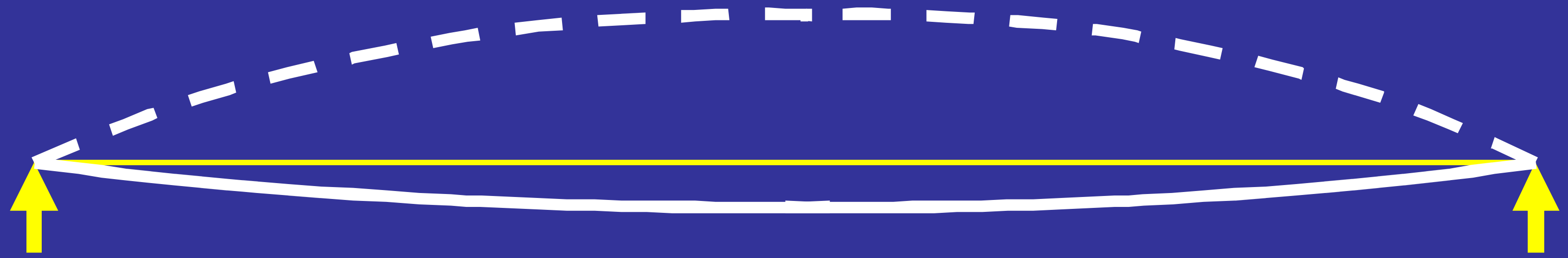
$$\delta_5 = - \delta_3 + [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx}$$



..then self weight creeps down, 2nd term is the creep

$$\delta_5 = - \delta_3 + [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx}$$

$$\boldsymbol{\delta_6 = + \delta_4 + 5 w_1 \varphi_{28} L^4 / 384 E_{cm} I_{xx}}$$



..followed by finishes, dead loads w_2 after 28 days

$$\delta_5 = -\delta_3 + [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx}$$

$$\delta_6 = +\delta_4 + 5 w_1 \varphi_{28} L^4 / 384 E_{cm} I_{xx}$$

$$\boldsymbol{\delta_7 = +(1 + \varphi_{28}) 5 w_2 L^4 / 384 E_{cm} I_{xx}}$$

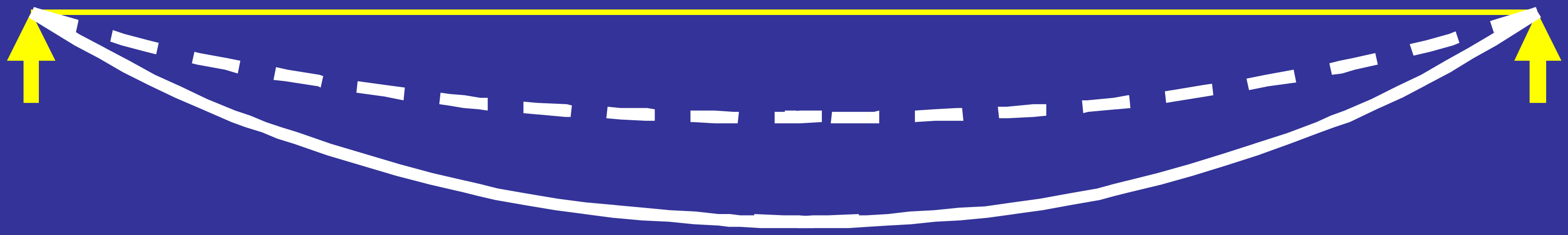
..and finally live loads ψ_2 w_3 over infinity time

$$\delta_5 = - \delta_3 + [\varphi_{28} P_{pmi} – (P_{pmi} – P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx}$$

$$\delta_6 = + \delta_4 + 5 w_1 \varphi_{28} L^4 / 384 E_{cm} I_{xx}$$

$$\delta_7 = + (1 + \varphi_{28}) 5 w_2 L^4 / 384 E_{cm} I_{xx}$$

$$\boldsymbol{\delta_8 = + (1 + 0.8 \varphi_{\infty}) 5 \psi_2 w_3 L^4 / 384 E_{cm} I_{xx}}$$

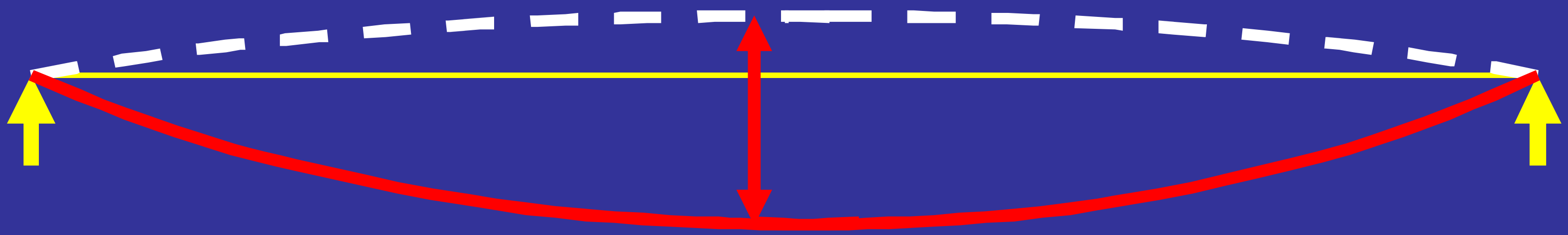


Active deflections due to creep effects and live loads takes parts of the previous equations

$$\delta_9 = [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx}$$

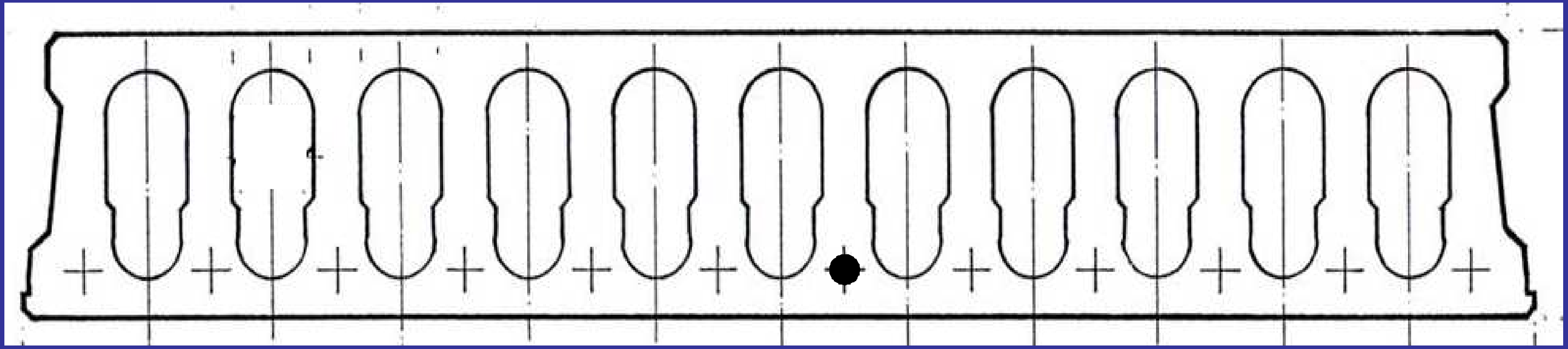
$$+ \varphi_{28} 5 (w_1 + w_2) L^4 / 384 E_{cm} I_{xx}$$

$$+ (1 + 0.8 \varphi_\infty) 5 \psi_2 w_3 L^4 / 384 E_{cm} I_{xx}$$



For composite design, replace I_{xx} with $I_{xx,c}$

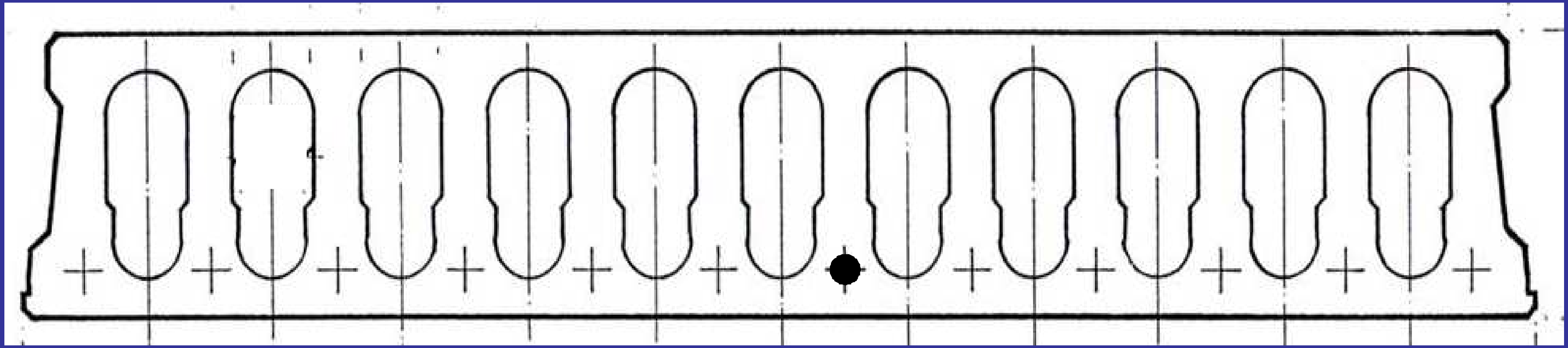
Worked example (continued)



Calculate camber, installation and long-term deflection

8.0 m effective span

Worked example



Self weight = 182791 x 24.5 x 10^{-6} = <u>4.48 kN/m</u>

Dead loads = 3.0 kN/m^2 = <u>3.60 kN/m</u> per unit

Use of floor = offices, then ψ_2 = 0.3

Live load = 0.3 x 4.0 = 1.2 kN/m^2 = <u>1.44 kN/m</u> per unit

Worked example

Camber at transfer

$$\delta_1 = \frac{739.0 \text{ x } 10^3 \text{ x } 82.75 \text{ x } 8000^2}{8 \text{ x } 32837 \text{ x } 1289 \text{ x } 10^6} = -\ 11.6 \text{ mm}$$

Self weight

Self weight hcu only

$$\delta_2 = \frac{5 \text{ x } 4.48 \text{ x } 8000^4}{384 \text{ x } 32837 \text{ x } 1289 \text{ x } 10^6} = +\ 5.7 \text{ mm}$$

<u>Net camber = - 5.9 mm</u> < length / 300 = 26 mm

Camber at installation

$$\varphi_1 = 2.5 \text{ x } (0.4 - 0.1) \text{ x } \frac{32837}{0.5 \text{ x } (32837 + 36283)} = 0.71$$

$$\delta_3 = - 11.6 \text{ x } (1 + 0.71) = - 19.8 \text{ mm}$$

Self weight at installation

$$\delta_4 = +5.7 \text{ x } (1 + 0.71) = +9.7 \text{ mm}$$

Long term camber

φ_{∞} = 0.8 x 2.5 = 2.0 for live load

φ_{28} = 2.0 x (1 – 0.4) = 1.2 for creep of camber and dead load

$$\delta_5 = -19.8 - \frac{[717.8 \times 1.2 - (717.8 - 614.3)] \times 82.75 \times 8000^2}{8 \times 36283 \times 1289 \times 10^6}$$

$$= -19.8 \ -10.7 = -30.5 \text{ mm}$$

Long term dead + live

$$\delta_6 = +9.7 + \frac{5 \times (\overset{\text{hcu + infill}}{1.2} \times 4.73 + \overset{\text{dead}}{2.2} \times 3.6 + \overset{\text{quasi-live}}{3.0} \times 1.44) \times 8000^4}{384 \times 36283 \times 1289 \times 10^6}$$

$$= +30.1 \text{ mm}$$

Final = -30.5 + 30.1 = -0.4 mm < span/250 = 26 mm

Conclusions to EC2 Prestress

1. Only 1 value for tension class = f_{ctm}
2. Zero tension if exposure > XC1
3. Prestress losses for initial relaxation and elastic shortening, plus shrinkage, creep and relaxation
4. Ultimate stress and strain equilibrium
5. Camber = immediate at transfer + creep
6. Deflections = static + creep