Flexural strength and camber

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Prestressed Concrete Hollow Core Units
Flexural strength and deflections

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Precast Concrete Structures 2nd ed.

700 pages with about 200 pages on precast floors

Figure 4.25 Principles of shear resistance for prestressed concrete elements.

Figure 4.26 Development of stresses in the transmission zone \( \tau_{pm} \) and anchorage zone \( \tau_{pm} \) of prestressed members at (1) release of tendons, (2) after all losses and (3) ultimate. (Based on BS EN 1992-1-1. Design of concrete structures – Part 1-1: General rules and rules for buildings. BSI, London. Figs. 8.16 and 8.17.)

Design rules recognise the fact that the critical shear plane may occur in the prestress development zone where \( \tau_{pm} \) is not fully developed. It is known that prestressing forces develop somewhere between linearly and parabolically, although BS EN 1992-1-1, Figs. 8.16 and 8.17 (combined here in Figure 4.26), adopts a linear development of stress in service and bi-linear at ultimate. Therefore, a reduced value \( \tau_{pm} \) is used up to \( \tau_{pm} \).
Syllabus

Syllabus


Prestress. Losses. Limit $f_{ctm}$. Moment of resistance.
Definitions.
Introduction.
Concrete and strands.
Cover.

Prestress.
Losses.
Limit $f_{ctm}$
Moment of resistance.

Ultimate strength.
Equilibrium.
Compatibility.
$M_{Rd}$.
Syllabus


Prestress. Losses. Limit $f_{ctm}$. Moment of resistance.


Serviceability stress check

Top stress $\sigma_t$

Bottom stress $\sigma_b$

Eccentricity = $z_{cp}$

Centroidal axis

Final force = $P_{po}$
Load or moment  vs  Deflection

Prestressed beam
Cracking occurs where
\(+\sigma_b - \frac{M}{Z_B} > -f_{ct}\)

Load or moment

Prestressed beam

Cracked stiffness

-ve camber

Deflection
Load or moment

Prestressed beam

Service moment of resistance

Ultimate moment of resistance

$M_{Rd}$

Deflection

-ve camber
Compressive cylinder strength $f_{ck}$

Design compressive strength

$$f_{cd} = 0.85 \frac{f_{ck}}{\gamma_c}$$

for example $0.85 \times 45 / 1.5 = 25.5 \text{ N/mm}^2$
Tensile strength due to flexure $f_{ctm}$

EC2-1-1, Table 3.1

$f_{ctm} = 0.3 f_{ck}^{2/3}$

e.g. $= 0.3 \times 45^{2/3} = 3.80 \text{ N/mm}^2$
EC2-1-1 values listed in Table 3.1

<table>
<thead>
<tr>
<th>Strength classes for concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck}$ (MPa)</td>
</tr>
<tr>
<td>$f_{ck,cube}$ (MPa)</td>
</tr>
<tr>
<td>$f_{cm}$ (MPa)</td>
</tr>
<tr>
<td>$f_{ctm}$ (MPa)</td>
</tr>
<tr>
<td>$f_{ctk,0,05}$ (MPa)</td>
</tr>
<tr>
<td>$f_{ctk,0,95}$ (MPa)</td>
</tr>
<tr>
<td>$E_{cm}$ (GPa)</td>
</tr>
</tbody>
</table>

Mean $f_{cm} = f_{ck} + 8$ N/mm$^2$
Early tensile stress at transfer

EC2-1-1, clause 3.1.2.

(9) The development of tensile strength with time is strongly influenced by curing and drying conditions as well as by the dimensions of the structural members. As a first approximation it may be assumed that the tensile strength \( f_{ctm}(t) \) is equal to:

\[
f_{ctm}(t) = (\beta_{cc}(t))^\alpha \cdot f_{ctm}
\]

where \( \beta_{cc}(t) \) follows from Expression (3.2) and

- \( \alpha = 1 \) for \( t < 28 \)
- \( \alpha = 2/3 \) for \( t \geq 28 \). The values for \( f_{ctm} \) are given in Table 3.1.

(6) The compressive strength of concrete at an age \( t \) depends on the type of cement, temperature and curing conditions. For a mean temperature of 20°C and curing in accordance with EN 12390 the compressive strength of concrete at various ages \( f_{cm}(t) \) may be estimated from Expressions (3.1) and (3.2).

\[
f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}
\]

\[
\therefore f_{ctm}(t) = f_{ctm} \times \frac{f_{cm}(t)}{f_{cm}}
\]

E.g. \( f_{ctm}(t) = 3.80 \times 38 / 53 = 2.72 \text{ N/mm}^2 \)
\[ E_{cm} = 22 \left( \frac{f_{cm}}{10} \right)^{0.3} \text{kN/mm}^2 \text{ for gravel and granite.} \]

For limestone \( x \times 0.9 \).

\[ \text{e.g.} = 22 \times \left( \frac{53}{10} \right)^{0.3} = 36.3 \text{kN/mm}^2 \]
Figure 3.10: Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)
e.g. $f_{pk} = 1770 \text{ N/mm}^2$

Figure 3.10: Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)
Durability = nominal cover \( c = c_{\text{min,dur}} + \Delta c_{\text{dev}} \)

For precast slabs > C30/37, use S2

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**Table 4.5N: Values of minimum cover, \( c_{\text{min,dur}} \), requirements with regard to durability for prestressing steel**

<table>
<thead>
<tr>
<th>Structural Class</th>
<th>Exposure Class according to Table 4.1</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>X0</td>
</tr>
<tr>
<td>S1</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>10</td>
</tr>
<tr>
<td>S3</td>
<td>10</td>
</tr>
<tr>
<td>S4</td>
<td>10</td>
</tr>
<tr>
<td>S5</td>
<td>15</td>
</tr>
<tr>
<td>S6</td>
<td>20</td>
</tr>
</tbody>
</table>

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Much better and recent information in BS 8500-1: 2015
Construction deviation $\Delta c_{\text{dev}} = 5$ mm if tendon positions are controlled

4.4.1.3 Allowance in design for deviation

(1)P To calculate the nominal cover, $c_{\text{nom}}$, an addition to the minimum cover shall be made in design to allow for the deviation ($\Delta c_{\text{dev}}$). The required minimum cover shall be increased by the absolute value of the accepted negative deviation.

**Note:** The value of $\Delta c_{\text{dev}}$ for use in a Country may be found in its National Annex. The recommended value is 10 mm.

(3) In certain situations, the accepted deviation and hence allowance, $\Delta c_{\text{dev}}$, may be reduced.

**Note:** The reduction in $\Delta c_{\text{dev}}$ in such circumstances for use in a Country may be found in its National Annex. The recommended values are:
- where fabrication is subjected to a quality assurance system, in which the monitoring includes measurements of the concrete cover, the allowance in design for deviation $\Delta c_{\text{dev}}$ may be reduced: $10$ mm $\geq \Delta c_{\text{dev}} \geq 5$ mm  \hspace{1cm} (4.3N)
Syllabus

Prestress.
Losses.
Limit $f_{ctm}$
Moment of resistance.
Prestressed concrete hollow core floor units
Calculate: Area $A_c$

Centroid height $y_b$ (and $y_t = h - y_b$)

2nd moment of area $I_{x-x}$

1st moment of area $S_{x-x}$ (for shear only)

Section modulus $Z_b = I_{x-x} / y_b$ and $Z_T = I_{x-x} / y_t$
Approx 28 days
16-24 hours
Approx 28 days
Ultimate
$P_p_{pi}$
$P_{pm0}$
$P_{pmi}$
$P_{po}$

20%-25% losses
Initial prestressing force $P_{pi} = \eta f_{pk} A_{ps}$

Eccentricity $z_{cp}$

Typically $0.7 \times 1770$

Immediate strand relaxation and elastic shortening at transfer ...
Transfer stress limits, e.g. based on $f_{ck}(t) = 30 \text{ N/mm}^2$

\[ \sigma_{t(t)} \geq - f_{ctm}(t) = -2.72 \text{ N/mm}^2 \]

\[ \sigma_{b(t)} \leq 0.6 f_{ck}(t) = +18.00 \text{ N/mm}^2 \]
Can be checked at the end of the transmission length $l_{pt}$

\[ \sigma_{t(t)} + \frac{M_{self}}{Z_t} \geq -f_{ctm}(t) = -2.72 \text{ N/mm}^2 \]

\[ \sigma_{b(t)} - \frac{M_{self}}{Z_b} \leq 0.6 f_{ck}(t) = +18.00 \text{ N/mm}^2 \]
Followed by long term losses due to creep, shrinkage and further strand relaxation

Results in a lower prestress $\sigma_b$ and $\sigma_t$
Adding prestress to imposed service stress =

\[ \sigma_t + \frac{M_s}{Z_t} - \sigma_b - \frac{M_s}{Z_b} = \sigma_b \]
Adding prestress to imposed service stress =

\[
\sigma_t M_s / Z_t \leq 0.45 f_{ck} = +20.25
\]

\[
\sigma_b M_s / Z_b \geq f_{ctm} = -3.80 \text{ N/mm}^2
\]
Service moment of resistance is lesser of

\[ M_{sd} = (\sigma_t + 0.45f_{ck}) Z_t \]

\[ \leq 0.45 f_{ck} = +20.25 \]
Service moment of resistance is lesser of

\[ M_{sd} = (\sigma_t + 0.45f_{ck}) Z_t \]

\[ M_{sd} = (\sigma_b + f_{ctm}) Z_b \]  Mostly critical

\[ \leq f_{ctm} = -3.80 \text{ N/mm}^2 \]
But for exposure > XC1, permissible tension $f_{ctm}$ is reduced (depending on each country)

(5) A limiting calculated crack width, $w_{max}$, taking into account the proposed function and nature of the structure and the costs of limiting cracking, should be established.

**Note:** The value of $w_{max}$ for use in a Country may be found in its National Annex. The recommended values for relevant exposure classes are given in Table 7.1N.

**Table 7.1N Recommended values of $w_{max}$ (mm)**

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<td>0,2</td>
</tr>
<tr>
<td>XC2, XC3, XC4</td>
<td></td>
<td>0,2$^2$</td>
</tr>
<tr>
<td>XD1, XD2, XS1, XS2, XS3</td>
<td>0,3</td>
<td>Decompression</td>
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**Note 1:** For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.

**Note 2:** For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.
For Class XC2-XC4 (external) no tension allowed. But can use quasi-permanent live load $x \psi_2$

(5) A limiting calculated crack width, $w_{\text{max}}$, taking into account the proposed function and nature of the structure and the costs of limiting cracking, should be established.

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**Note 2:** For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.
Worked example:
Worked example: XC1 exposure

1200 wide x 250 depth

12 no. 9.3 mm strands at 35 mm cover

Axis a = 39.6 mm

$f_{pk} = 1770 \text{ N/mm}^2$

Initial stressing to 70% = 1239 N/mm$^2$
$A_c = 182791 \text{ mm}^2$

$y_b = 122.4 \text{ mm}$

$I_{x-x} = 1270.3 \times 10^6 \text{ mm}^4$

$Z_b = \frac{1270.3}{122.4} = 10.378 \times 10^6 \text{ mm}^3$

$Z_T = 9.955 \times 10^6 \text{ mm}^3$

$A_{ps} = 12 \times 52 = 624 \text{ mm}^2$

$z_{cp} = 122.4 - 39.6 = 82.8 \text{ mm}$
Initial prestress $P_{pi} = 624 \times 1239 \times 10^{-3} = 773.1$ kN
Relaxation Class 2. 2.5% at 1000 hours
Immediate relaxation loss = 4.95 N/mm$^2$ (0.40%)

$E_p$ (strand) 195 kN/mm$^2$
$E_{cm}(t)$ (gravel aggregate) = 32.8 kN/mm$^2$
Elastic shortening loss = 49.67 N/mm$^2$ (4.01%)
Remaining force at transfer

\[ P_{pm0} = 739.0 \text{ kN (4.41\% loss)} \]

\[ \sigma_t(t) = -2.10 \geq -2.72 \text{ N/mm}^2 \]

\[ \sigma_b(t) = 9.94 \leq 18.0 \text{ N/mm}^2 \quad \text{OK} \]
Further loss of prestress at installation at (say) 28 days:

RH at transfer = 70%
Creep coefficient at 28 days = 0.84
Creep loss of stress at 28 days = 34.0 N/mm² (2.74%)

\[ P_{pmi} = 717.8 \text{ kN} \] (this is used later to determine camber after installation)
Final loss of prestress at 500,000 hours (57 years):

RH at service (indoor exposure) = 50%
Creep coefficient from installation to life = 1.60
Creep loss of stress = 61.8 N/mm² (4.99%)

Shrinkage $\varepsilon_{sh} = 420 \times 10^{-6}$
Shrinkage loss = 74.7 N/mm² (6.03%)

Long term relaxation loss = 40.3 N/mm² (3.25%)
After all losses

\[ P_{p0} = 614.3 \text{ kN (20.5\% loss)} \]

\[ \sigma_b = 8.26 \text{ N/mm}^2 \]

\[ \sigma_t = -1.746 \text{ N/mm}^2 \]
But creep losses may be reduced by reversal of prestress due to self weight and dead loads.

Creep loss at support = 95.8 N/mm²

Creep loss at mid-span = 63.4 N/mm²

Self weight M = 26.87 kNm
Final $\sigma_b = 8.634 \text{ N/mm}^2$ (increase of 4.5%)

$\sigma_t = -1.825 \text{ N/mm}^2$
\[ M = -1.825 \]  
\[ +8.634 \]  
\[ MS / Zt \]  
\[ MS / Zb \]  
\[ \leq 0.45 f_{ck} = +20.25 \]  
\[ \geq f_{ctm} = -3.80 \text{ N/mm}^2 \]
But ! compound values may be used for \( I_{xx} \), \( Z_b \) and \( Z_t \) based on the transformed area of the strands:

\[
m = \frac{E_p}{E_{cm}} = \left( \frac{195}{36.3} \right) - 1 = 4.37
\]

Then \( I_{xx} = 1289 \times 10^6 \text{ mm}^4 \)

\( Z_{b,co} = 10.634 \times 10^6 \text{ mm}^3 \) (increase of 2.5%)
Service moment of resistance is lesser of

\[ M_{Sd} = (20.25 + 1.825) \times 10.004 = 220.8 \text{ kNm} \]
\[ M_{Sd} = (8.634 + 3.80) \times 10.634 = 132.2 \text{ kNm} \]

\[ = -1.825 \]
\[ M_{sd} / Z_{t,co} \leq 0.45 f_{ck} = +20.25 \]
\[ \geq f_{ctm} = -3.80 \text{ N/mm}^2 \]
Transmission length

At mid-span \( M_{sd} = (\sigma_b + f_{ctm}) Z_{b,co} \)

\( M_{sd} = (0 + f_{ctm}) Z_{b,co} \)

129.3 kNm

132.2 kNm
Syllabus

4 point bending test of prestressed hollow core slab.

Initial camber = -27 mm

1200 x 320 deep x 11.0 m span
Self-imposed $M_{Ed} = \text{service moment of resistance}$

Deflection = +17 mm
Camber = -10 mm
Self + imposed $M_{Ed} = M_{Rd}$ (ultimate resistance)

First cracking
Deflection approx 25 mm
Self + imposed $M_{Ed} = 1.25 \ M_{Rd}$ (ultimate load + 25%)

Cracks widening and increasing
Deflection approx 35 mm
Limit = span / 250 = 44 mm
Ultimate moment of resistance

Strains

Stress
Strain development from initial prestress to ultimate

First is the pre-strain due to final prestress after all losses =

\[ \varepsilon_{po} = \frac{\sigma_{po}}{E_p} \]
Strain development from initial prestress to ultimate

Bending strain added to pre-strain
Strain development from initial prestress to ultimate

Bending strain now overtake the pre-strain
Strain development from initial prestress to ultimate

Final ultimate strains

\[ \varepsilon_{cu} = 0.0035 \]

\[ \varepsilon_p < \varepsilon_{ud} \text{ code value 0.02} \]
Strain development from initial prestress to ultimate

Total strain

\[ \varepsilon_p = \varepsilon_{po} + 0.0035 \frac{(d - x)}{x} \]

Now find \( x \) and the stresses

Final ultimate strains

\[ \varepsilon_{cu} = 0.0035 \]

\[ \varepsilon_p < \varepsilon_{ud} \text{ code value 0.02} \]
Constitutive relationship stress v strain

\[ \varepsilon \text{ p} \rightarrow d - x \rightarrow \varepsilon \text{ cu} \]
Constitutive relationship stress v strain

Equilibrium

\[ F_c = F_s \]

\[ 0.567 f_{ck} b 0.8 x = f_p A_p \]

and compatibility

\[ \frac{x}{(d - x)} = \frac{\varepsilon_{cu}}{\varepsilon_p} \]
Combining;

\[ f_p = 0.567 f_{ck} b 0.8 (d-x) \frac{\varepsilon_{cu}}{A_p \varepsilon_p} \]

or stress = inverse of strain
1. Equilibrium of forces gives inverse stress vs strain relationship.
2. Stress \( v \) strain curve for strand

\[
\begin{align*}
\text{Stress } f_p & \quad \text{Strain } \varepsilon_p \\
\text{Strains} & \\
\text{Stress } F_c & \quad \text{Stress } F_s
\end{align*}
\]
Stress v strain diagram, e.g. for strand with $f_{pk} = 1770 \text{ N/mm}^2$

$E_p = 195 \text{ kN/mm}^2$

Strain

Stress

1385
1517
1539

0.0071
0.02
0.0222
Stress v strain diagram, e.g. for strand with $f_{pk} = 1770 \text{ N/mm}^2$

This line becomes Eq. (1)

$E_p = 195 \text{ kN/mm}^2$

0.0071  0.02  

1385  1517
Total strain = pre-strain + compatibility concrete strain $\varepsilon_p$

$$= \varepsilon_{po} + \varepsilon_{cu} \left( \frac{d}{x} - 1 \right) \quad \text{...(2)}$$

where, pre-strain after losses

$$\varepsilon_{po} = \frac{\sigma_{po}}{E_p}$$

Force equilibrium

$$F_s = F_c$$

$$f_p A_p = 0.567 f_{ck} b 0.8 x \quad \text{...(3)}$$
Combining 3 equations gives the quadratic solution:

\[ 0.567f_{ck} \cdot 0.8b \cdot (\varepsilon_{uk} - \varepsilon_{LOP}) \cdot x^2 \]
\[ -[0.9(\varepsilon_{uk} - \varepsilon_{LOP}) + 0.1(\varepsilon_{po} - \varepsilon_{cu} - \varepsilon_{LOP})] \cdot A_p \cdot f_{pd} \cdot x \]
\[ -0.1 \varepsilon_{cu} \cdot d \cdot A_p \cdot f_{pd} = 0 \]

Solving yields \( x \)

Then \( \varepsilon_p \) and \( f_p \) are found
Check that $f_p$ is not greater than the maximum allowed, e.g. $f_{pk,\text{max}} = 1517 \text{ N/mm}^2$

Check $0.8x < \text{depth of top flange}$

Determine the centroid of the compression block, $d_n = 0.4x$

Lever arm $z = d - d_n$
Ultimate moment of resistance

\[ M_{Rd} = f_p A_p z \]
Worked example (continued)

\[ A_p = 624 \text{ mm}^2 \]
\[ d = 250 - 39.6 = 210.4 \text{ mm} \]
\[ \varepsilon_{po} = \frac{984.5}{195000} = 0.00505 \]
Pre-strain before ultimate take place
The rest is ultimate compatibility
Worked example

The quadratic terms are:
$$369.6 x^2 - 12515 x - 70708 = 0$$

$$x = 38.8 \text{ mm}$$

Compression depth = \(0.8 \times 38.8 = 31.0 \text{ mm}\)

< top flange depth = 35 mm

\(d_n = 0.4 \times 38.8 = 15.5 \text{ mm}\)

\(z = 210.4 - 15.5 = 194.9 \text{ mm}\)
Worked example

Then $\varepsilon_p = 0.202025 > 0.02$

$\therefore f_p = 1517 \text{ N/mm}^2$

$M_{Rd} = 624 \times 1517 \times 194.9 \times 10^{-6} = 184.4 \text{ kNm}$

Remember $M_{sd} = 132.2 \text{ kNm}$  $\therefore M_{Rd} / M_s = 1.39$

A good margin for most dead and live load combinations
Pre-camber, here 3 days after transfer
Camber & Deflections

1. Pre-camber at transfer < L/300 ±50%
2. Deflection due to self weight at installation < L/250
3. Long-term total deflection < L/250*
4. Active deflection (after installation) < L/500* (or L/350 if non-brittle finishes)

* EC2-1-1 limits
Upward camber due to transfer force

\[ \delta_1 = - \frac{P_{pm0} z_{cp} L^2}{8 E_{cm}(t) I_{xx}} \]
Upward camber due to transfer force

\[ \delta_1 = - P_{pm0} z_{cp} L^2 / 8 E_{cm}(t) I_{xx} \]

plus downward due to self weight

\[ \delta_2 = +5 w_o L^4 / 384 E_{cm}(t) I_{xx} \]

Stock-yard condition at 1 day
Camber & Deflections

Creep of concrete causes a reduction in Young’s modulus, but at the same time the concrete is gaining strength and stiffness to 28 days.
Camber & Deflections

Creep of concrete causes a reduction in Young’s modulus, but at the same time the concrete is gaining strength and stiffness to 28 days.

Creep coefficient $\phi_\infty = 2.5$

Coefficient of development at:
- transfer $= 0.1$
- 15 days $= 0.3$
- 28 days $= 0.4$
- 2 months $= 0.5$
- 3 months $= 0.6$
- $\infty = 1.0$

Values from ASSAP, Italy
Camber & Deflections

Creep coefficient $\phi_\infty = 2.5$

Coefficient of development at:
transfer $= 0.1$
28 days $= 0.4$

So the net effect is to average the 1 and 28 day values

$\phi_1 = \frac{E_{cm}(t)}{0.5 \times [E_{cm} + E_{cm}(t)]}$
Camber & Deflections

Creep coefficient $\varphi_\infty = 2.5$

Coefficient of development at:
- transfer $= 0.1$
- 28 days $= 0.4$

\[ \varphi_1 = \frac{E_{cm}(t)}{0.5 \times [E_{cm} + E_{cm}(t)] \times 2.5 \times (0.4 - 0.1)} \]

\[ = 0.75 \times \frac{E_{cm}(t)}{0.5 \times [E_{cm} + E_{cm}(t)]} \]
At 28 days, - creep camber + a bit for the small change in prestress force + creep deflection =

\[ \delta_3 = - (1 + \varphi_1) \delta_1 + (P_{p_{m0}} - P_{p_{mi}}) z_{cp} L^2 / 8 E_{cm} I_{xx} \]

plus downward due to self weight

\[ \delta_4 = + (1 + \varphi_1) \delta_2 \]
fim Manual

Camber at installation for 300 mm deep hcu

Elliott calc $f_{bc} = 14.4 \text{ N/mm}^2$

Elliott calc $f_{bc} = 10.1 \text{ N/mm}^2$
Long-term changes from $E_{cm}$ to $E_{cm} / (1 + \varphi_\infty)$

$$0.8 \varphi_\infty = 0.8 \times 2.5 = 2.0$$

0.8 is a long-term concrete aging coefficient

For loads after installation

$$\varphi_{28} = 2.0 \times (1.0 - 0.4) = 1.20$$

Final long-term deflection from many sources
First, camber increases upwards, less a bit for the change in prestress

\[ \delta_5 = -\delta_3 + \left[ \varphi_{28} P_{pmi} - (P_{pmi} - P_{po}) \right] z_{cp} \frac{L^2}{8 E_{cm} I_{xx}} \]
..then self weight creep's down, 2\textsuperscript{nd} term is the creep

\[ \delta_5 = -\delta_3 + [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] z_{cp} L^2 / 8 E_{cm} I_{xx} \]

\[ \delta_6 = +\delta_4 + 5 w_1 \varphi_{28} L^4 / 384 E_{cm} I_{xx} \]
..followed by finishes, dead loads $w_2$ after 28 days

\[
\delta_5 = - \delta_3 + \left[ \varphi_{28} P_{pmi} - (P_{pmi} - P_{po}) \right] z_c p L^2 / 8 E_{cm} I_{xx}
\]

\[
\delta_6 = + \delta_4 + 5 w_1 \varphi_{28} L^4 / 384 E_{cm} I_{xx}
\]

\[
\delta_7 = + (1 + \varphi_{28}) 5 w_2 L^4 / 384 E_{cm} I_{xx}
\]
..and finally live loads $\psi_2 w_3$ over infinity time

\[
\begin{align*}
\delta_5 & = -\delta_3 + \left[ \varphi_{28} P_{pmi} - (P_{pmi} - P_{po}) \right] z_{cp} \frac{L^2}{8 E_{cm} I_{xx}} \\
\delta_6 & = +\delta_4 + 5 w_1 \varphi_{28} \frac{L^4}{384 E_{cm} I_{xx}} \\
\delta_7 & = + (1 + \varphi_{28}) 5 w_2 \frac{L^4}{384 E_{cm} I_{xx}} \\
\delta_8 & = + (1 + 0.8 \varphi_{\infty}) 5 \psi_2 w_3 \frac{L^4}{384 E_{cm} I_{xx}}
\end{align*}
\]
Active deflections due to creep effects and live loads takes parts of the previous equations

\[ \delta_9 = [\varphi_{28} P_{pmi} - (P_{pmi} - P_{po})] \frac{z_{cp} L^2}{8 E_{cm} I_{xx}} + \varphi_{28} 5 (w_1 + w_2) \frac{L^4}{384 E_{cm} I_{xx}} + (1 + 0.8 \varphi_\infty) 5 \psi_2 w_3 \frac{L^4}{384 E_{cm} I_{xx}} \]

For composite design, replace \( I_{xx} \) with \( I_{xx,c} \)
Calculate camber, installation and long-term deflection
8.0 m effective span
Worked example

Self weight = 182791 x 24.5 x 10^{-6} = 4.48 kN/m

Dead loads = 3.0 kN/m^2 = 3.60 kN/m per unit

Use of floor = offices, then \( \psi_2 = 0.3 \)

Live load = 0.3 x 4.0 = 1.2 kN/m^2 = 1.44 kN/m per unit
Worked example

Camber at transfer

$$\delta_1 = \frac{739.0 \times 10^3 \times 82.75 \times 8000^2}{8 \times 32837 \times 1289 \times 10^6} = -11.6 \text{ mm}$$

Self weight

Self weight hcu only

$$\delta_2 = \frac{5 \times 4.48 \times 8000^4}{384 \times 32837 \times 1289 \times 10^6} = +5.7 \text{ mm}$$

Net camber = -5.9 mm < length / 300 = 26 mm
Camber at installation

\[ \varphi_1 = 2.5 \times (0.4 - 0.1) \times \frac{32837}{0.5 \times (32837 + 36283)} = 0.71 \]

\[ \delta_3 = -11.6 \times (1 + 0.71) = -19.8 \text{ mm} \]

Self weight at installation

\[ \delta_4 = +5.7 \times (1 + 0.71) = +9.7 \text{ mm} \]
Long term camber

\[ \varphi_\infty = 0.8 \times 2.5 = 2.0 \quad \text{for live load} \]

\[ \varphi_{28} = 2.0 \times (1 - 0.4) = 1.2 \quad \text{for creep of camber and dead load} \]

\[ \delta_5 = -19.8 - \frac{[717.8 \times 1.2 - (717.8 - 614.3)] \times 82.75 \times 8000^2}{8 \times 36283 \times 1289 \times 10^6} \]

\[ = -19.8 - 10.7 = -30.5 \text{ mm} \]
Long term dead + live

\[ \delta_6 = +9.7 + \frac{5 \times (1.2 \times 4.73 + 2.2 \times 3.6 + 3.0 \times 1.44) \times 8000^4}{384 \times 36283 \times 1289 \times 10^6} \]

= +30.1 mm

Final = -30.5 + 30.1 = -0.4 mm < span/250 = 26 mm
Conclusions to EC2 Prestress

1. Only 1 value for tension class = $f_{\text{ctm}}$

2. Zero tension if exposure > XC1

3. Prestress losses for initial relaxation and elastic shortening, plus shrinkage, creep and relaxation

4. Ultimate stress and strain equilibrium

5. Camber = immediate at transfer + creep

6. Deflections = static + creep