Design of hollowcore cross-sections

Gösta Lindström
Strängbetong, Sweden

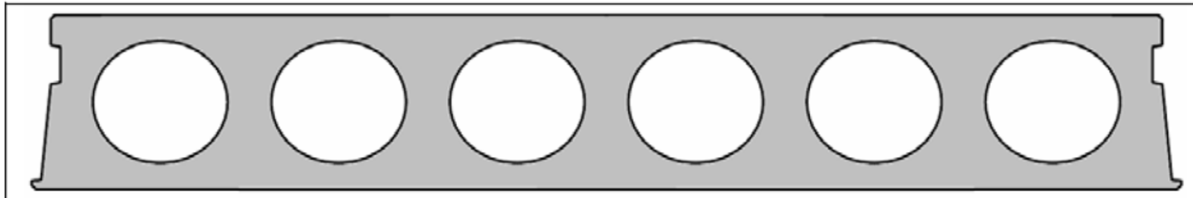
Topics to be presented *(from coming fib guide)* **:**

- **Shear capacity calculation**
  - Shear flexure
  - Shear tension
- **Anchorage**
- **Bending**
- **Torsion**
- **Interaction effects**
  - Shear and torsion
  - Shear and bending.

Is there one optimized cross section ?

- **Important aspects**
  - Spans
    - Maximize (load capacity) / (slab weight)
  - Sound properties
    - Sufficient weight
  - Fire protection
    - Strands protected for sufficient bending moment capacity
    - Shear capacity ?
  - Minimum of floor depth
    - Homogenous hollow cores ?
  - Integrated installations
    - Heating / Cooling
    - Electricity
    - Piping.

Design of hollow core cross-sections



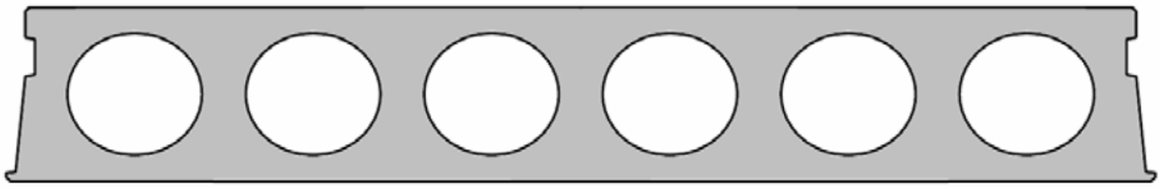
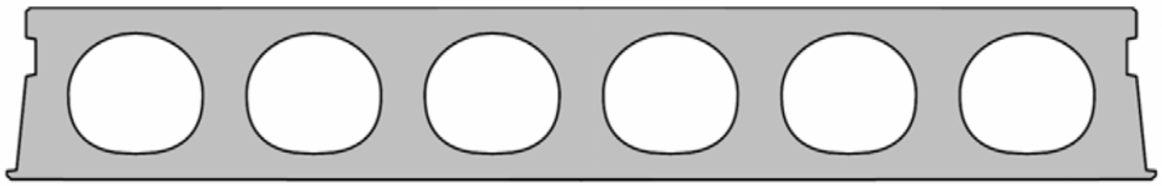
Height = 200 mm
Tfl_top = 30 mm
Tfl_bot = 30 mm
Bw = 45.67 mm
$N_{top} = N_{bot} = 2$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

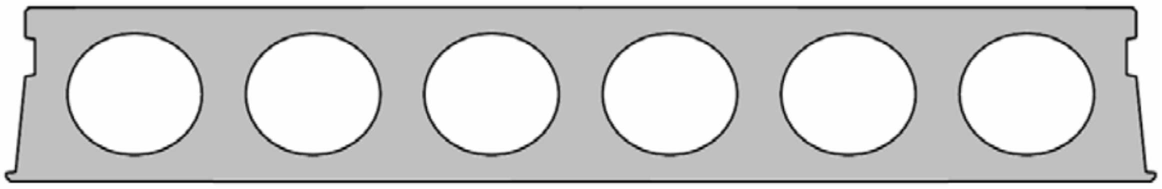
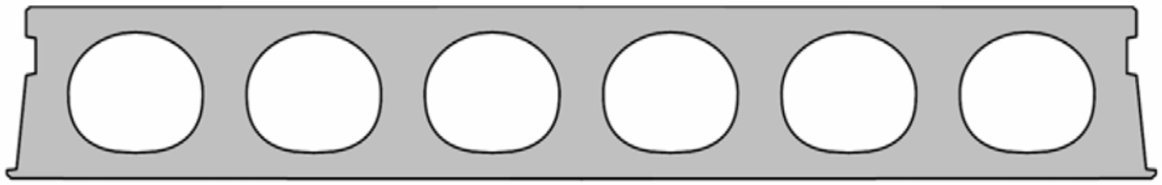
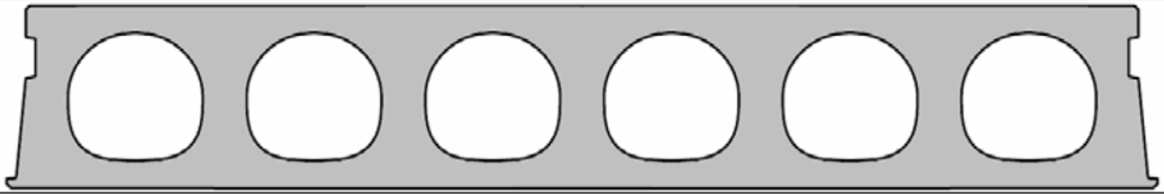
	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

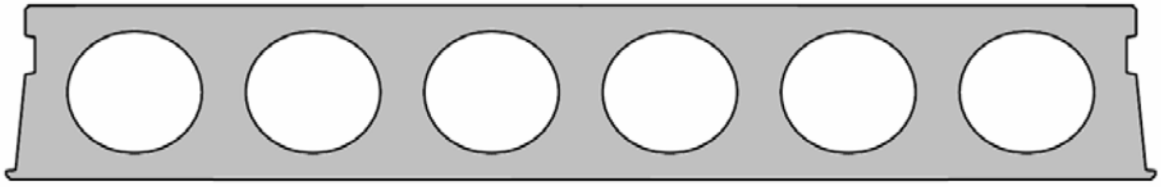
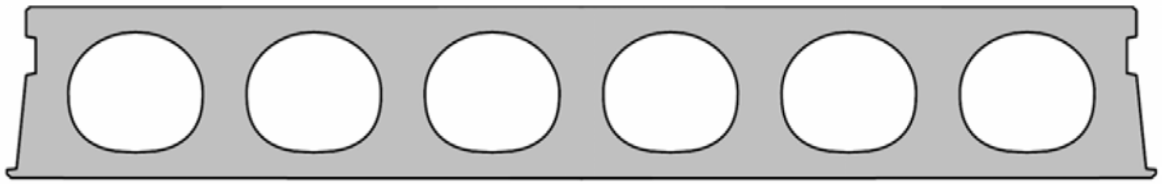
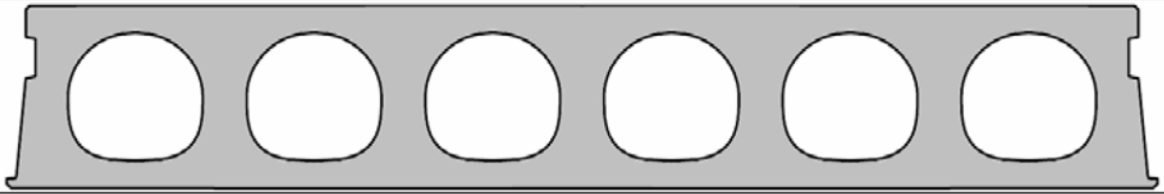
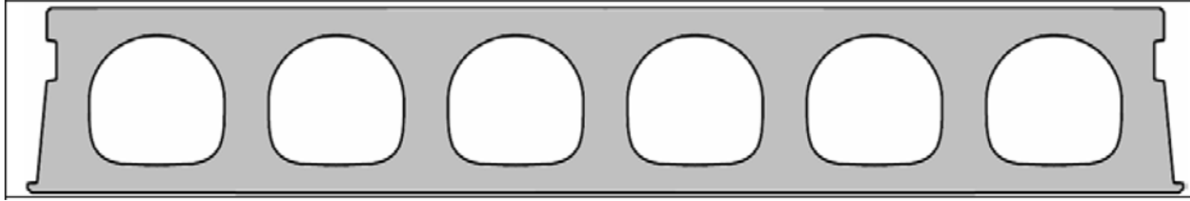
	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$
	$N_{top} = 2$ $N_{bot} = 3$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

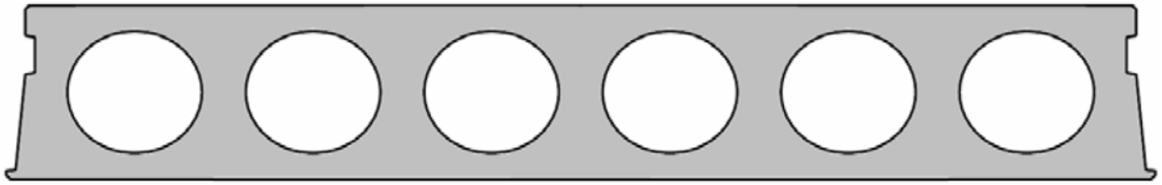
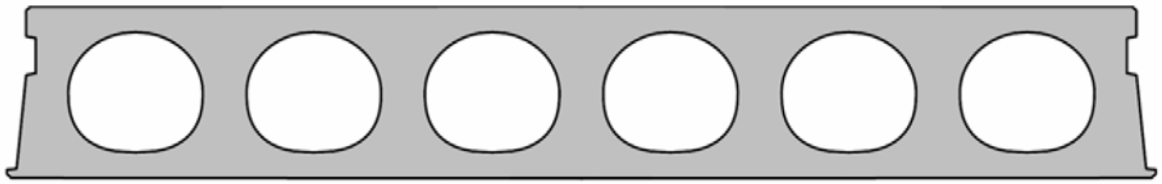
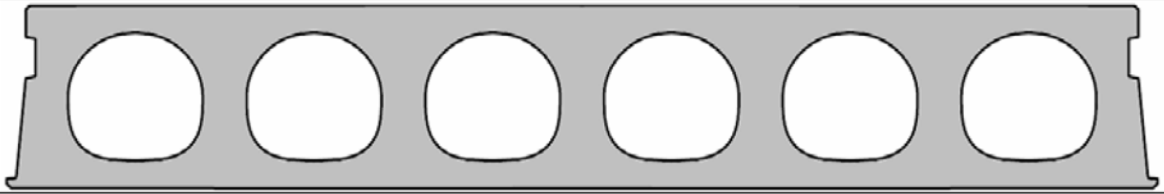
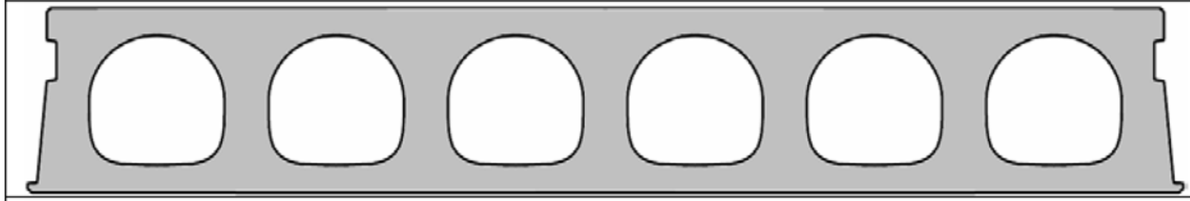
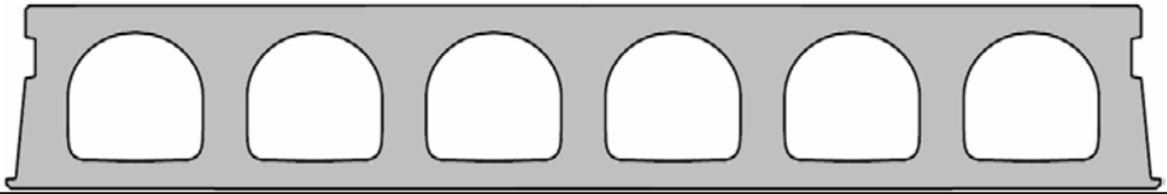
	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$
	$N_{top} = 2$ $N_{bot} = 3$
	$N_{top} = 2$ $N_{bot} = 4$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

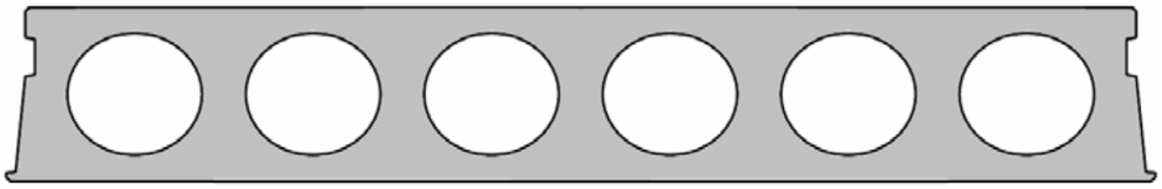
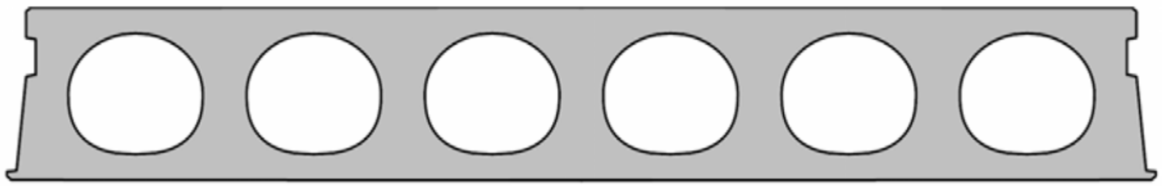
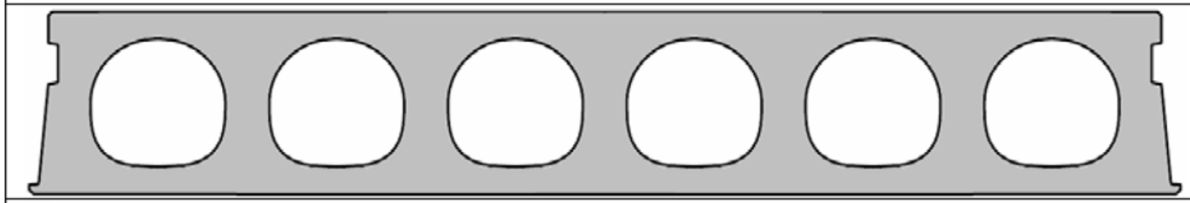
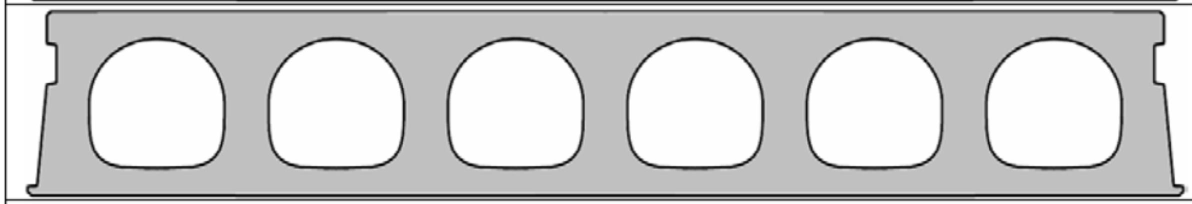
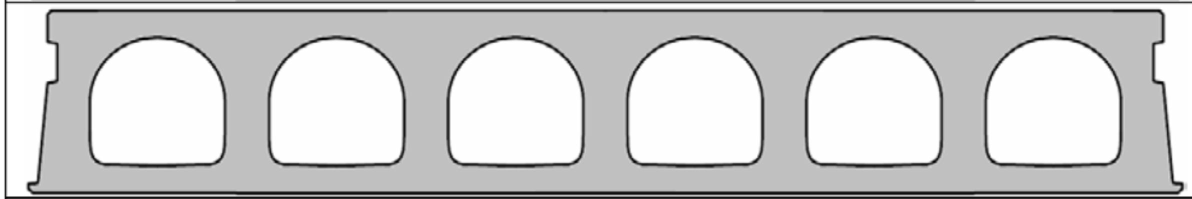
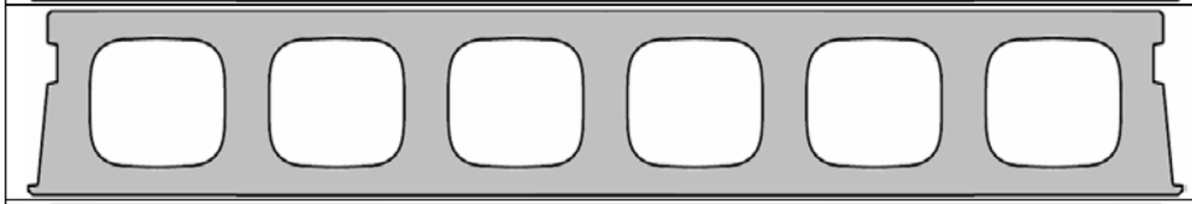
	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$
	$N_{top} = 2$ $N_{bot} = 3$
	$N_{top} = 2$ $N_{bot} = 4$
	$N_{top} = 2$ $N_{bot} = 8$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

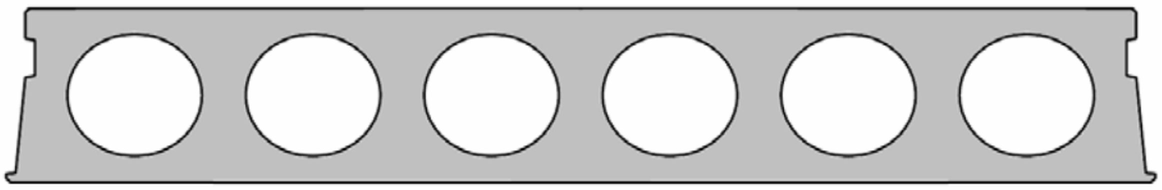
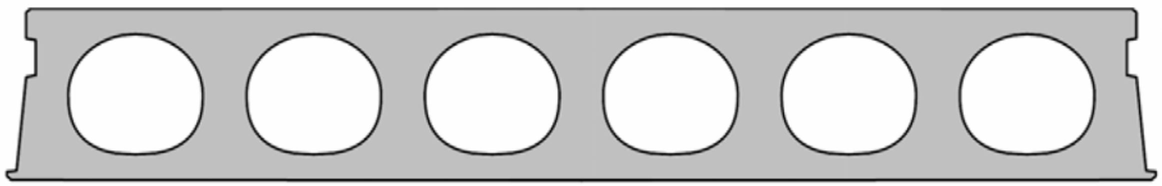
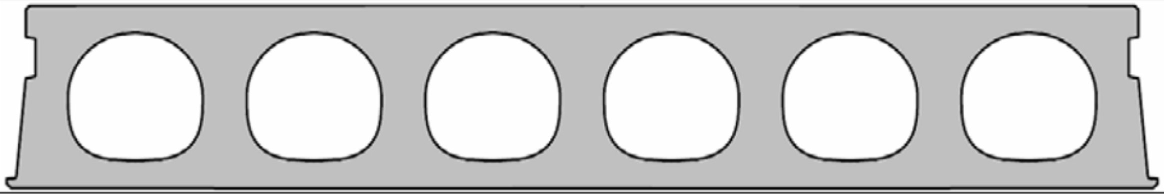
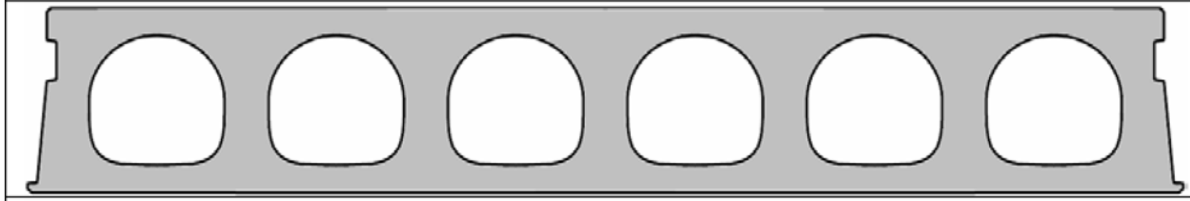
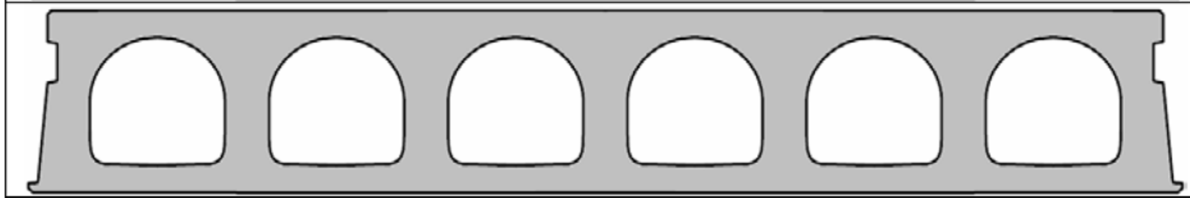
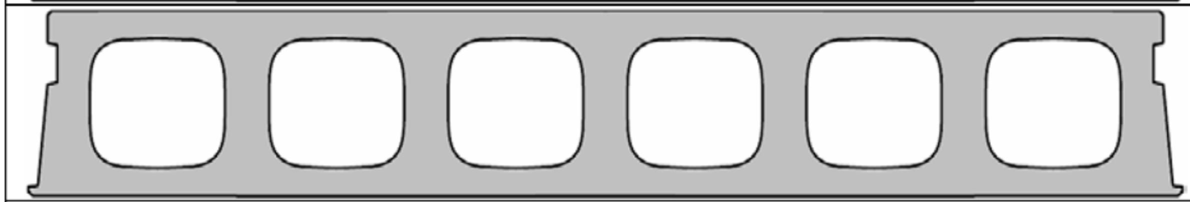
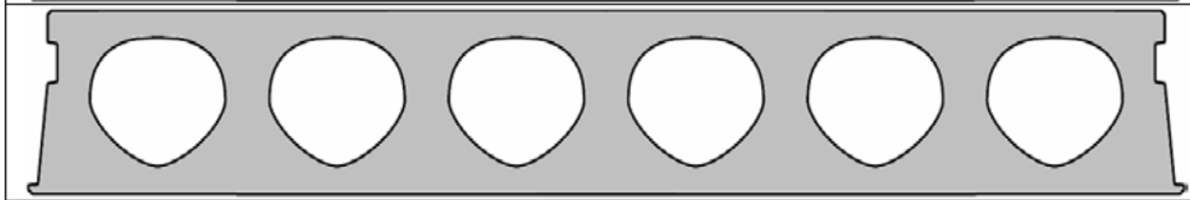
	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$
	$N_{top} = 2$ $N_{bot} = 3$
	$N_{top} = 2$ $N_{bot} = 4$
	$N_{top} = 2$ $N_{bot} = 8$
	$N_{top} = 4$ $N_{bot} = 4$

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Design of hollow core cross-sections

	Height = 200 mm Tfl_top = 30 mm Tfl_bot = 30 mm Bw = 45.67 mm $N_{top} = N_{bot} = 2$
	$N_{top} = 2$ $N_{bot} = 2.5$
	$N_{top} = 2$ $N_{bot} = 3$
	$N_{top} = 2$ $N_{bot} = 4$
	$N_{top} = 2$ $N_{bot} = 8$
	$N_{top} = 4$ $N_{bot} = 4$
	$N_{top} = 2.5$ $N_{bot} = 1.5$

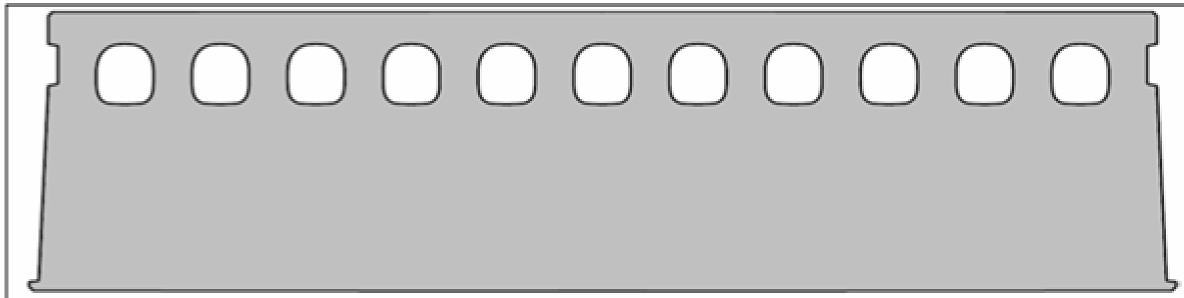
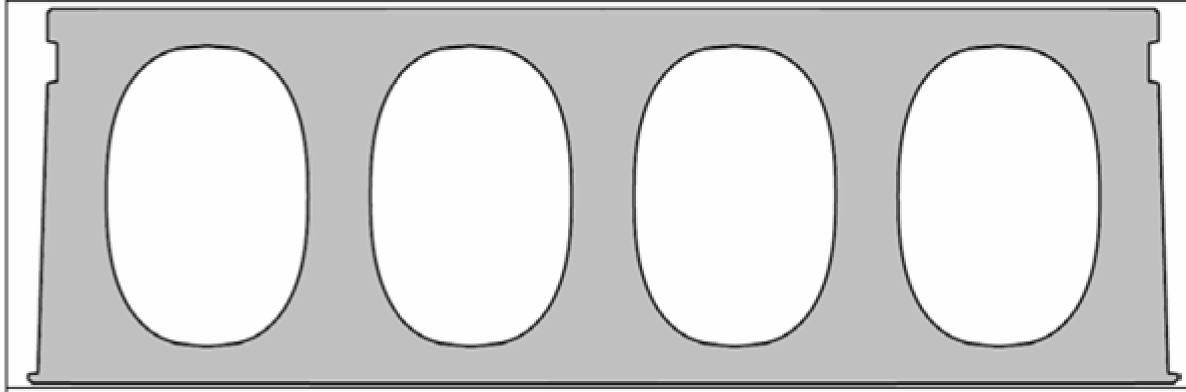
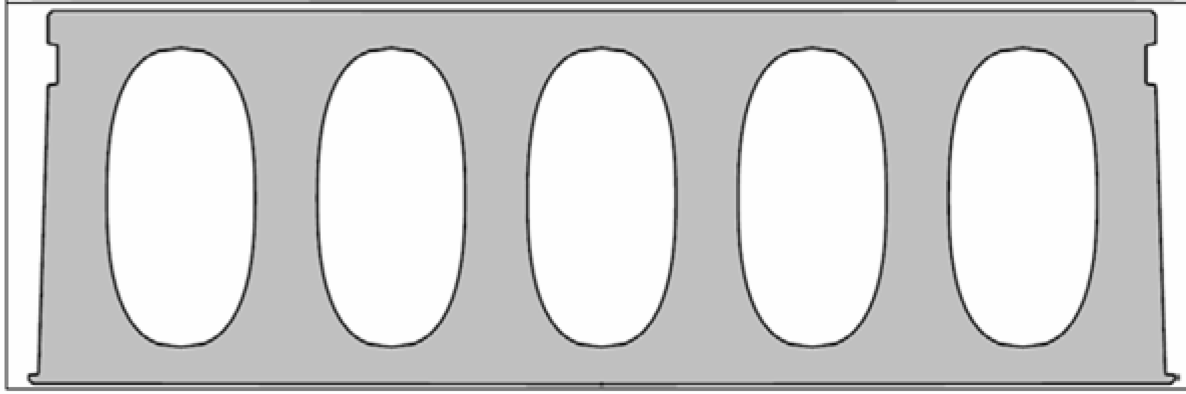
$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

STRÄNGBETONG

Design of hollow core cross-sections

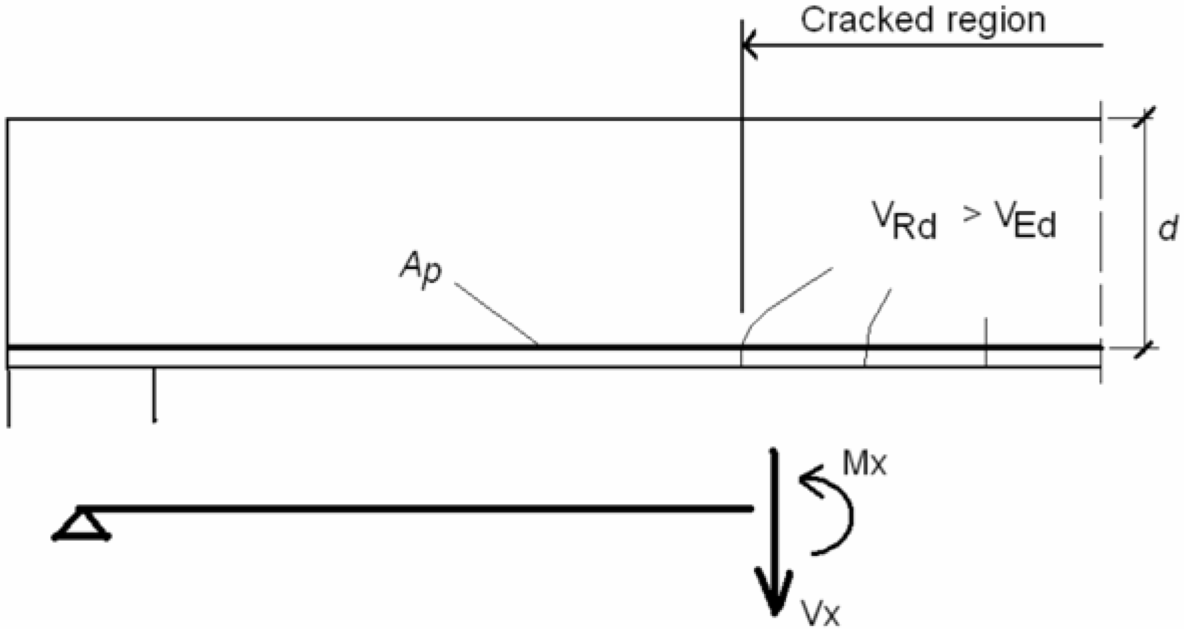
	H = 300 mm Tfl_top = 35 mm Tfl_bot = 200 mm Bw = 40 mm N_{top} = 2.5 N_{bot} = 4
	H = 400 mm tfl_top = 40 mm tfl_bot = 40 mm Bw = 65 mm N_{top} = 2.5 N_{bot} = 2.5
	H = 400 mm tfl_top = 40 mm tfl_bot = 40 mm Bw = 65 mm N_{top} = 2.5 N_{bot} = 2.5

$$\left(\frac{x}{r_H}\right)^N + \left(\frac{z}{r_V}\right)^N = 1$$

Core shape description

- **Radius**
  - Vertical
  - Horizontal
- **Shape coefficient**
  - N

Shear capacity in regions **cracked** in flexure

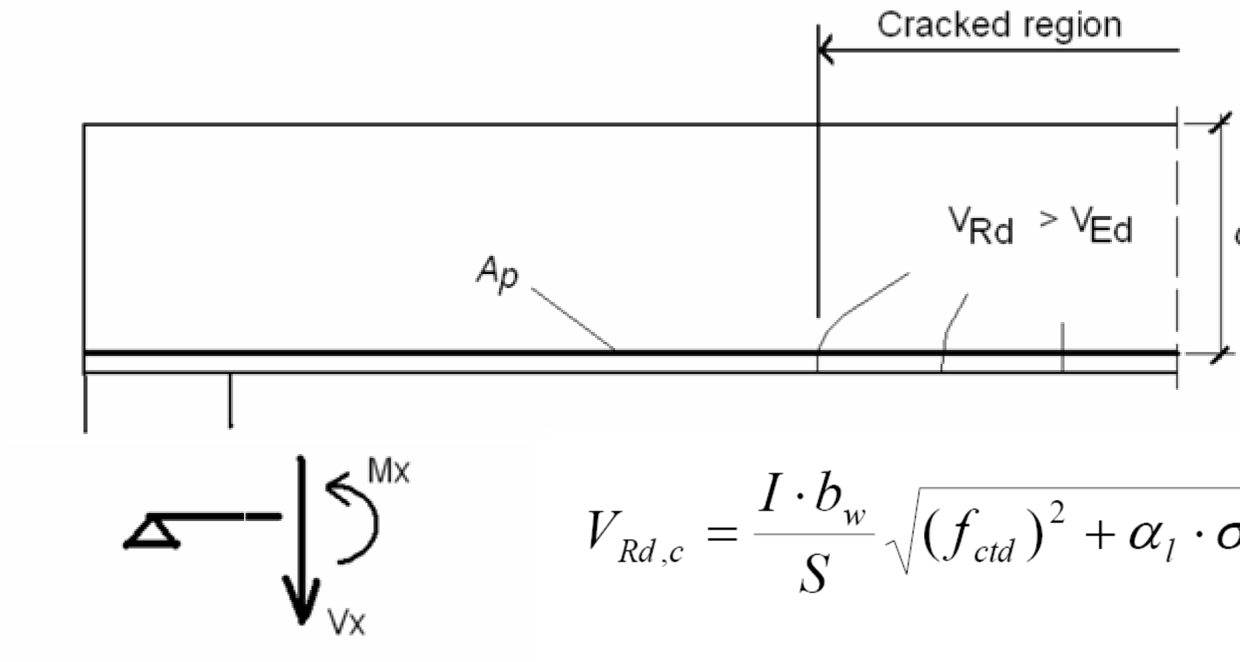


$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w d$$

From EC2

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} \cdot \min\left\{ \left(1 + \sqrt{\frac{0.200}{d}} \right); 2.0 \right\} \cdot \left(100 \cdot \min\left\{ \frac{A_p}{b_w d}; 0.02 \right\} \cdot f_{ck} \right)^{\frac{1}{3}} + 0.15 \cdot \min\left\{ \left(\frac{N_{Ed}}{A_c} \right); 0.2 \cdot f_{cd} \right\} \right]$$

Shear capacity in regions **uncracked** in flexure

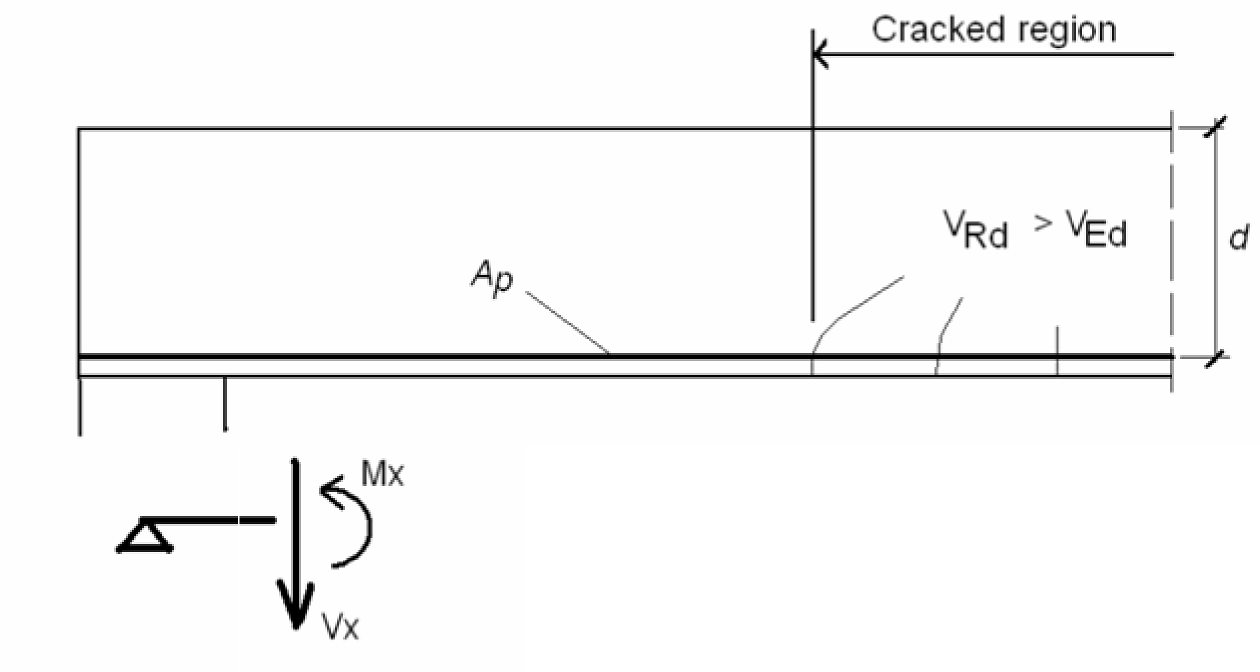


$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \cdot \sigma_{cp} \cdot f_{ctd}}$$

From EC2

I	is the second moment of area of the cross-section with respect of the centroidal axis
b_w	is the width of the cross-section at the centroidal
S	is the first moment of area for the part of the cross-section above the centroidal axis evaluated with respect of the centroidal axis.
f_{ctd}	tensile strength of concrete expressed as a design value ($= f_{ctk}/\gamma_c$)
α_l	$= l_x / l_{pt2} \leq 1.0$ for pretensioned tendons where
l_x	is the distance of section considered from the starting point of transmission length
l_{pt2}	is the upper bound of the transmission length ($= 1.2\ l_{pt}$)
σ_{cp}	is the concrete compressive stress at the centroidal axis

*Shear capacity in regions **uncracked** in flexure*



From EC2

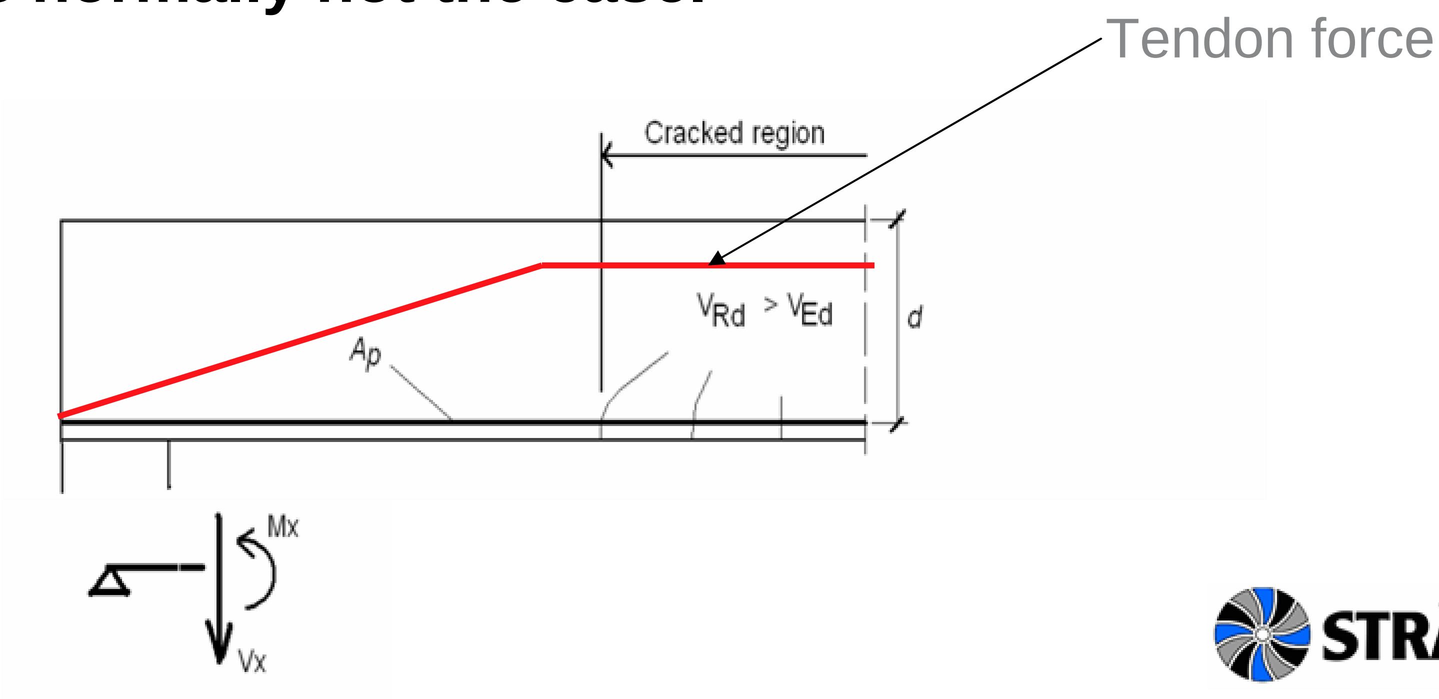
I	is the second moment of area of the cross-section with respect of the centroidal axis
b_w	is the width of the cross-section at the centroidal
S	is the first moment of area for the part of the cross-section above the centroidal axis evaluated with respect of the centroidal axis.
f_{ctd}	tensile strength of concrete expressed as a design value ($= f_{ctk}/\gamma_c$)
α_l	$= l_x / l_{pt2} \leq 1.0$ for pretensioned tendons where
l_x	is the distance of section considered from the starting point of transmission length
l_{pt2}	is the upper bound of the transmission length ($= 1.2\ l_{pt}$)
σ_{cp}	is the concrete compressive stress at the centroidal axis

*Shear capacity in regions **uncracked** in flexure*

EC2 formula OK for Hollow core cross-sections provided the following is fulfilled:

- **Minimum section width at centroidal axis**
- **X-section where prestressing force full transferred**

But, this is normally not the case.



Shear capacity in regions ***uncracked*** in flexure

Background for suggested calculation of Shear Tension Capacity

Outside the development length the strain distribution over cross-section height is linear and the concrete stress in uncracked region is given by the by the equation

$$\sigma_c = \frac{-P_x}{A} + \frac{-P_x \cdot e + M_x}{I} \cdot z \qquad (5.2)$$

In equation (5.2) the stress in the concrete cross-section is defined as positive in tension and

P_x	is the tendon force at the location along the slab (positive value)
e	is eccentricity of tendon force with respect of the centroidal axis (positive below the centroidal axis)
M_x	bending moment due to self weight and external loading at cross-section considered with positive moment resulting in tensile stresses in bottom fibre of cross-section
z	is the position of stress evaluation in the cross-section with respect of the centroidal axis (z is positive below the centroidal axis)
A	is the cross-section area
I	is the second moment of area of the cross-section with respect of the centroidal axis

*Shear capacity in regions **uncracked** in flexure*

Background for suggested calculation of Shear Tension Capacity

By assuming that the stress distribution also in the transfer region can be expressed by equation (5.2) the shear stress in the transfer region is by (Yang 1994) determined as

$$\tau = \frac{1}{b_w}\left[\left(\frac{A_{cp}}{A} - \frac{S_{cp}\cdot e}{I}\right)\frac{dP}{dx} + \frac{S_{cp}}{I}\cdot V\right] \qquad (5.3)$$

Where the shear stress ($\tau = \tau(x,z)$) is evaluated at location x along the hollow core axis and position z in vertical position in the cross-section. In equation (5.3) some additional definitions are introduced to be interpreted as follows

A_{cp} is the cross-section area above the position z with respect of the centroidal axis

S_{cp} is the first moment of the area (A_{cp}) for the part of the cross-section above the position z with respect to centroidal axis and evaluated with respect of the centroidal axis.

Shear capacity in regions **uncracked** in flexure

INTERNATIONAL PRESTRESSED HOLLOWCORE ASSOCIATION

Technical Seminar 2007

Background for suggested calculation of Shear Tension Capacity

At a location x,z with the normal stress σ_c and the shear stress τ the maximum principal stress σ_I is determined as

$$\sigma_I = \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + \tau^2} \qquad (5.4)$$

Which can be rearranged to express the shear stress as

$$\tau = \sqrt{\sigma_I^2 - \sigma_I \cdot \sigma_c} = \sigma_I \sqrt{1 - \frac{\sigma_c}{\sigma_I}} \qquad (5.5)$$

The condition for shear tension failure is reached when the maximum principal stress is equal to the concrete tensile strength f_{ct} resulting in the failure criteria in a design situation

$$\tau = f_{ctd} \sqrt{1 - \frac{\sigma_c}{f_{ctd}}} \qquad (5.6)$$

Shear capacity in regions ***uncracked*** in flexure

Background for suggested calculation of Shear Tension Capacity

By combining equation (5.3) and (5.6) the shear tension capacity V can be determined from the equation

$$V_{Rd} = \boxed{\left(e - \frac{I \cdot A_{cp}}{S_{cp} \cdot A}\right)\frac{dP}{dx}} + \boxed{\frac{I \cdot b_w}{S_{cp}} \cdot f_{ctd}\sqrt{1 - \frac{\sigma_c}{f_{ctd}}}} \qquad (5.7)$$

= EC2

Only in transfer region

Shear capacity in regions **uncracked** in flexure

Background for suggested calculation of Shear Tension Capacity

Close to the support:

$$M_x = R \cdot x = V_x \cdot x \qquad (5.8)$$

Inserting this into the expression for σ_c in (5.7) results in the possibility to express the shear capacity as

$$V_{Rd} = \left(e - \frac{I \cdot A_{cp}}{S_{cp} \cdot A}\right)\frac{dP}{dx} - \frac{I \cdot b_w^2}{S_{cp}^2} \cdot \frac{f_{ctd} \cdot x \cdot z}{2} + \frac{I \cdot b_w}{S_{cp}} \cdot$$

$$\cdot \sqrt{\left(\frac{b_w}{S_{cp}} \cdot \frac{f_{ctd} \cdot x \cdot z}{2}\right)^2 - \left(e - \frac{I \cdot A_{cp}}{S_{cp} \cdot A}\right)\frac{dP}{dx} \cdot \frac{f_{ctd} \cdot x \cdot z}{2} + f_{ctd}\left(f_{ctd} + \frac{P_x}{A} + \frac{P_x \cdot e \cdot z}{I}\right)} \qquad (5.9)$$

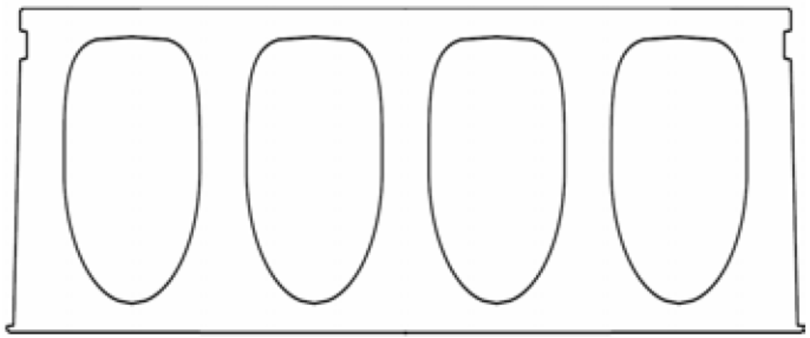
This is the expression suggested for EN 1168

For H > 400 mm

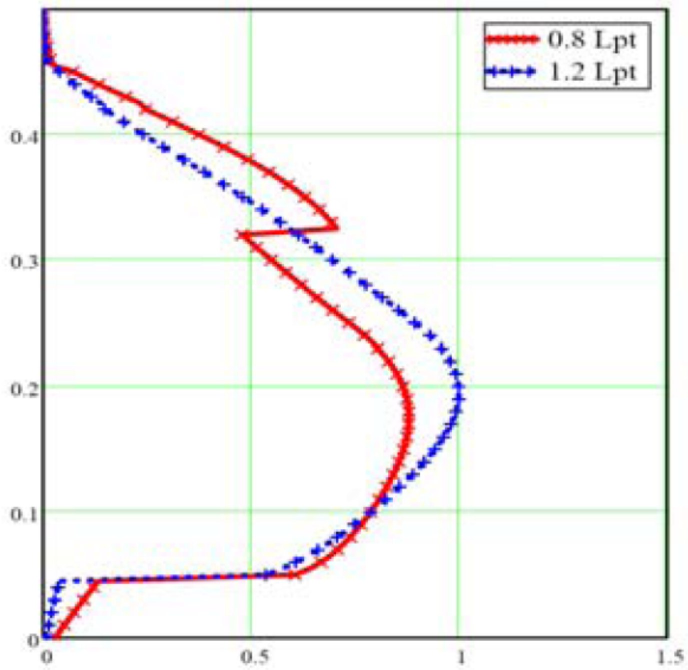
A reduction 0.9

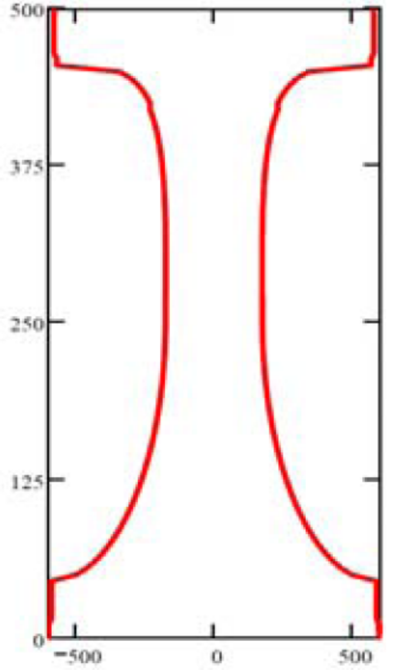
The minimum value of shear tension capacity are than evaluated with equation (5.9) using $z = z_{CG} - x \cdot \tan(35^o)$, where z_{CG} is the distance from bottom fibre to centre of gravity of the cross-section. Minimum is found by checking the range with z varying over the total web height.

Shear capacity in regions **uncracked** in flexure



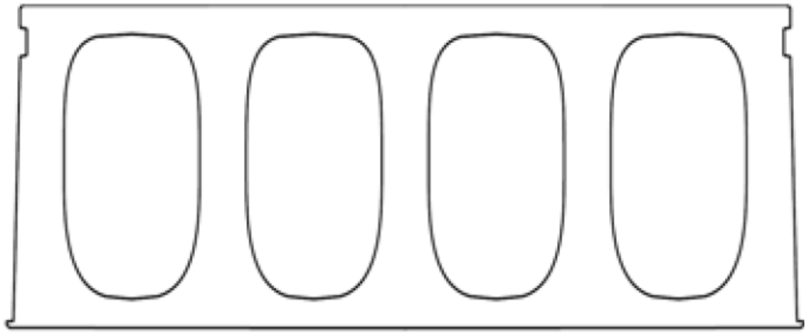
Shape C, V_{Rd} = 390.8 kN



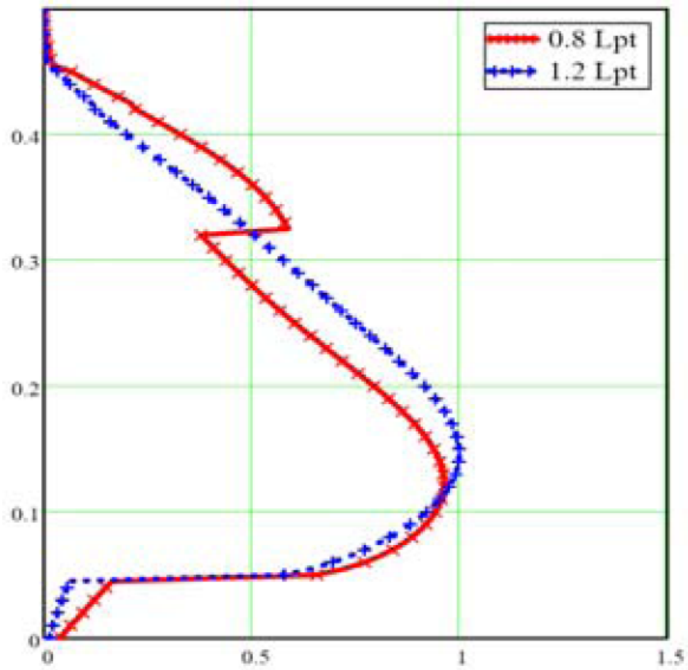


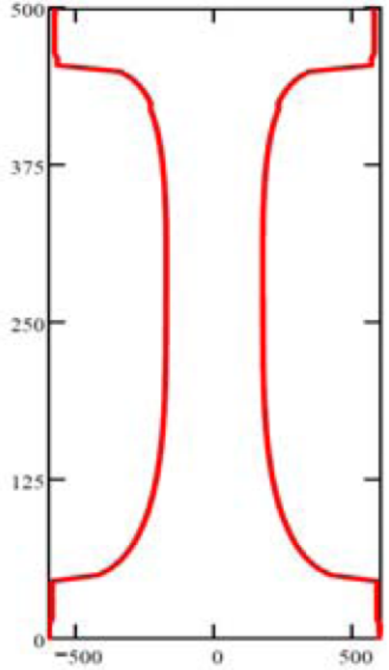
V_{RdEC2} = 383 kN $\qquad$ V_{Rd}/V_{RdEC2} = 1.02

Shear capacity in regions ***uncracked*** in flexure



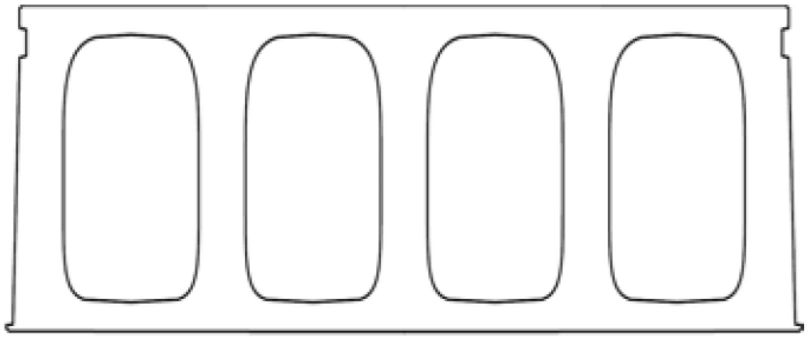
Shape B, $V_{Rd} = 346.0$ kN



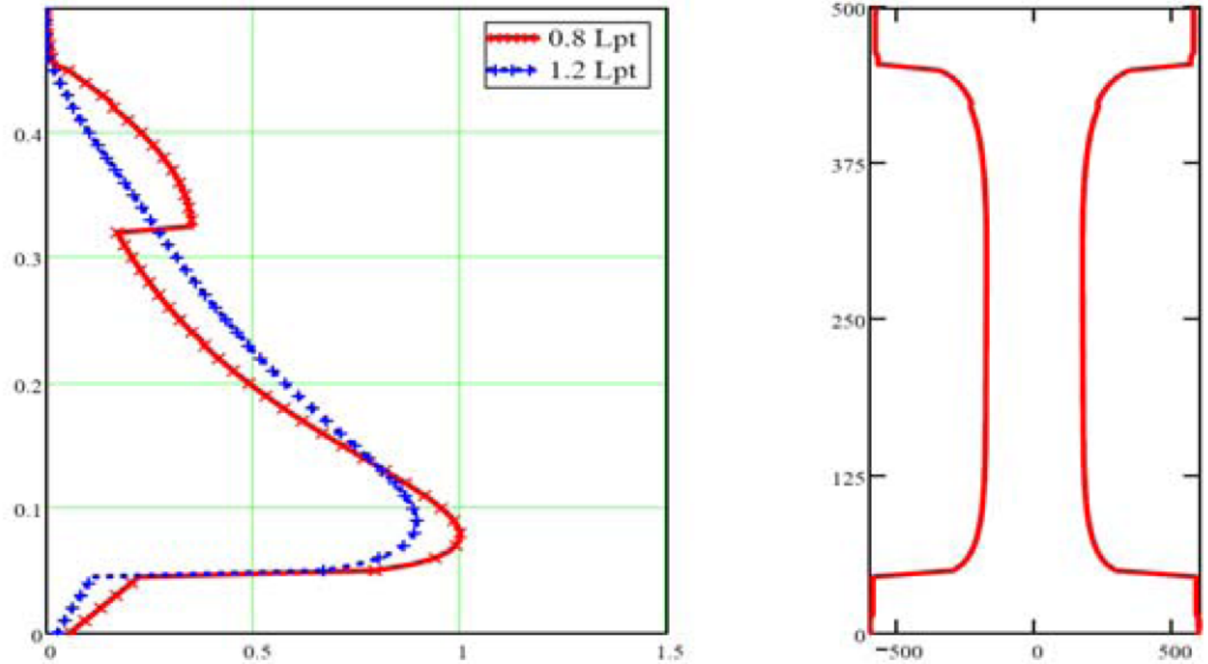


$V_{RdEC2} = 389.2$ kN $\qquad V_{Rd}/V_{RdEC2} = 0.89$

Shear capacity in regions ***uncracked*** *in flexure*

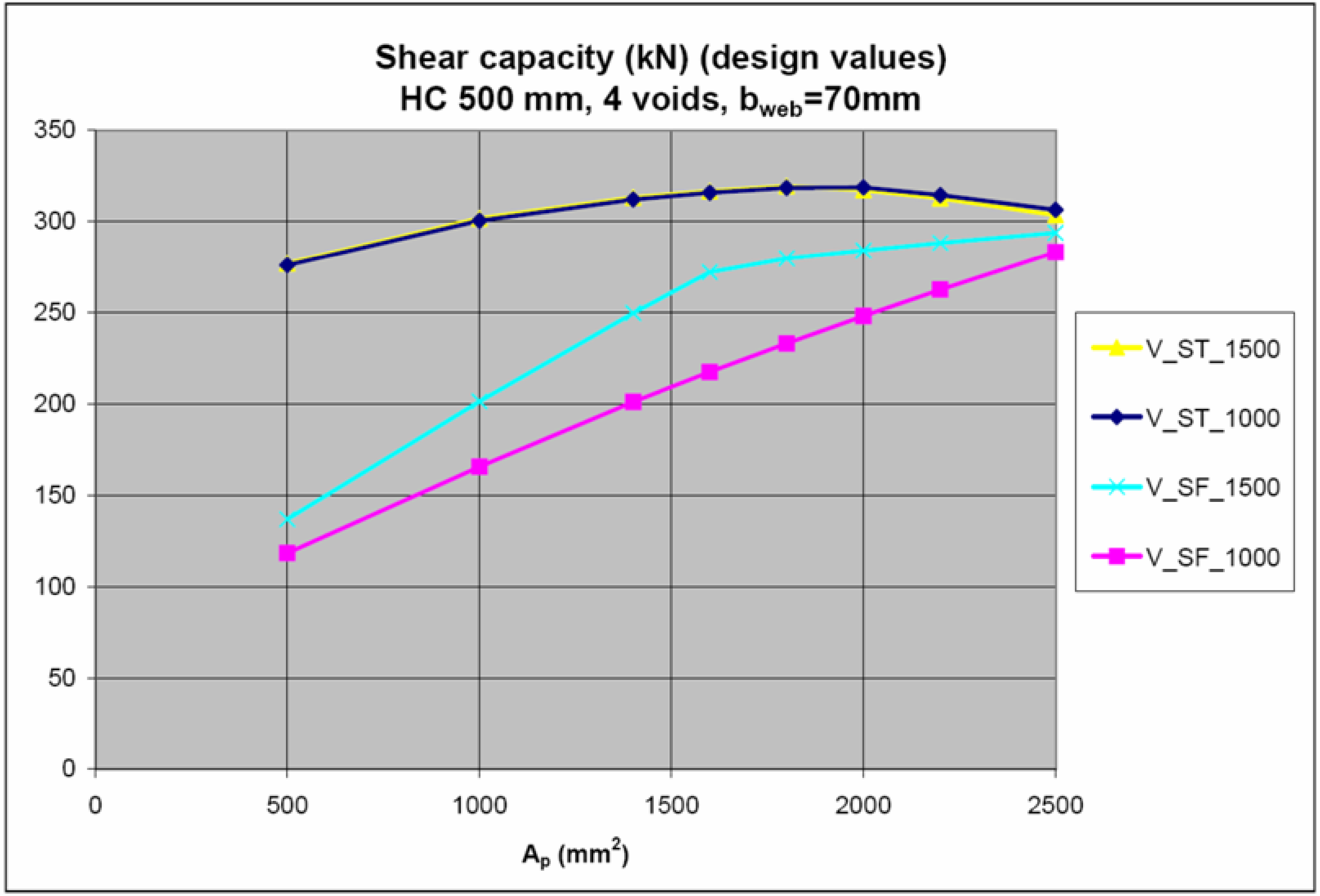


Shape A, V_{Rd} = 243.6 kN



V_{RdEC2} = 396 kN **V_{Rd}/V_{RdEC2} = 0.61**

Shear resistance



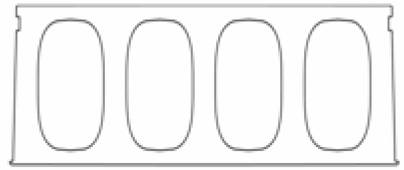
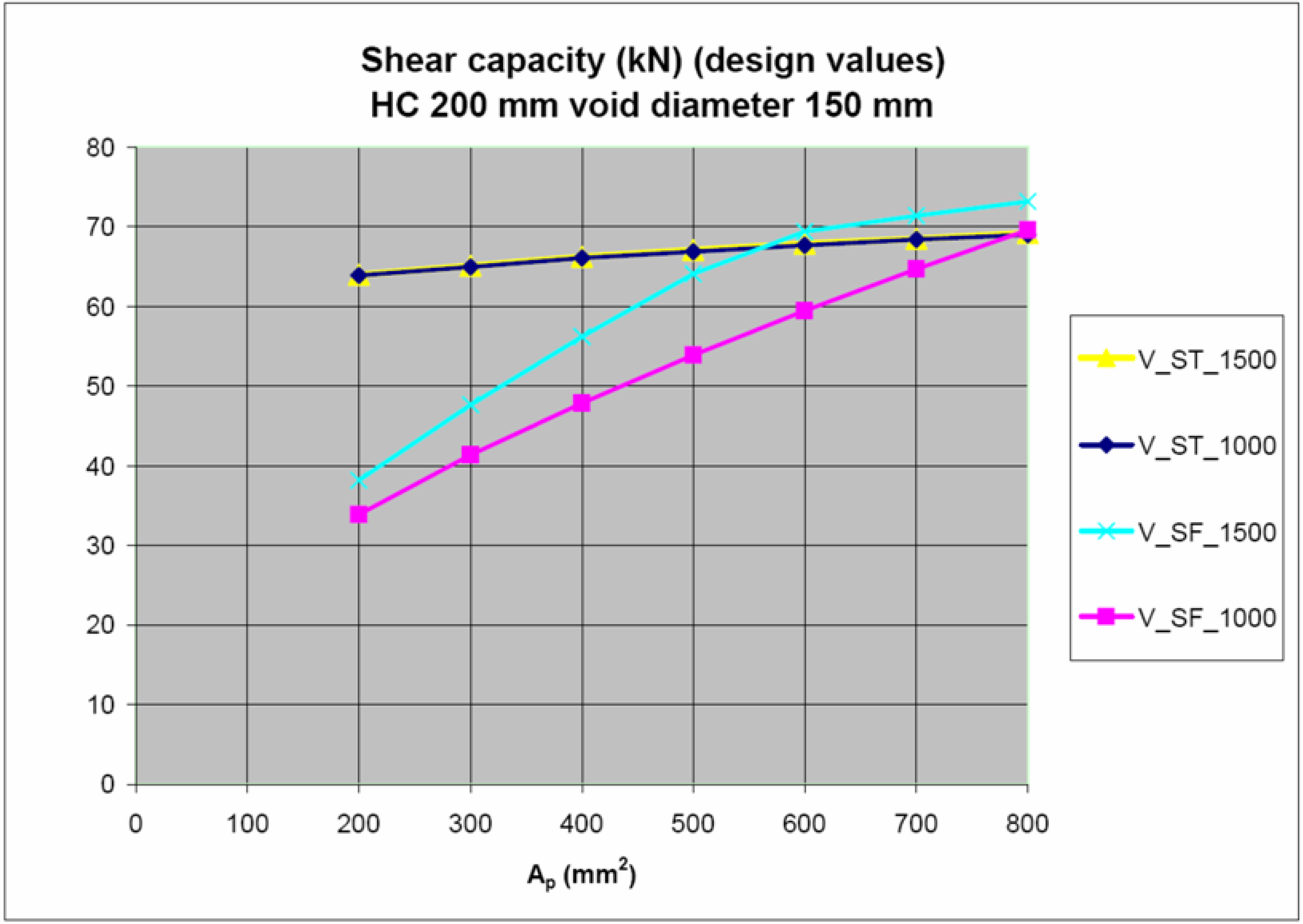


Figure 5.5 Design value of shear capacity (kN) for hollow core of height 500 mm, 4 voids, minimum web thickness 70 mm and concrete grade C 50/60. Capacity due to shear flexure (V_SF) and shear tension (V_ST) are presented for different initial prestressing levels 1500 and 1000 MPa. Resulting prestressing force is located 46.5 mm above bottom surface.

Shear resistance



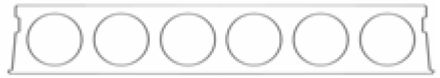


Figure 5.4 Design value of shear capacity (kN) for hollow core of height 200 mm, circular voids diameter 150 mm, minimum web thickness 35 mm and concrete grade C 40/50. Capacity due to shear flexure (V_SF) and shear tension (V_ST) are presented for different initial prestressing levels 1500 and 1000 MPa. Resulting pretressing force is located 37 mm above bottom surface.

Shear resistance for Hollow core 200 mm

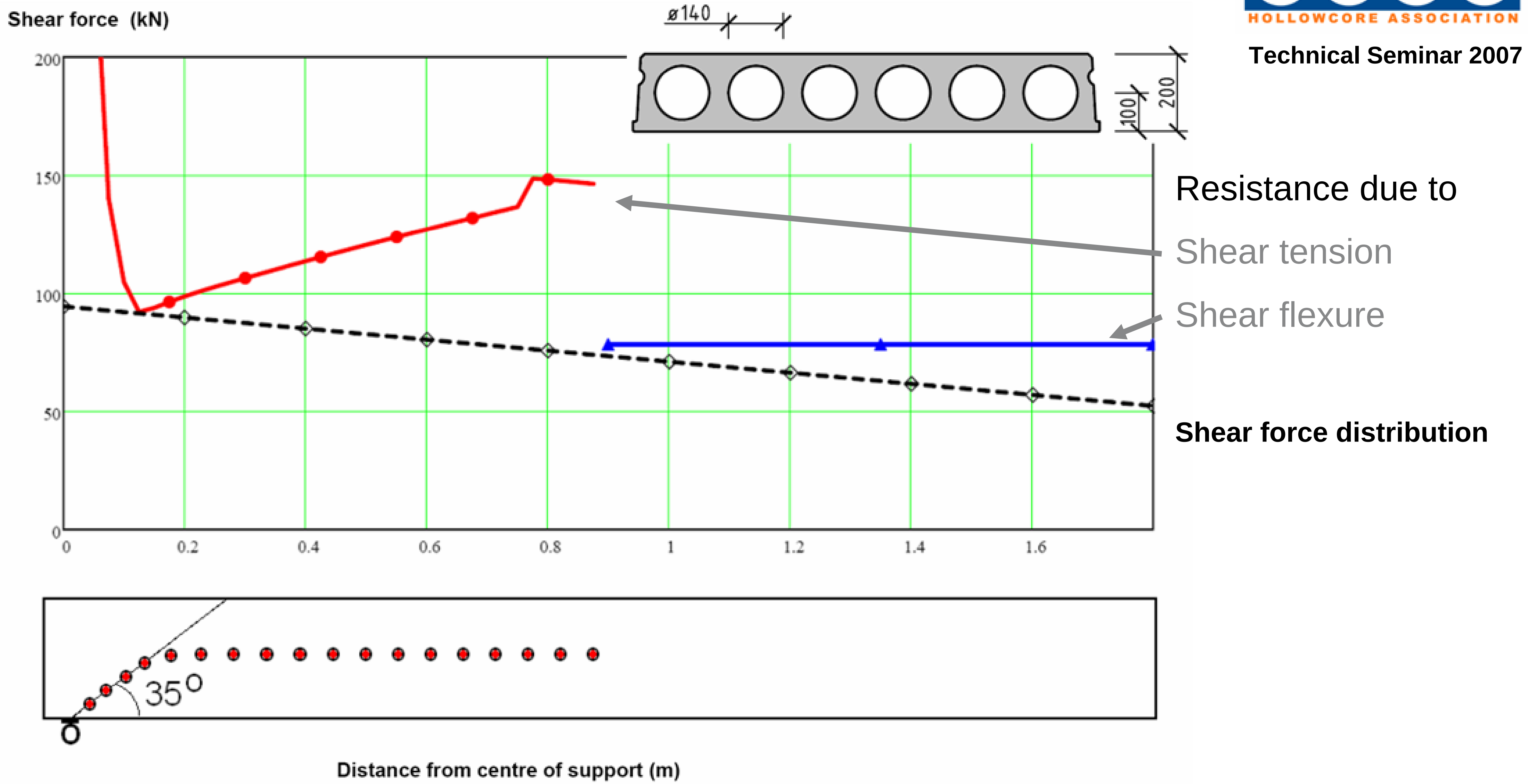
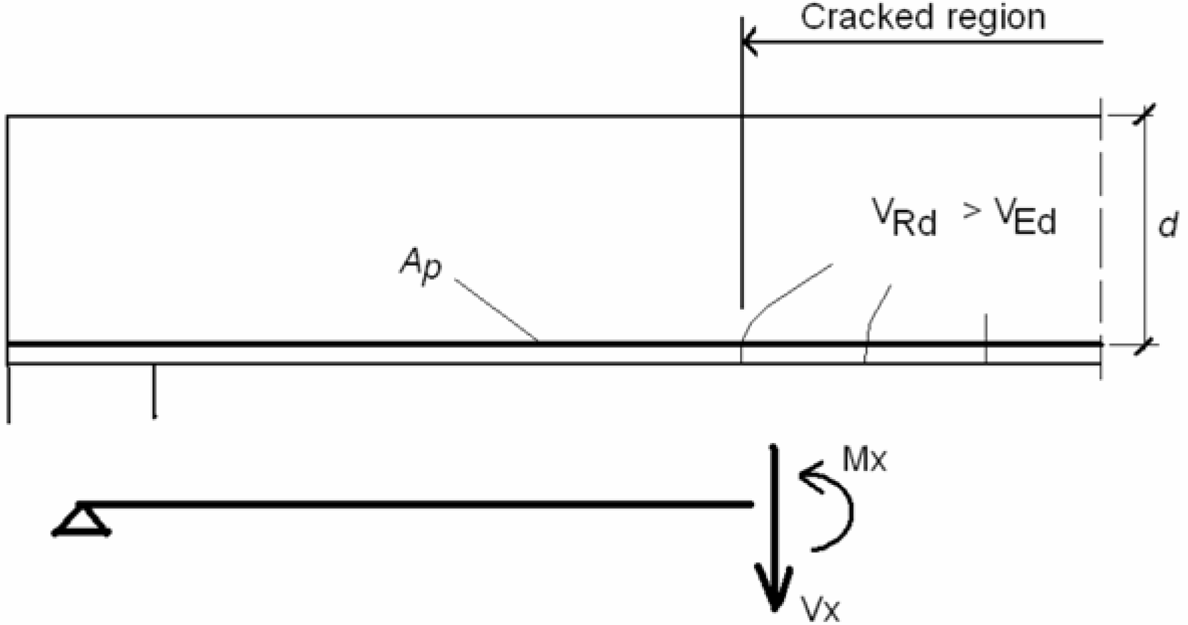


Figure 5.2 Shear resistance distribution (design value) close to the support for hollow core cross-section of height 200 mm with 6 circular voids of diameter 140 mm, concrete grade C40/50. Prestressing using 7 strands of diameter 12.9 mm with effective prestressing 850 MPa (after losses).

Anchorage capacity



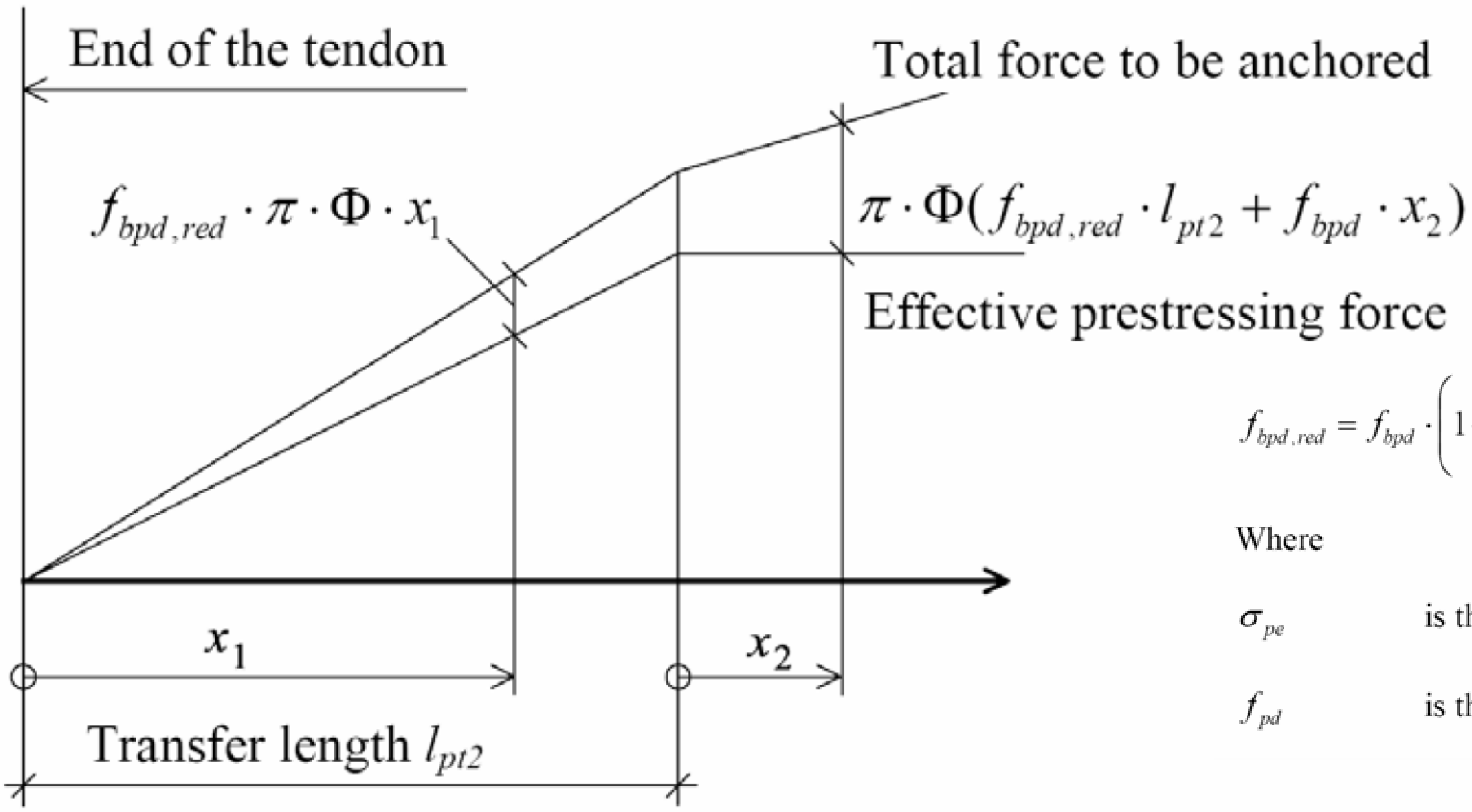
$$P_{xa\_req} = \min\left\{\left(\frac{M_x}{0.9d} + 1.5 \cdot V_x\right), \left(\frac{M_{\max}}{0.9d}\right)\right\} = \min\left\{\left(\frac{M_x(x+1.35d)}{0.9d}\right), \left(\frac{M_{\max}}{0.9d}\right)\right\}$$

Conservative approach is suggested

-Increase due to inclined shear crack (18.4 degrees)

-limited by the maximum tendon force needed in the span

Anchorage capacity



$$f_{bpd,red} = f_{bpd} \cdot \left(1 - \frac{\sigma_{pe}}{f_{pd}}\right)$$

Where

σ_{pe} is the effective prestress

f_{pd} is the prestressing strength $f_{p0.1k}/\gamma_s$

Anchorage capacity

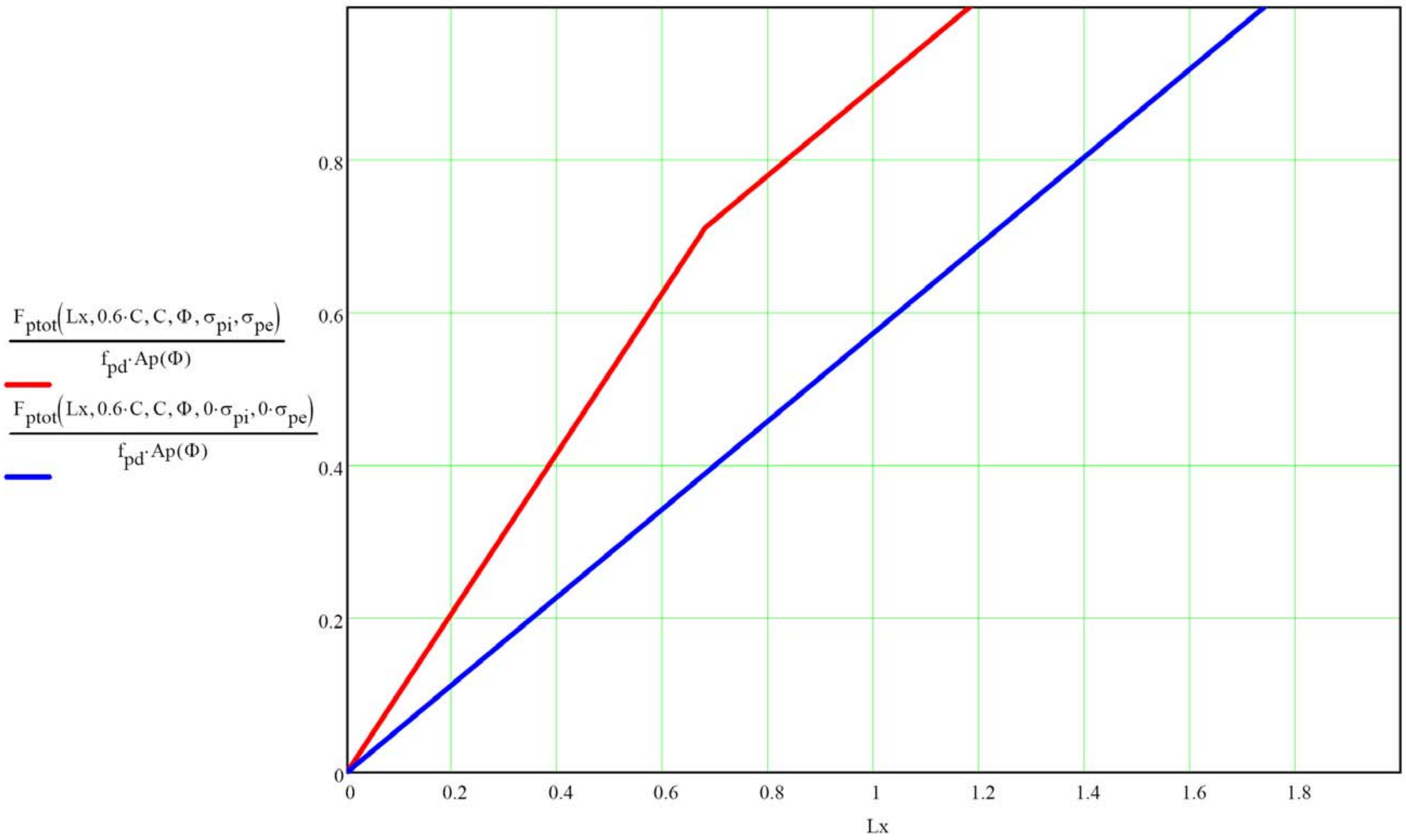
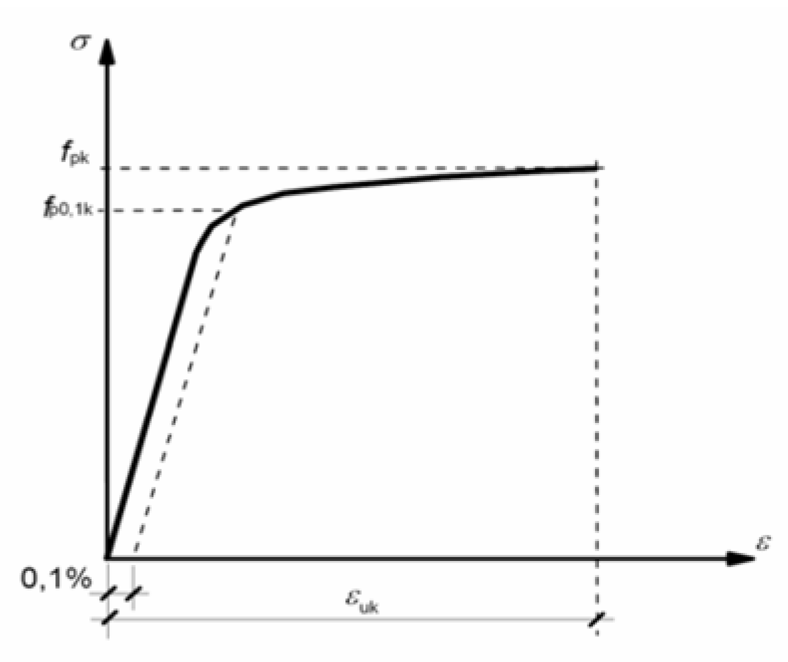


Figure 5.11 Strand anchorage with or without prestressing. The curves indicate the design value of anchorage length needed for a load level expressed as relation to strand strength. The data used are strand diameter 12.9 mm, initial prestressing 1000 MPa and effective prestressing 850 MPa and concrete grade C50/60. Strand with $f_{pk} = 2060$ and $f_{pd} = 1612$ MPa.

Bending moment



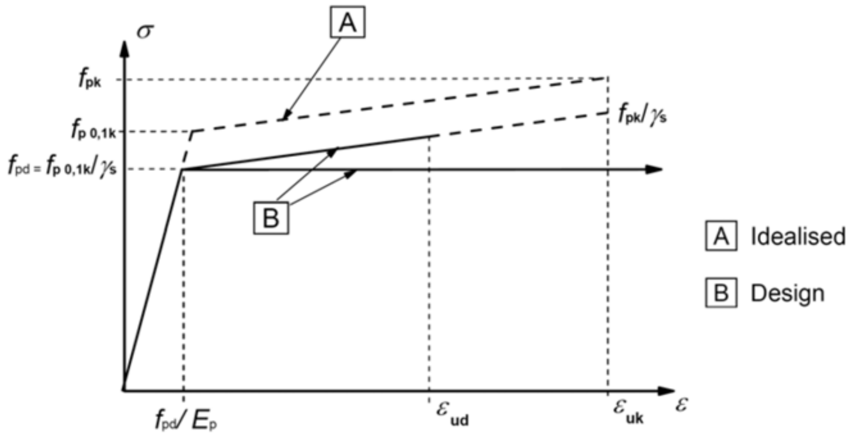


Figure 5.2.2-2 Idealised stress-strain relationship for prestressing steel and the change to design values.

Bending moment (as EC2)

-Calculated from:

- -Material stress-strain relationship
- -Axial equilibrium
- -Rotational equilibrium
- -Strain compatibility.

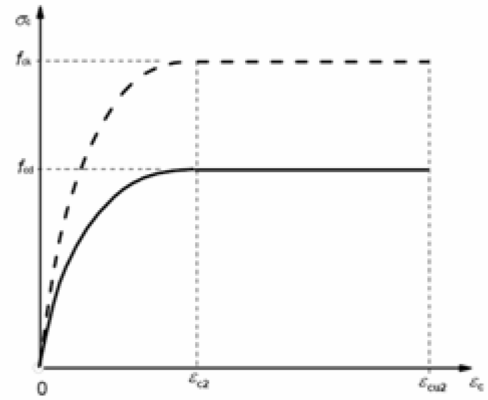


Figure 5.2.2-3 Parabola-rectangle diagram under compression (EC2).

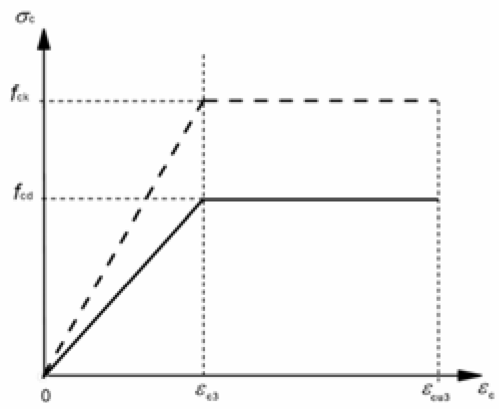


Figure 5.2.2-4 Bi-linear stress-strain relation (EC2).

Torsion

For a rectangular or circular cross-section subjected to torsional moment (T) the rotation gradient $(d\phi/dx)$ of the cross-section is defined by the torsional stiffness C by the equation

$$\frac{d\phi}{dx} = \frac{T}{C} = \frac{T}{GK_T}$$

Where C is the torsional rigidity GK_T where

G is the modulus of elasticity in shear $= \dfrac{E}{2(1+\nu)}$ where ν is the poisson's ratio.

K_T is the cross-sectional factor for torsional rigidity (m^4). For a circular section it is the same as the second area of moment along the polar central axis.

Torsion

For a solid rectangular cross-section [1] of width B larger then the depth H this cross-sectional factor is

$$K_{T,solid} = \frac{BH^3}{3}\left(1 - 0.630\frac{H}{B}\right)$$

The shear stress in a cross-section subjected to pure torsion (Saint-Venant torsion) can be calculated as

$$\tau_T = \frac{T}{W_T} \qquad \text{where}$$

W_T is the torsion resistance of the cross-section (m^3)

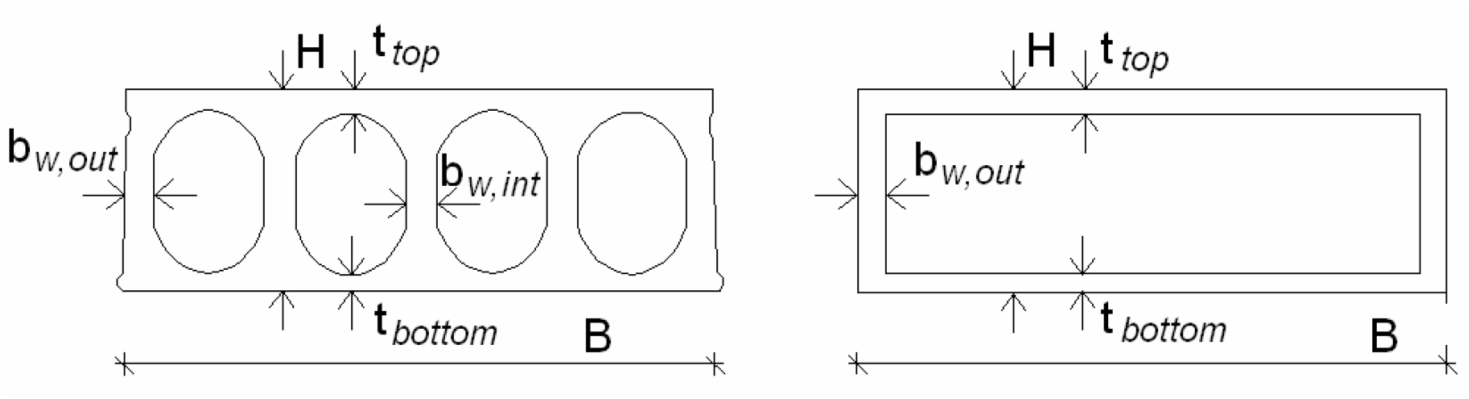


Figure 5.2.3-1 Transformation of a hollow core cross-section into a tubular cross-section for calculation of torsional cross-sectional properties.

For a thin walled rectangular cross-section with total width B and total depth H and with wall thickness t_{top} and t_{bottom} of the flanges and $b_{w,out}$ of the webs the cross-sectional factor is expressed as

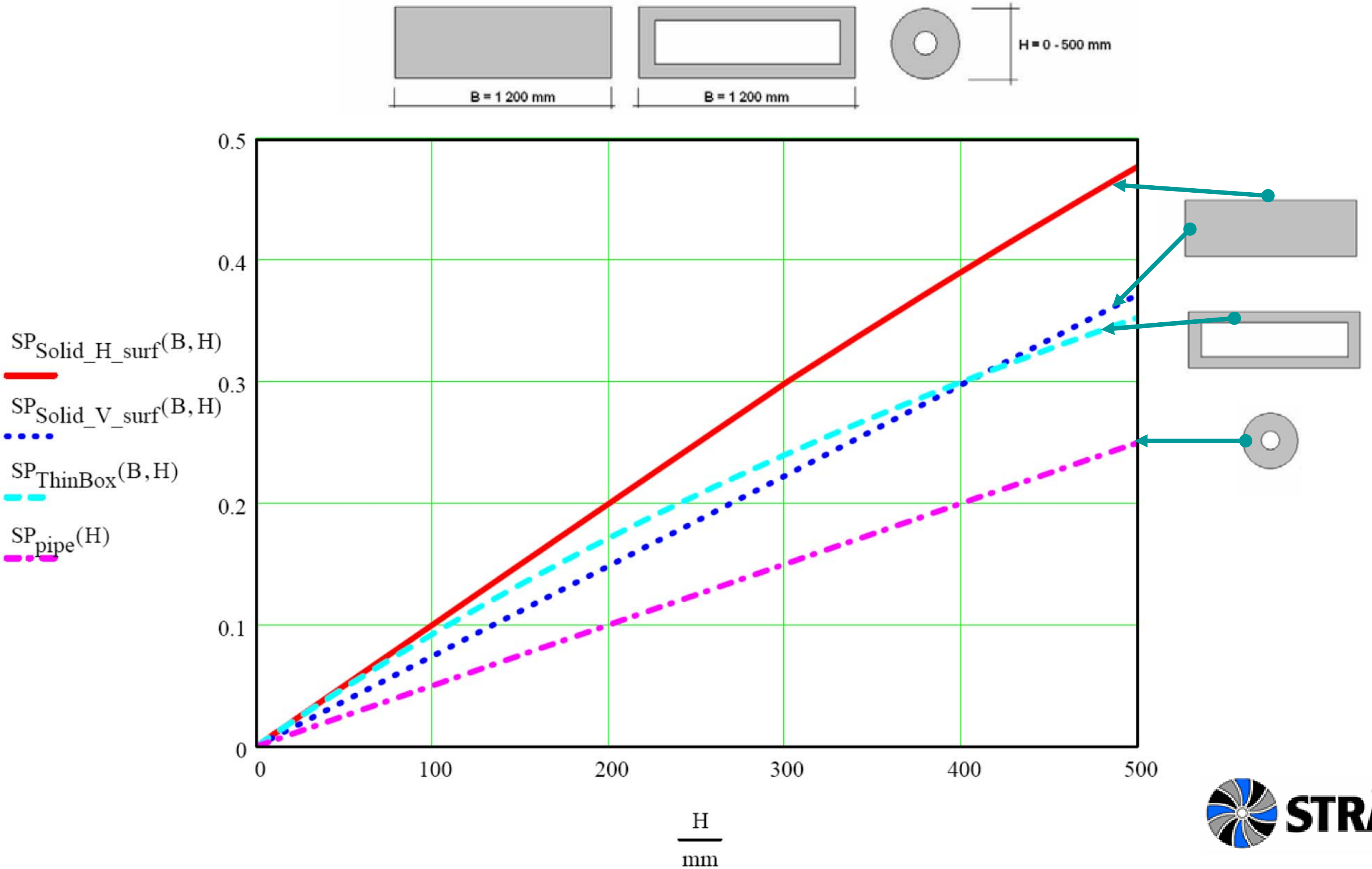
$$K_{T,tube} = \frac{4(B - b_{w,out})^2 (H - 0.5(t_{top} + t_{bottom}))^2}{(B - b_{w,out})\left(\frac{1}{t_{top}} + \frac{1}{t_{bottom}}\right) + (H - 0.5(t_{top} + t_{bottom}))\left(\frac{2}{b_{w,out}}\right)}$$

The torsional resistance for the same tubular cross-section can be expressed

$$W_T(t) = 2(H - 0.5(t_{top} + t_{bottom}))(B - b_{w,out}) \cdot t$$

$$\tau_T = G\frac{d\phi}{dx}\cdot\frac{K_T}{W_T}$$

Relative shear stress for different X-sections



For the **tubular cross-section** the wall thickness should be limited by

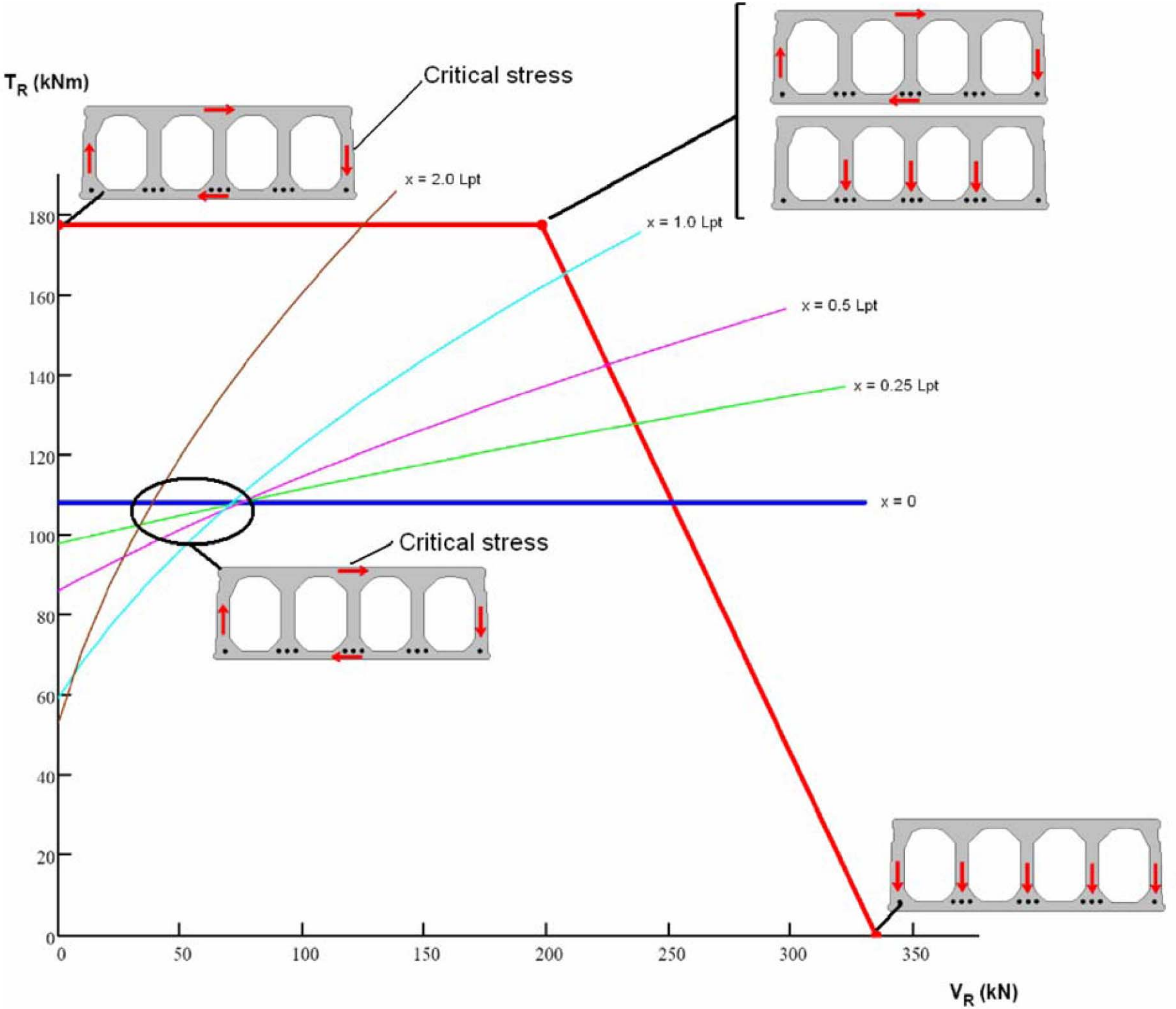
$b_{w,out} \leq 0.265H$ when using $W_{T,web} = W_T(b_{w,out})$

$t_{flange} \leq 0.175H$ when using $W_{T,top} = W_T(t_{top})$ or $W_{T,bottom} = W_T(t_{bottom})$

For hollow core cross-section with larger wall thickness use the resistance for the solid section when evaluating the shear stress in the actual "wall".

Interaction effect: Shear and torsion

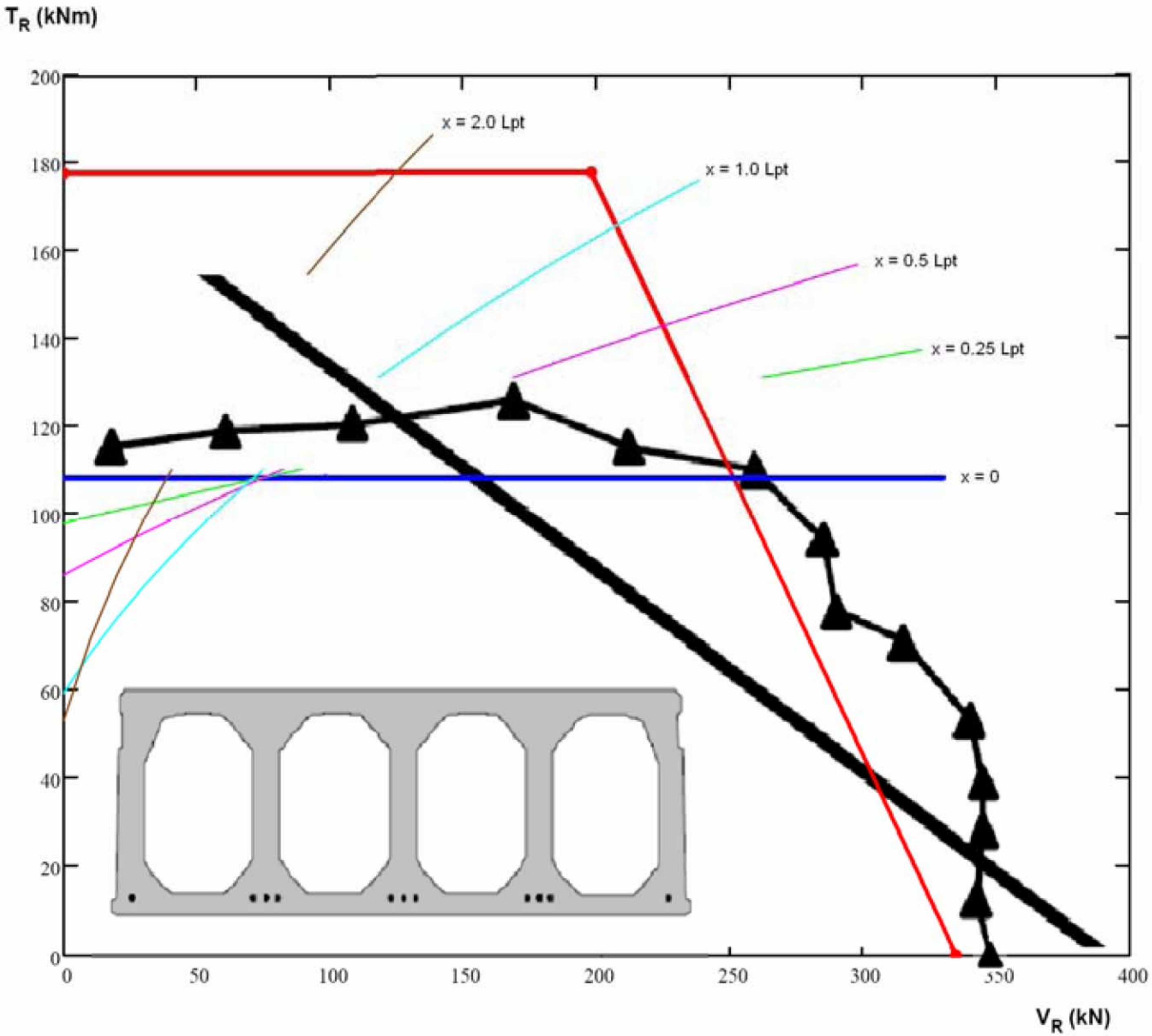
Technical Seminar 2007



Interaction effect: Shear and torsion

INTERNATIONAL PRESTRESSED HOLLOWCORE ASSOCIATION

Technical Seminar 2007



Verification with results from HOLCOTORS

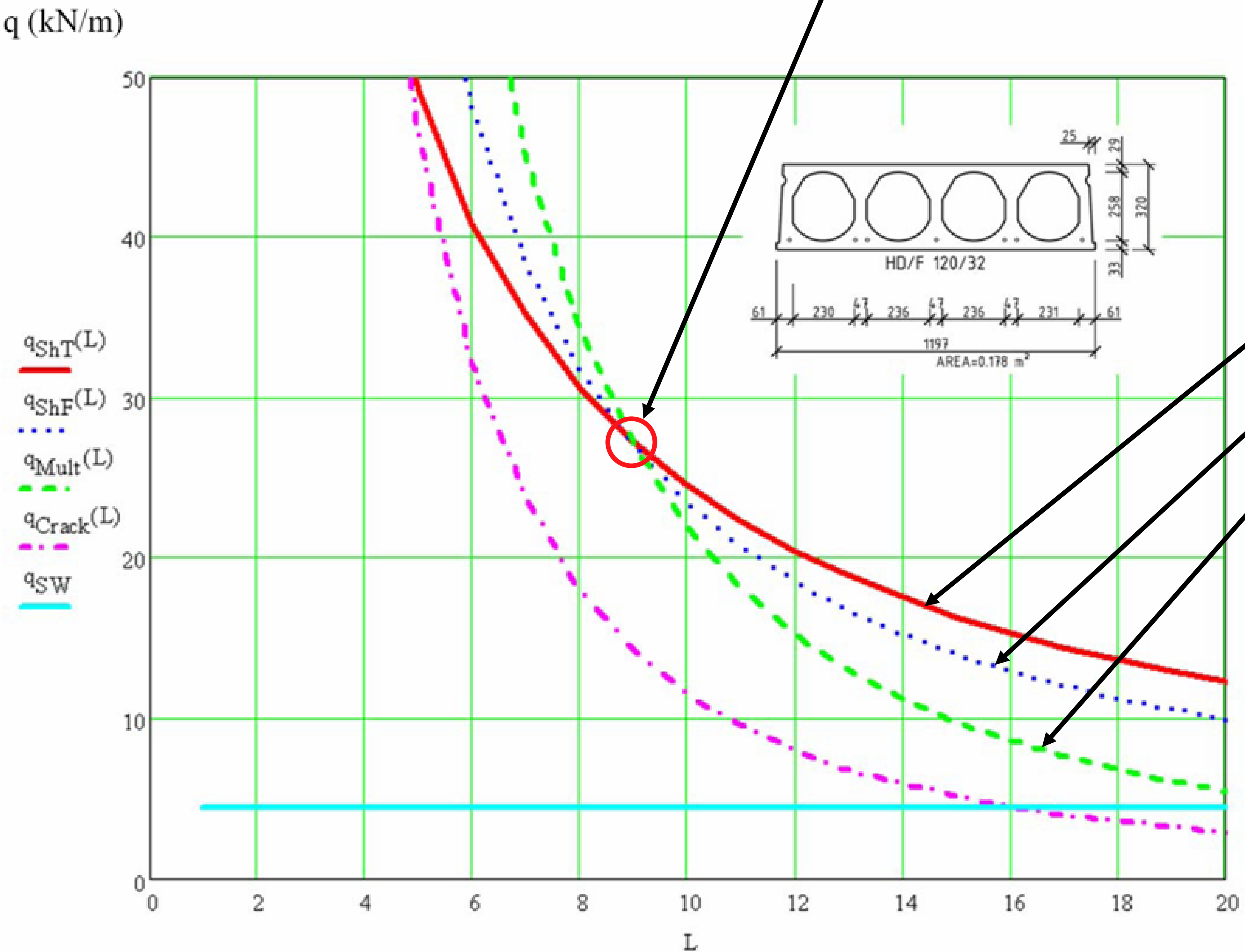
FE – Analysis

EN 1168

STRÄNGBETONG

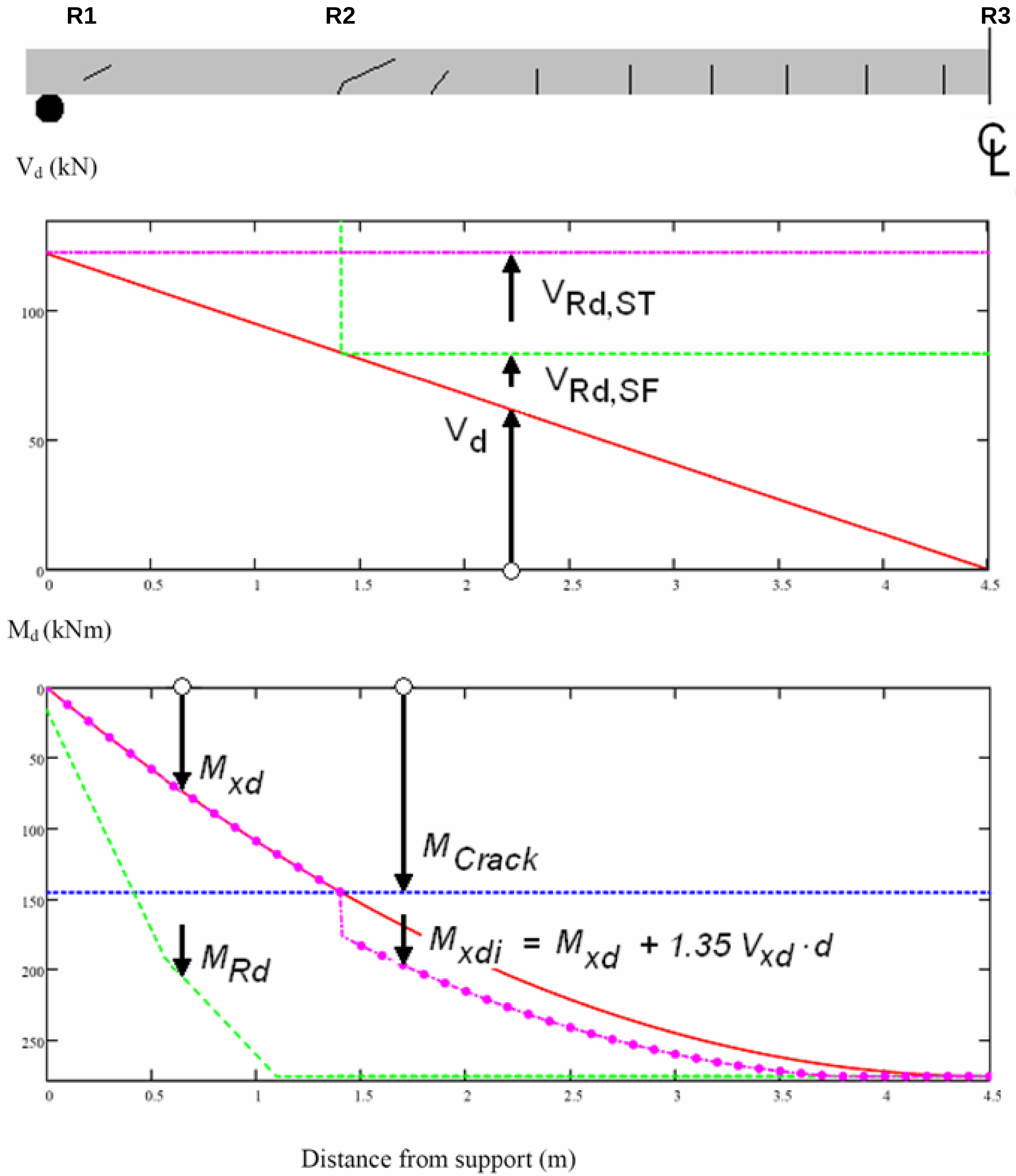
Interaction effect: Shear and bending

Is there an interaction effect ?



Resistance due to:

- Shear tension
- Shear flexure
- Bending moment



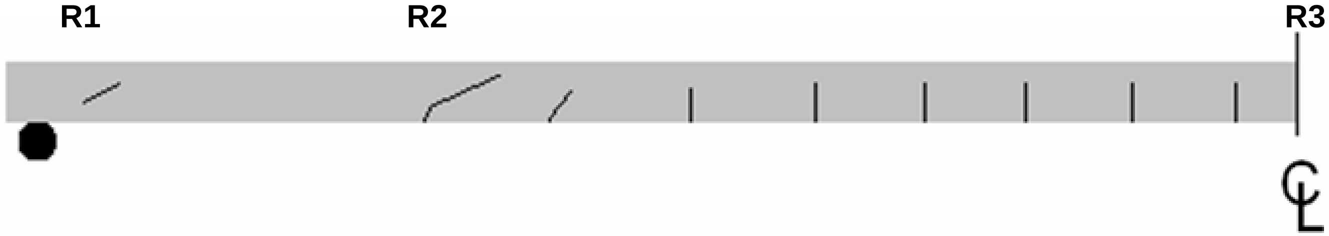
Technical Seminar 2007

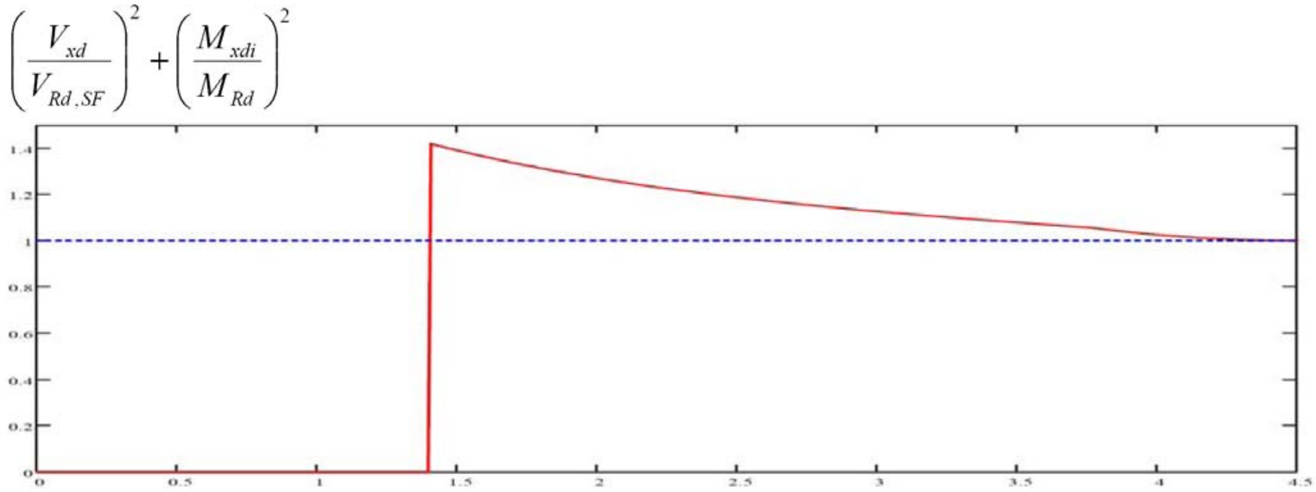
Resistance due to:

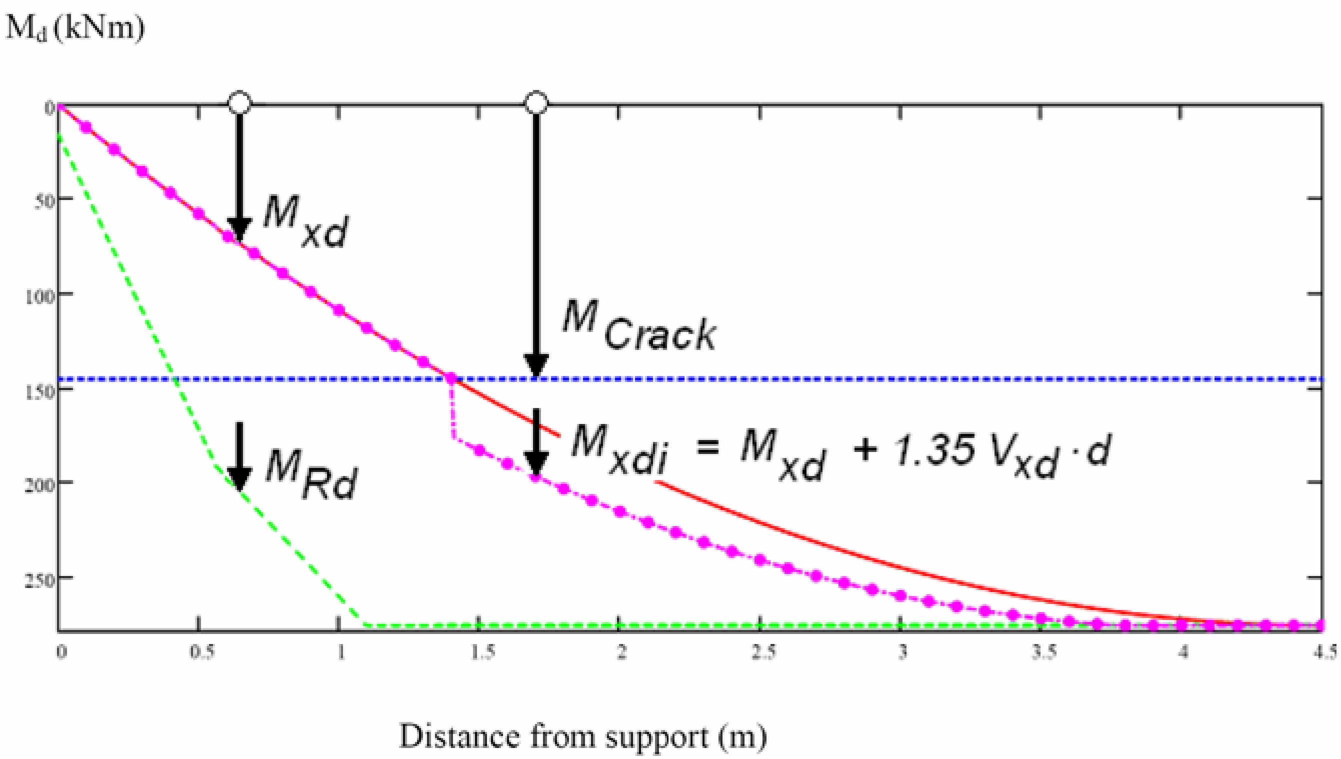
R1 Shear tension

R2 Shear flexure

R3 Bending moment

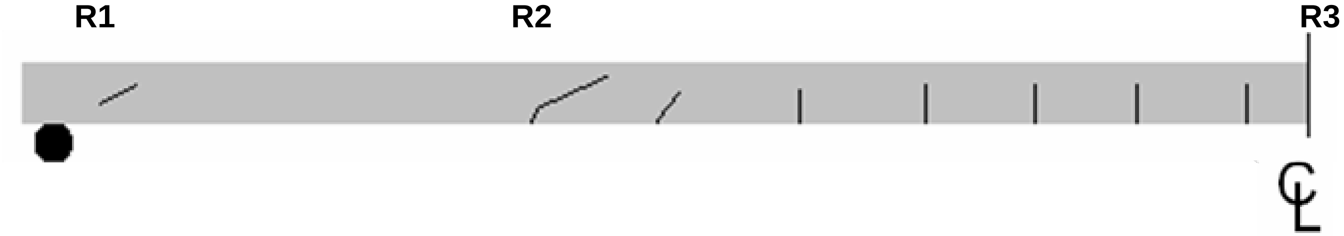


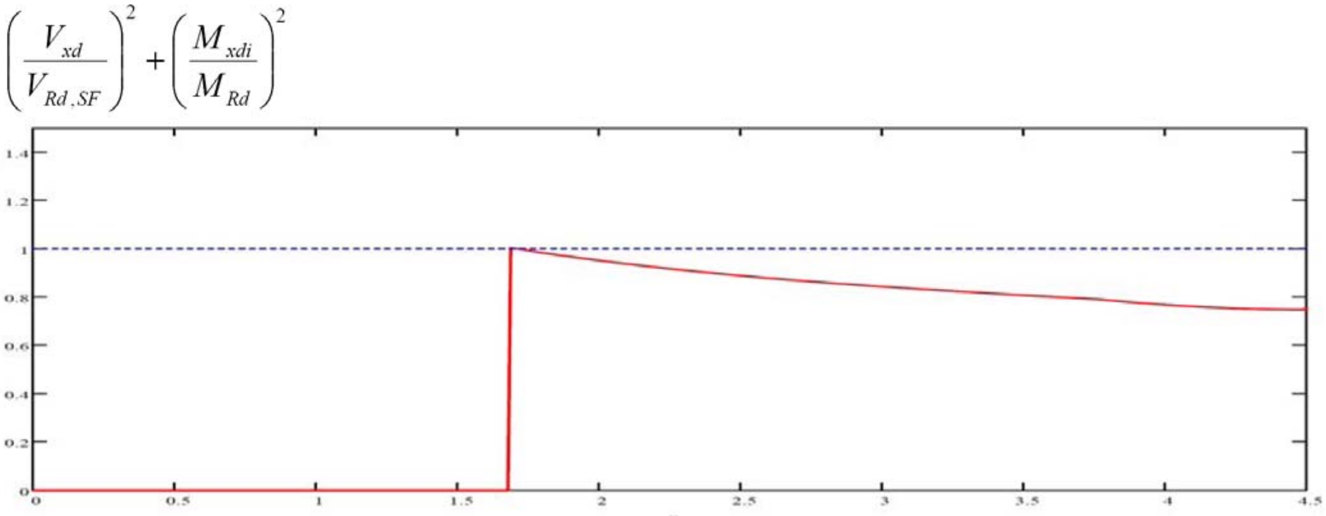




Suggested Interaction formula:

$$\left(\frac{V_{xd}}{V_{Rd,SF}}\right)^2 + \left(\frac{M_{xdi}}{M_{Rd}}\right)^2 \leq 1$$





Suggested Interaction formula:

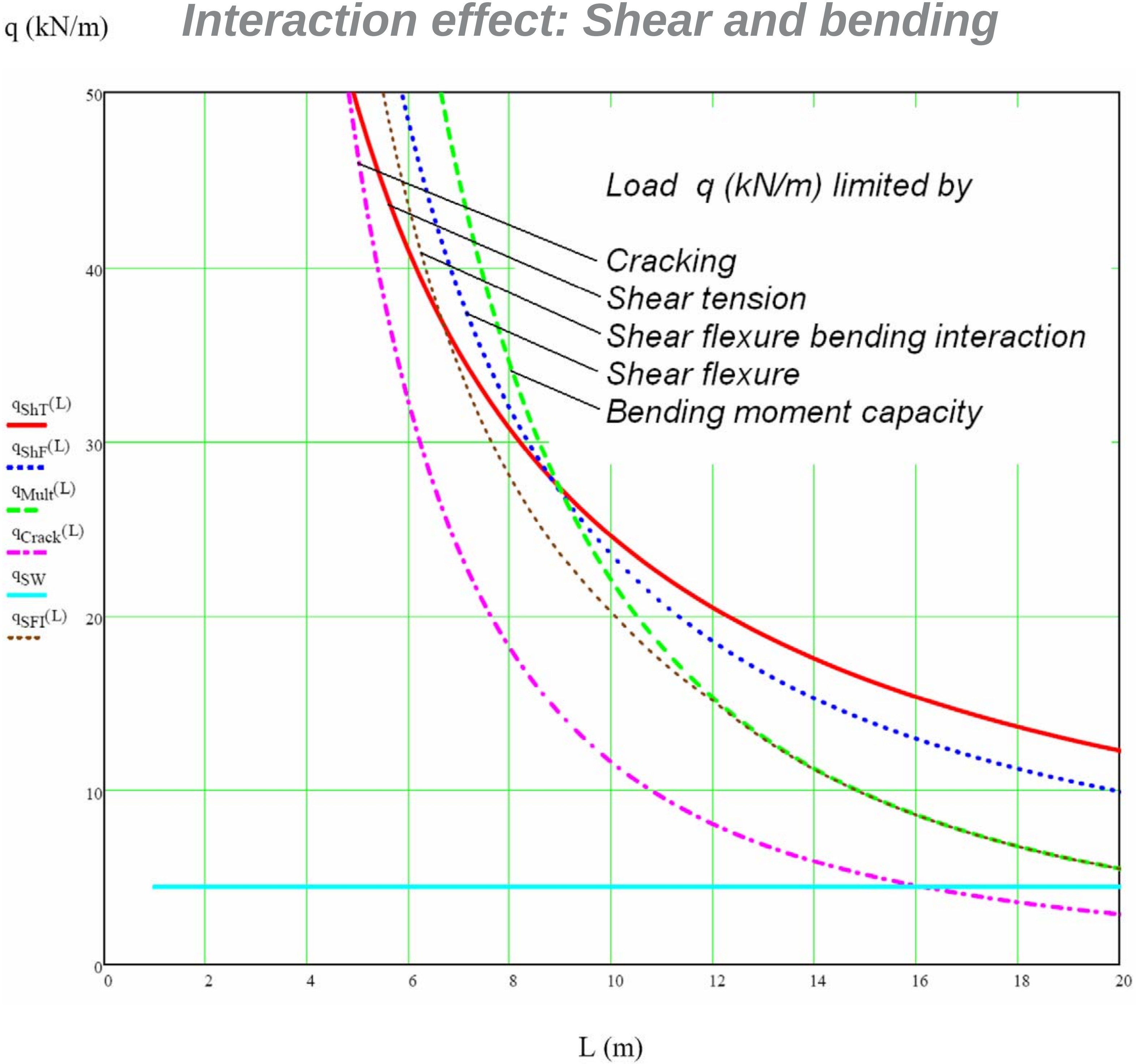
$$\left(\frac{V_{xd}}{V_{Rd,SF}}\right)^2 + \left(\frac{M_{xdi}}{M_{Rd}}\right)^2 \leq 1$$

The distributed load reduced:

From 27.2kN/m to 23.5 kN/m

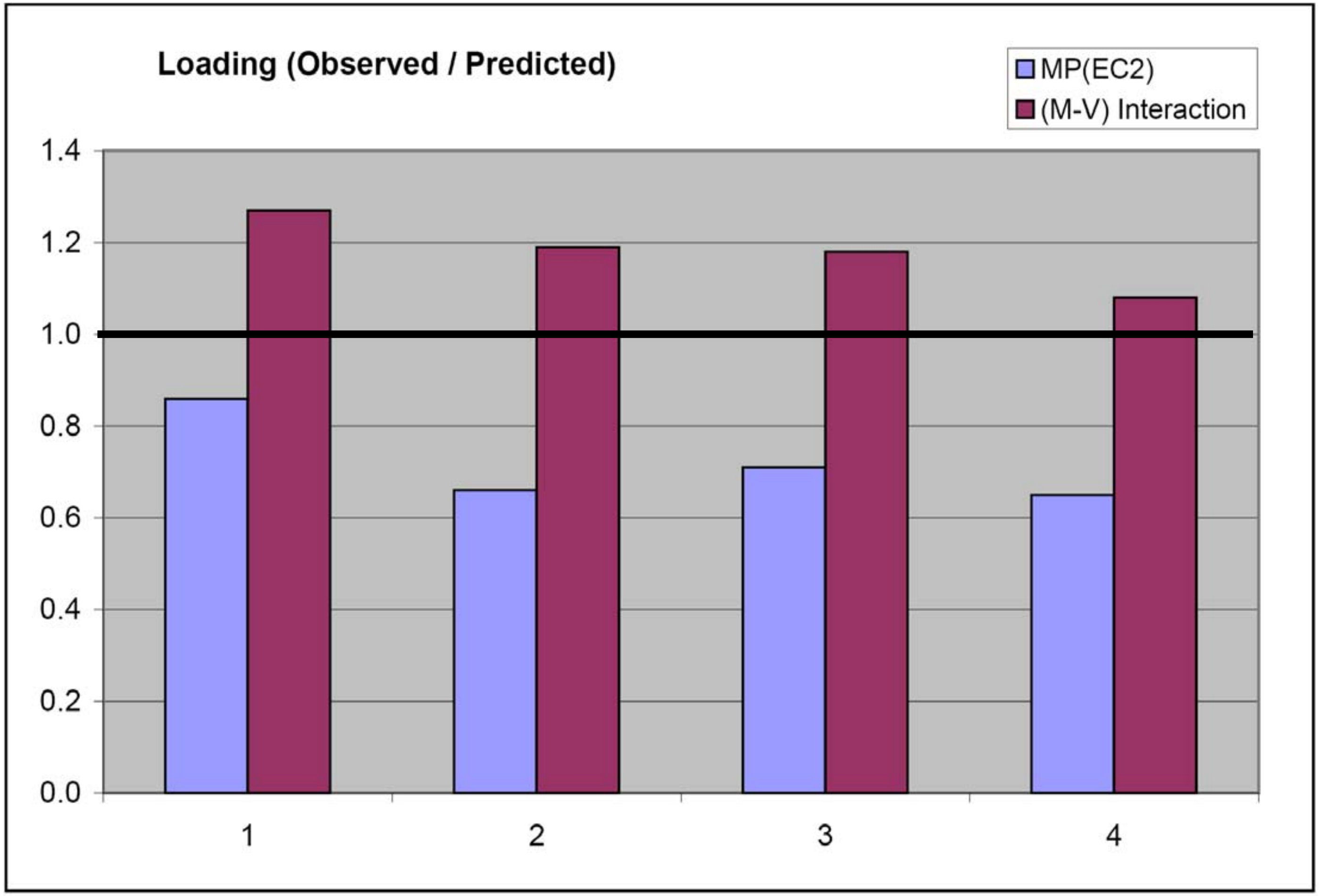
Interaction effect: Shear and bending

Technical Seminar 2007



Interaction effect: Shear and bending

M-V interaction for HC 500 mm
Comparison with available test results (Matti P.)



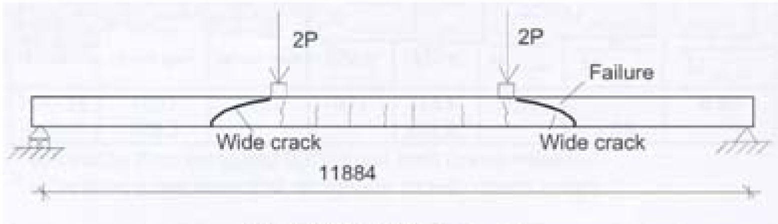


Fig. 23. P50-16. Failure mode.

Setup test 1

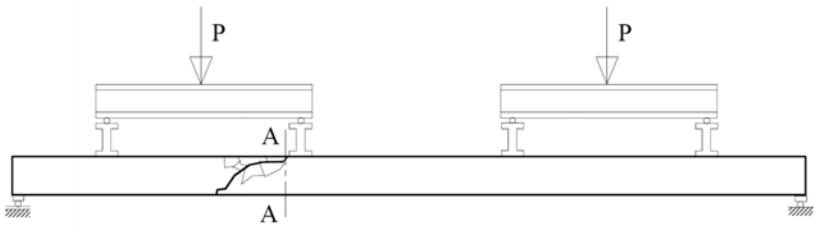


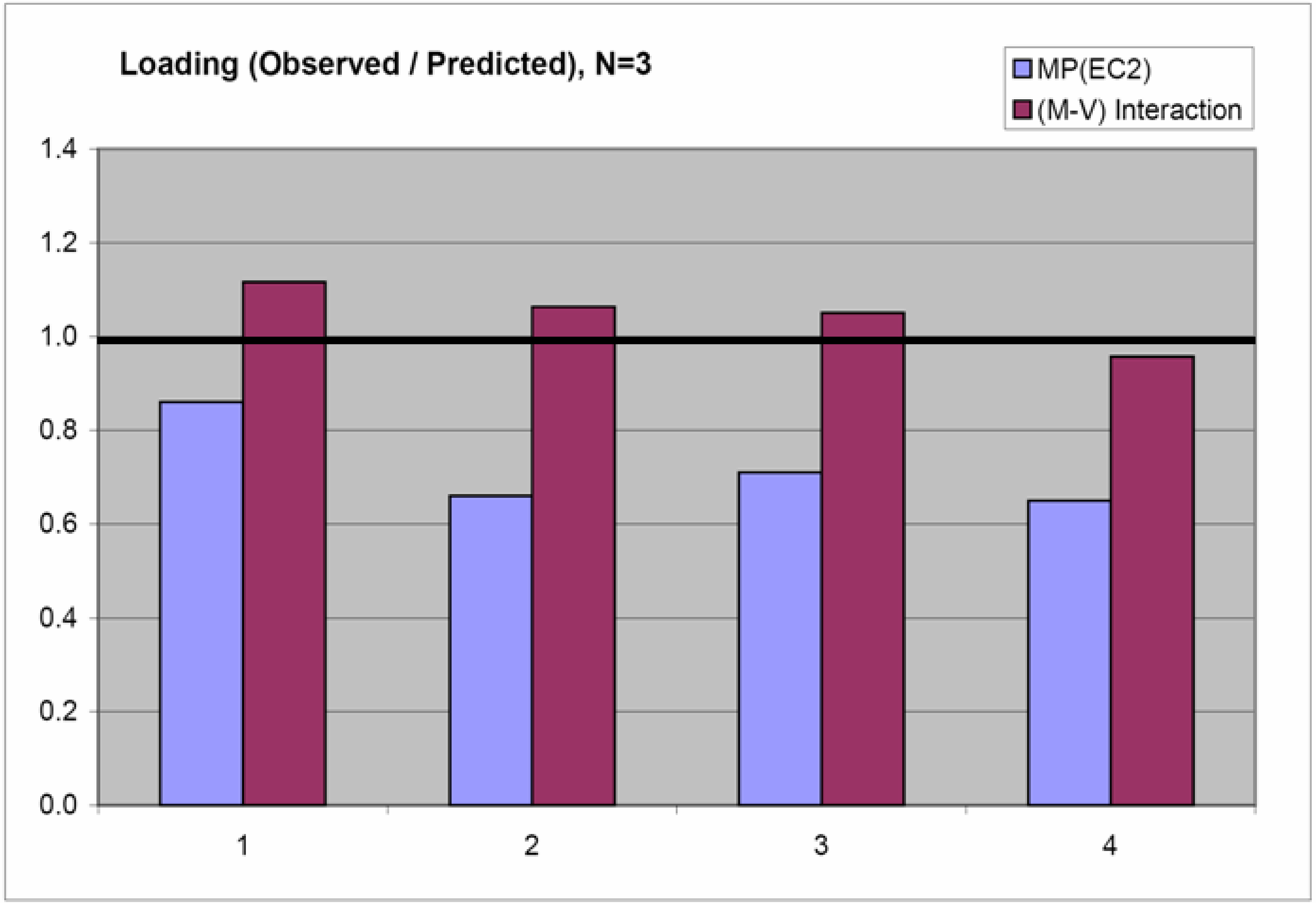
Fig. 2. Failure mode and critical cross-section A - A.

Setup test 2-4

$$\left(\frac{V_{xd}}{V_{Rd,SF}}\right)^2 + \left(\frac{M_{xdi}}{M_{Rd}}\right)^2 \leq 1$$

Interaction effect: Shear and bending

M-V interaction for HC 500 mm
Comparison with available test results (Matti P.)



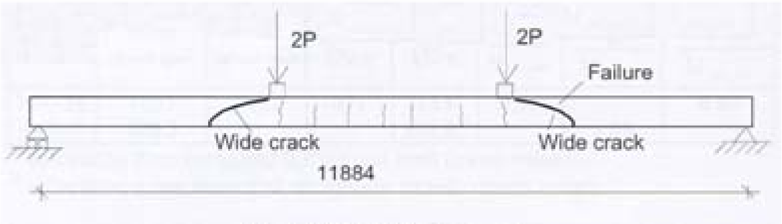


Fig. 23. P50-16. Failure mode.

Setup test 1

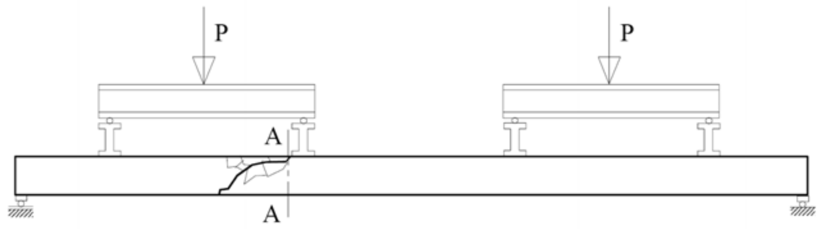


Fig. 2. Failure mode and critical cross-section A - A.

Setup test 2-4

$$\left(\frac{V_{xd}}{V_{Rd,SF}}\right)^3 + \left(\frac{M_{xdi}}{M_{Rd}}\right)^3 \leq 1$$

Tanks for the attention…