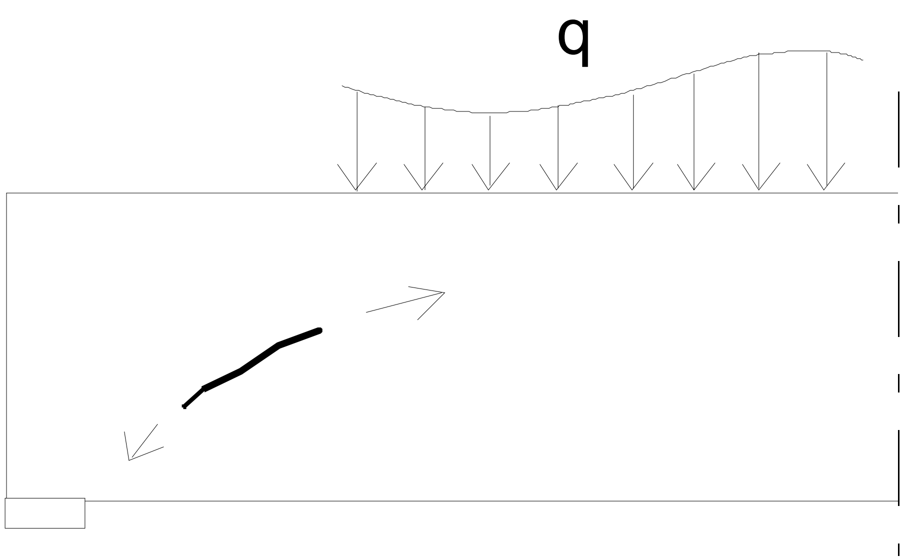
Resistance of PHC slabs against web shear failure

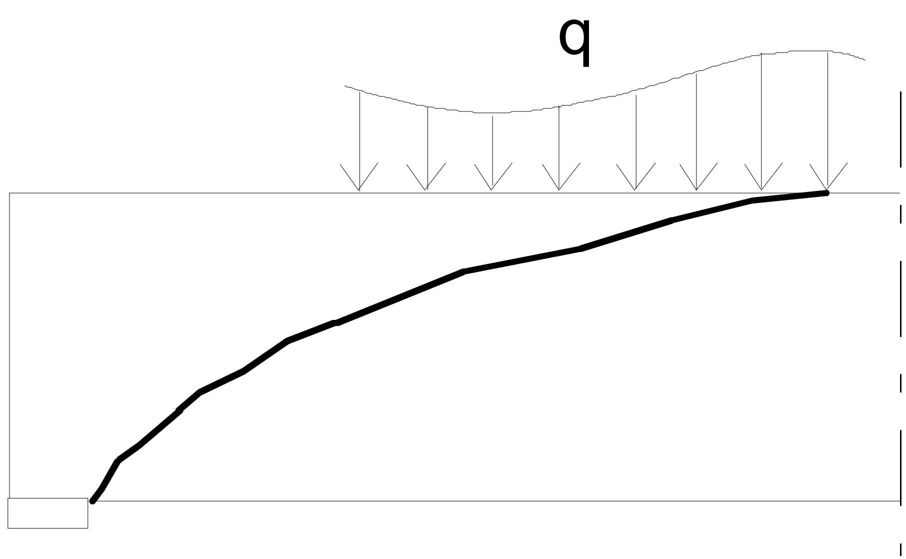
Comparison of experimental and theoretical results

Matti Pajari

Web shear failure



Progress of cracking



Failure

Traditional design method (EC2)

- Maximum principal stress:

$$\sigma_I = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

  - Axial normal stress: $\sigma = \frac{-P}{A}$

  - Shear stress: $\tau = \frac{VS}{Ib_w}$

- Failure criterion: $\sigma_I = f_{ct}$

Numerical example: 2D FE-Analysis

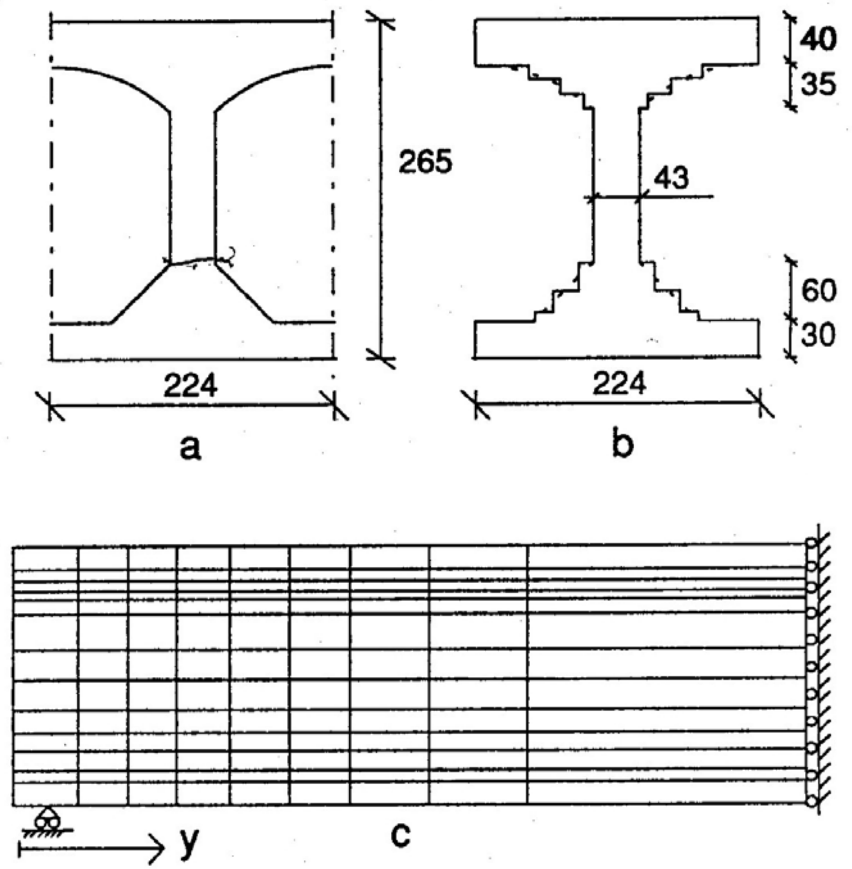
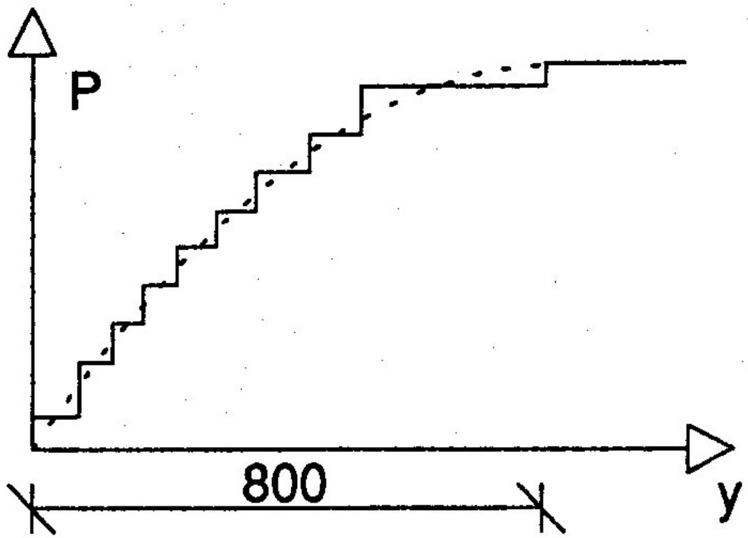


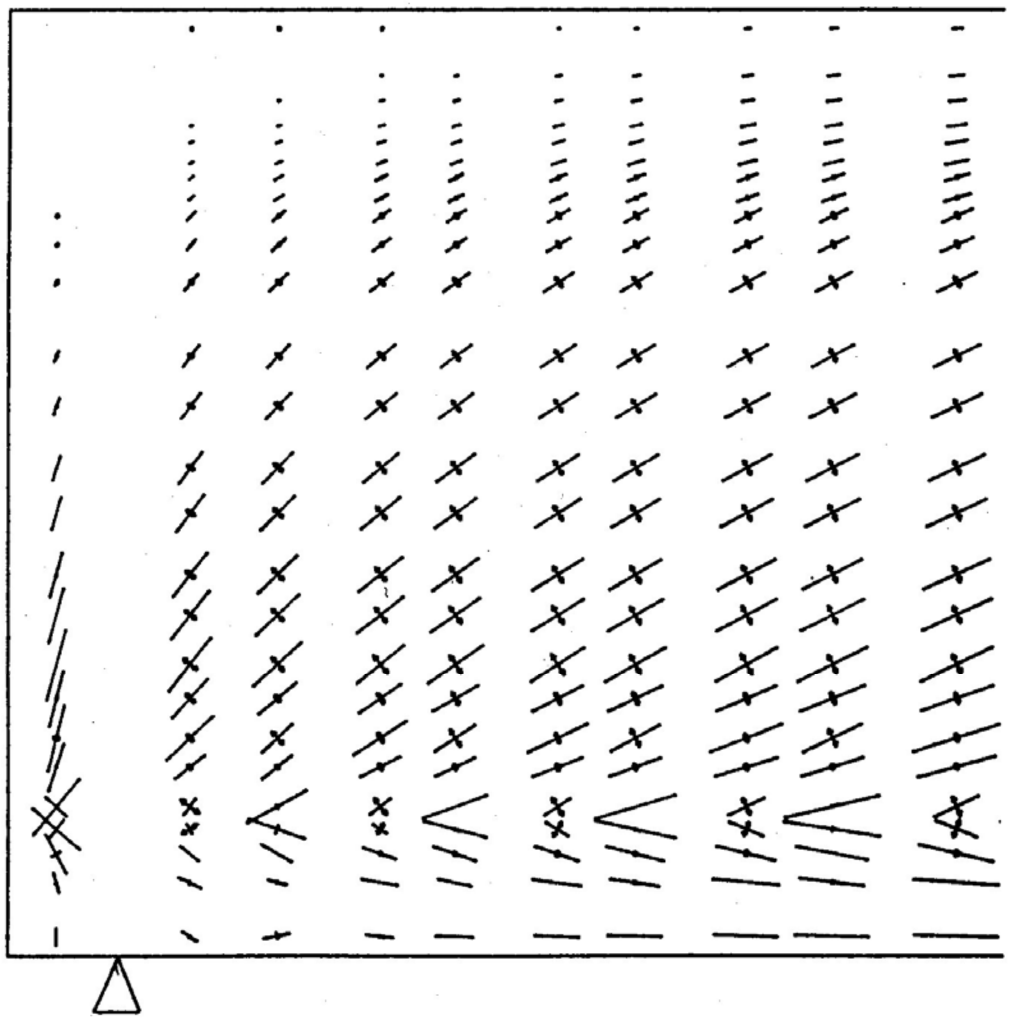
Fig. 1. a) Cross-section of one web and the flanges on both sides. b) Approximate cross-section for two dimensional FEM-model. c) Side view on FEM-model. There is a vertical point load at the right end of the model corresponding to 200 kN / 1,2 m.

Transfer of prestress



Assumed transfer of prestressing force (dashed line) and as modelled (continuous stepwise line).

Principal stresses



Principal stresses illustrated as vectors. Tensile stresses are indicated by arrows.

Conclusions

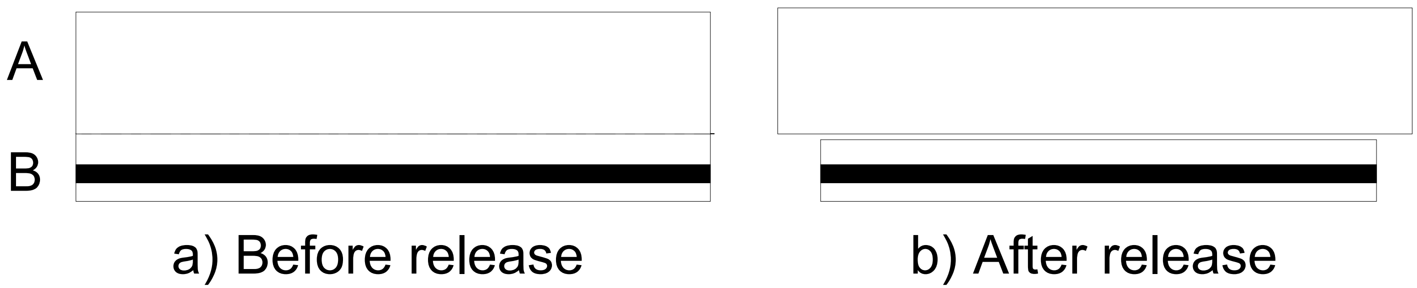
- The critical point for the considered section is at the junction of web and flange
- The tensile stress at the critical point is tens of percent higher than at the centroidal axis

Question:

- 1. Why?

Release of prestressing force

If no contact between A and B:

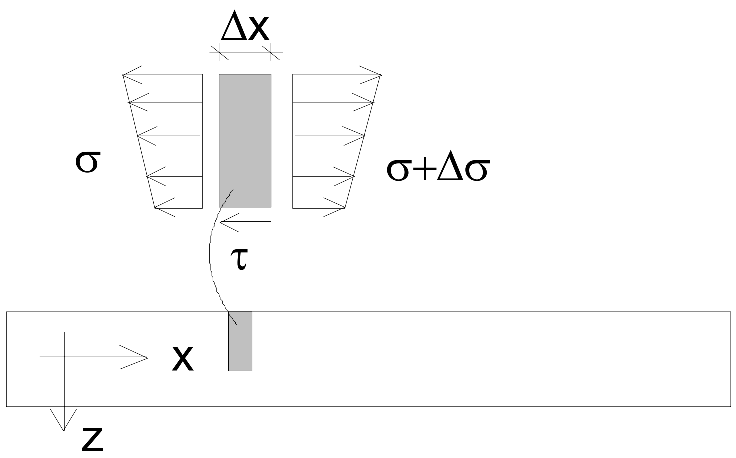


Since there is a strong bond between A and B, the interface is subjected to shear stresses within the transfer zone

The shear stresses due to the transfer depend on the location of the considered horizontal interface

Navier's bending theory:

$$\sigma = \frac{-P}{A} + \frac{-Pe + M}{I} z$$



If P = constant and M = constant: $\Delta\sigma = 0 \Rightarrow \tau \equiv 0$

Else: $\tau \equiv 0$ is not true

Navier's bending theory:

$$\sigma = \sigma_N + \sigma_B$$ or

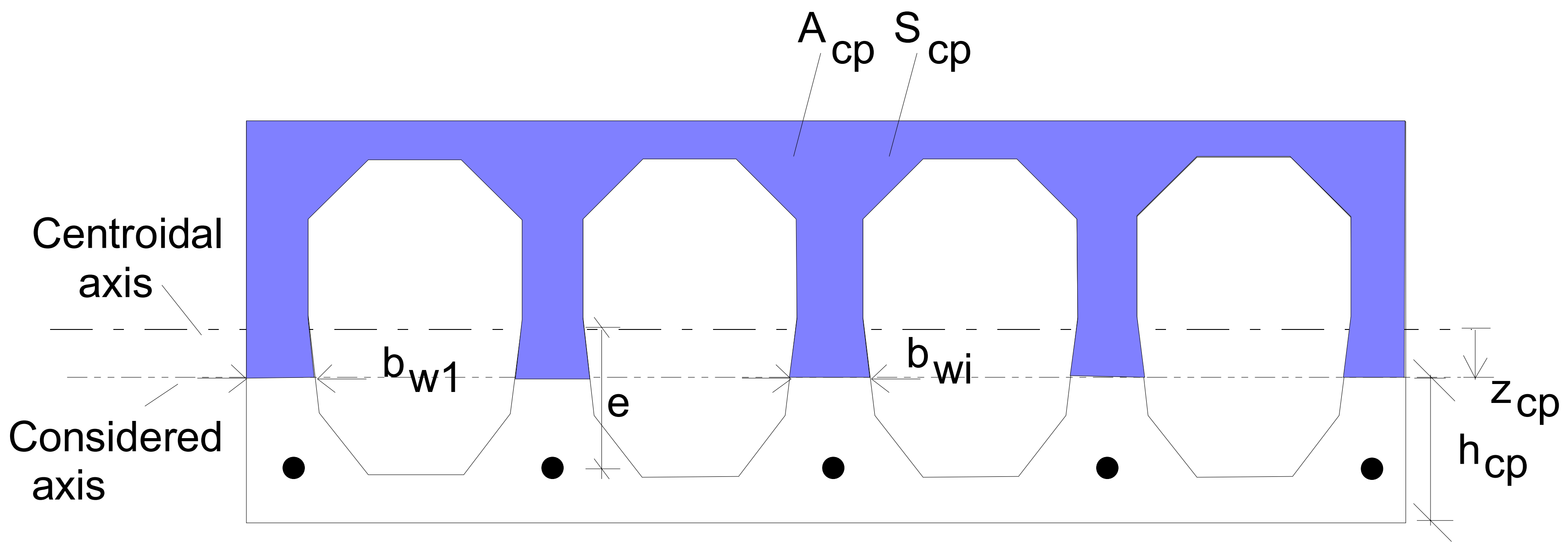
$$\sigma = \frac{-P}{A} + \frac{-Pe + M}{I} z$$

$$\tau = \tau_V + \tau_{dP}$$ or

$$\tau = \frac{1}{b_w} \frac{S}{I} V + \frac{1}{b_w} \left(\frac{A_{cp}}{A} - \frac{Se}{I} \right) \frac{dP}{dx}$$

(Yang's formulation)

Meaning of A_{cp} and S_{cp} = S



Common for EC2, German Zulassung and Yang:

- Maximum principal stress: $$\sigma_I = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$
- Failure criterion: $$\sigma_I = f_{ct}$$

Differences in:

- calculation of σ and τ
- location of critical point
- transfer of prestressing force
- etc.

Navier's bending theory:

$$\sigma = \sigma_N + \sigma_B$$ or $$\tau = \tau_V + \tau_{dP}$$ or

$$\sigma = \frac{-P}{A} + \frac{-Pe + M}{I} z$$

$$\tau = \frac{1}{b_w} \frac{S}{I} V + \frac{1}{b_w} \left(\frac{A_{cp}}{A} - \frac{Se}{I} \right) \frac{dP}{dx}$$

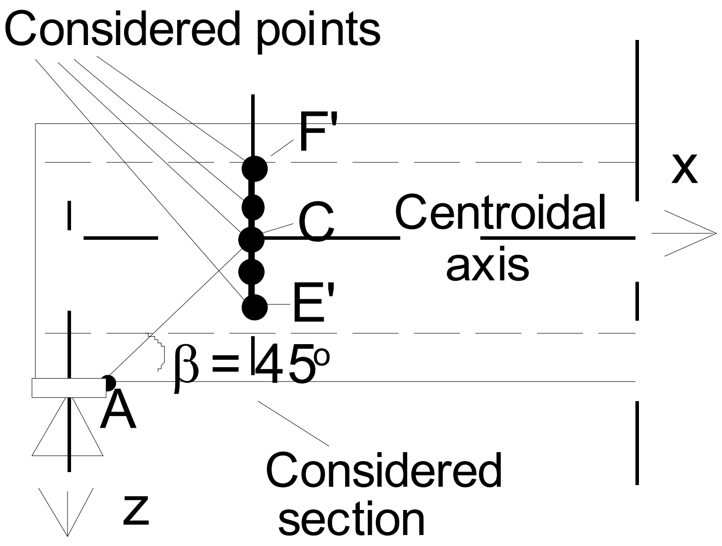
EC2: σ_B and τ_{dP} ignored

German Zulassung: σ_B ignored

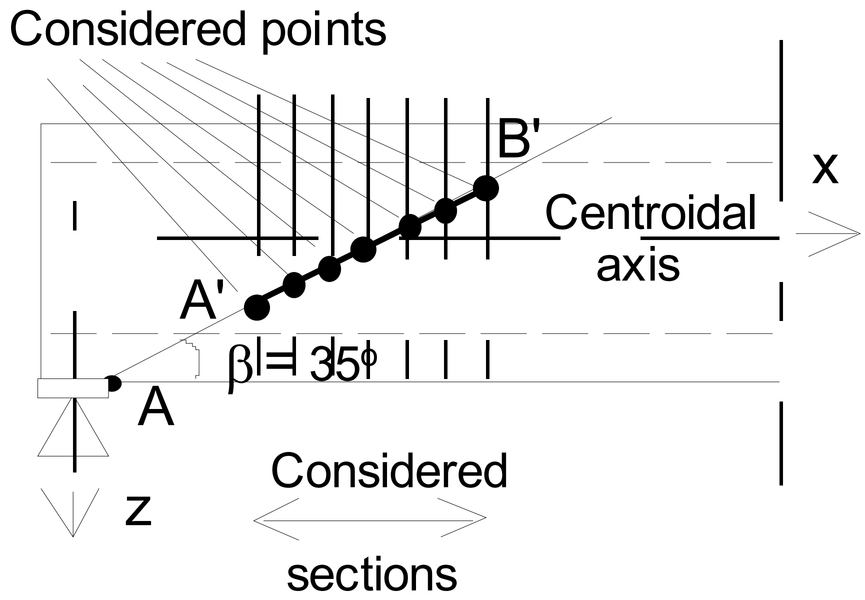
Yang: ”Nothing” ignored in Navier's sense

Considered sections and points

EC2 and Zulassung



Yang



Transfer of prestressing force

EC2:	Linear
Zulassung:	Parabolic
Yang:	Any model

Possible reasons to bad fit (non-conservativeness) when theoretical and experimental resistances differ

- Tensile strength overestimated
- Stresses underestimated
- Failure criterion not accurate

Tests

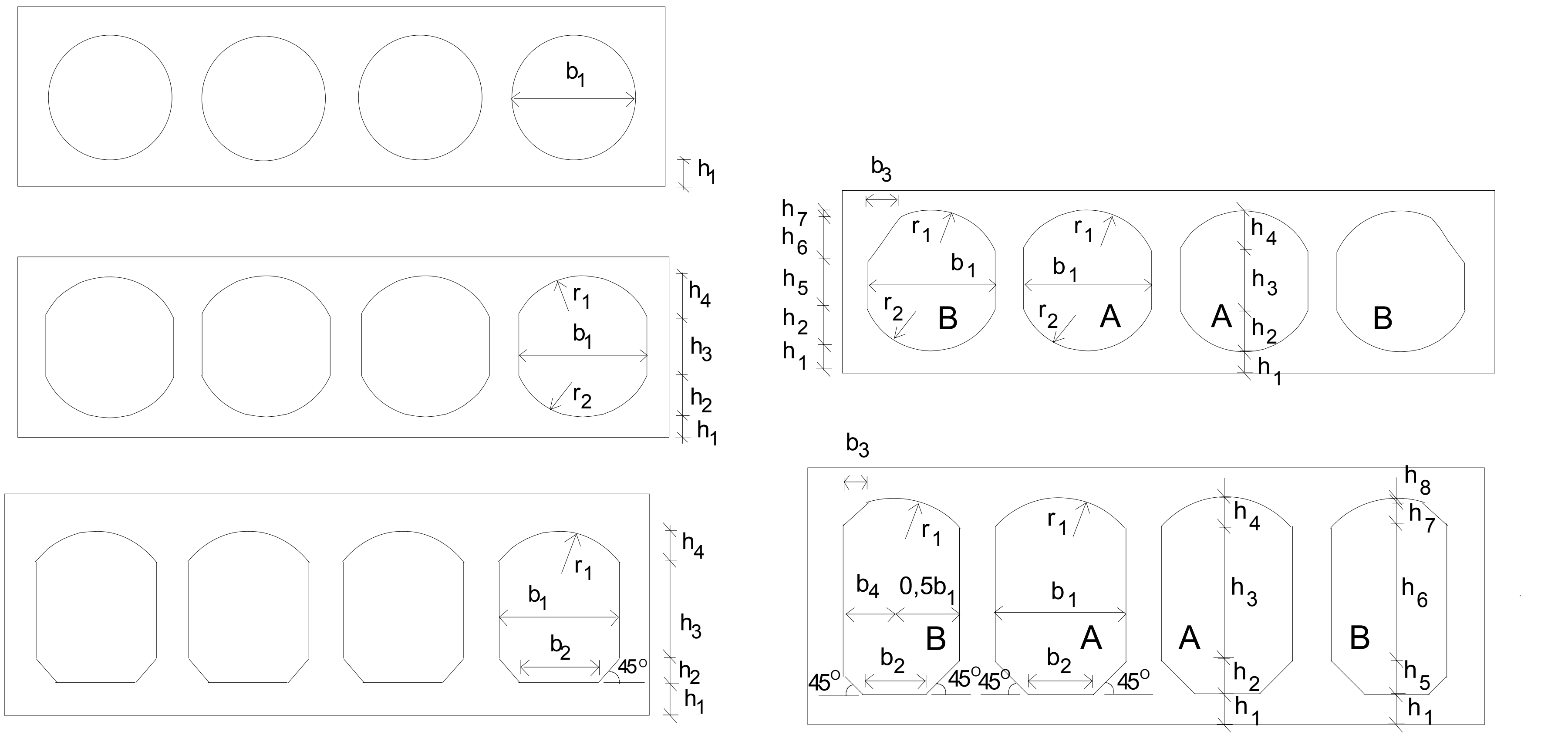
- All tests on single PHC slabs carried out by VTT 1990 – 2004
- Only those with web shear failure included
- Slabs with missing data rejected
- Slabs with shear span < 2.4H rejected
- Slabs with excessive bond slip rejected

Error in App. B.

Reads: $\Delta l_0 = \varepsilon_p d$

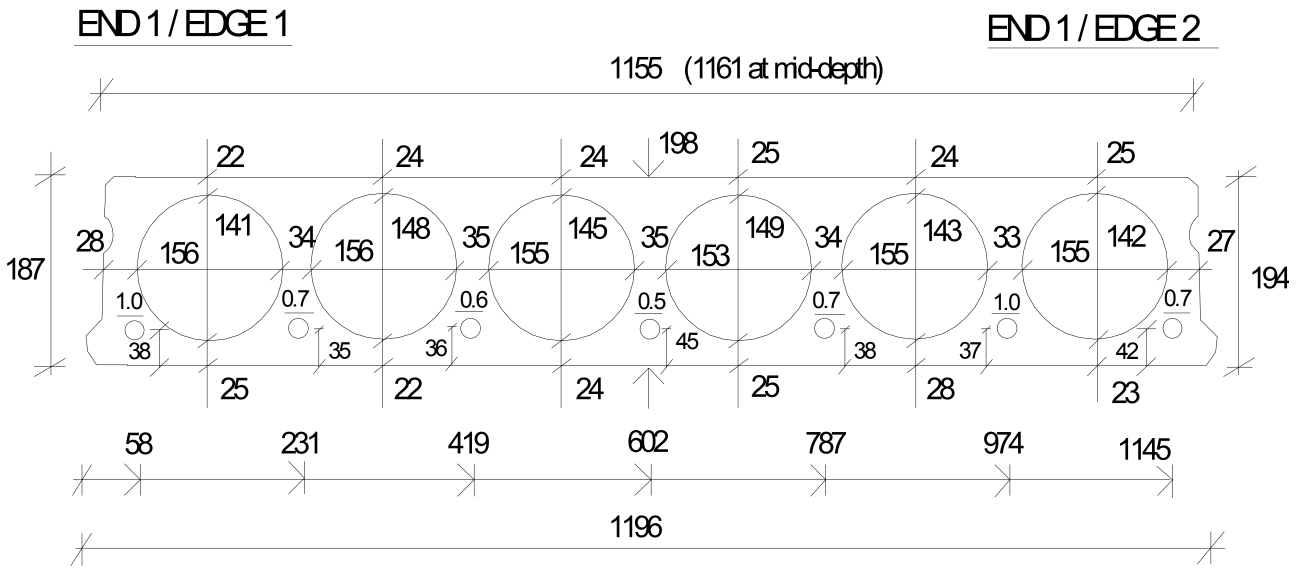
Should read: $\Delta l_0 = 32\varepsilon_p d$

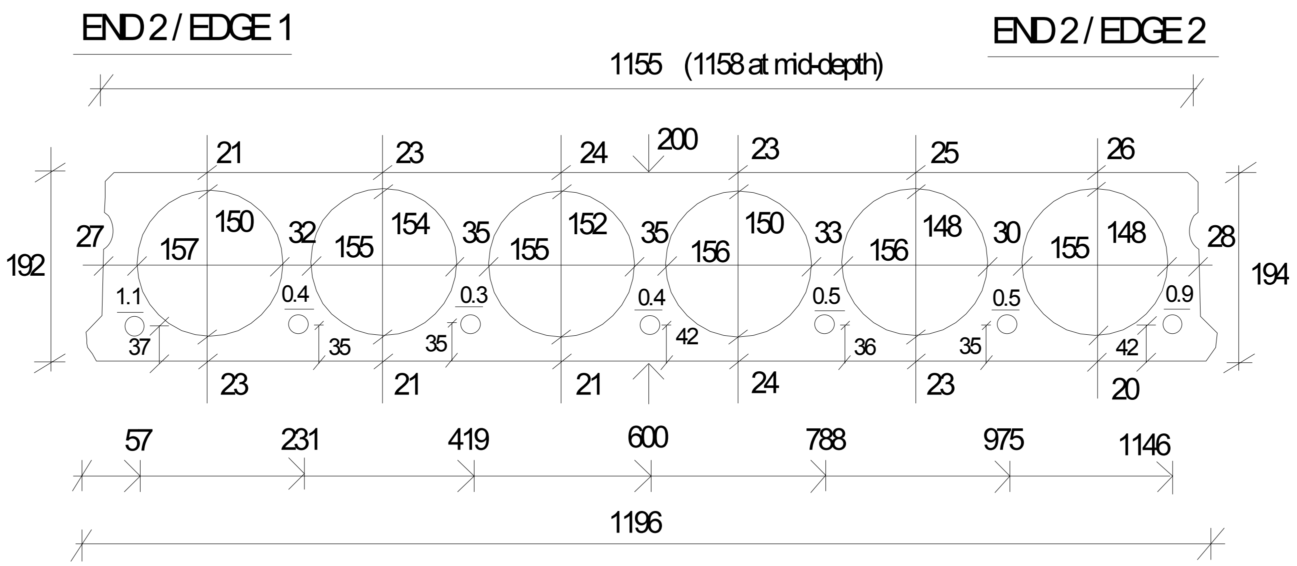
Types of hollow cores in test specimens



Lower strands : 7 ϕ 12,5 prestress = 900 MPa

Length : 5010 mm
Mass : 1440 kg





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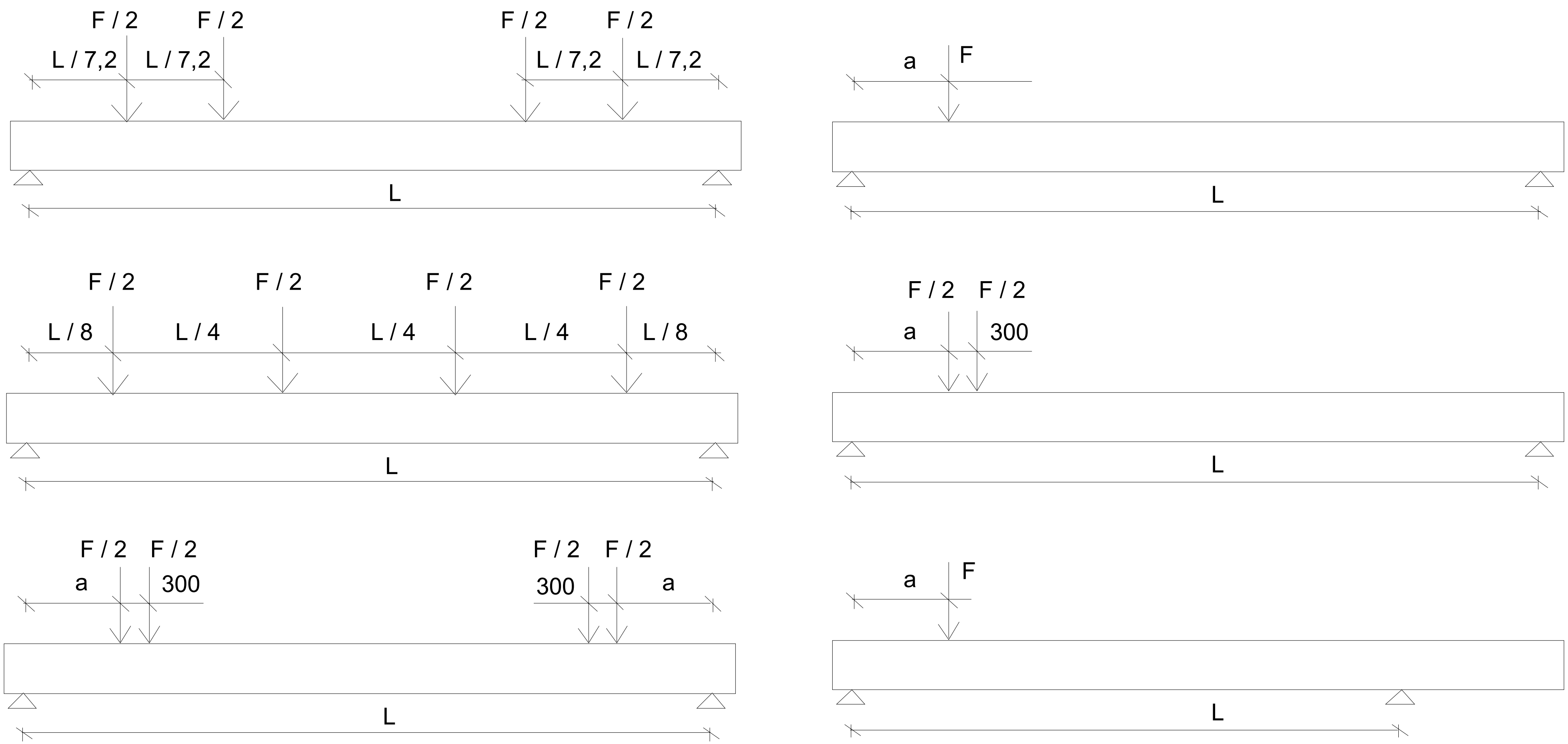
F / 2
F / 2
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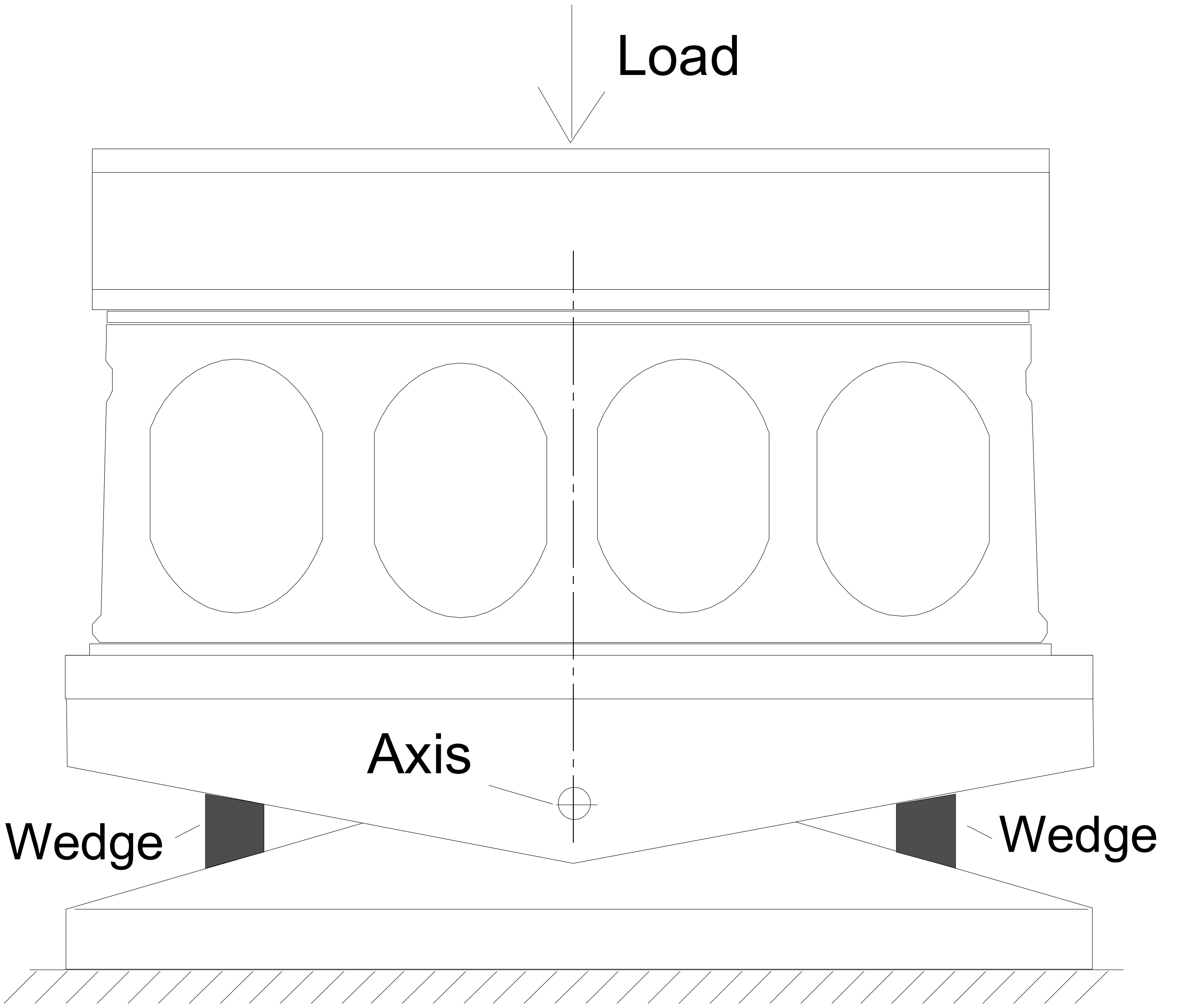

Load
Axis
Wedge
Wedge

Number of acceptable and rejected tests

Thickness mm	Accepted tests	Rejected tests	Note
200	4	5	
265	20	7	
320	10	4	5 with circular voids 5 with non-circular voids
370	2	3	
400	7	2	
500	6	2	
Total	49	23 (32%)	

Calculations

Measured properties for

- self weight
- **strength of concrete**
- location of strands,
- width of slab and
- **web width b_w**

Nominal dimensions for geometry except for those mentioned above

Two assumed losses of prestress: 5% and 15%

Strength?

- Strength measured from six 50 mm cores
- Conversion $f_{c,C50} \rightarrow f_{c,C150}$ is not specified in EN standards

Decision:

$f_{c,K150} = f_{c,C50}$ *(core strength)*

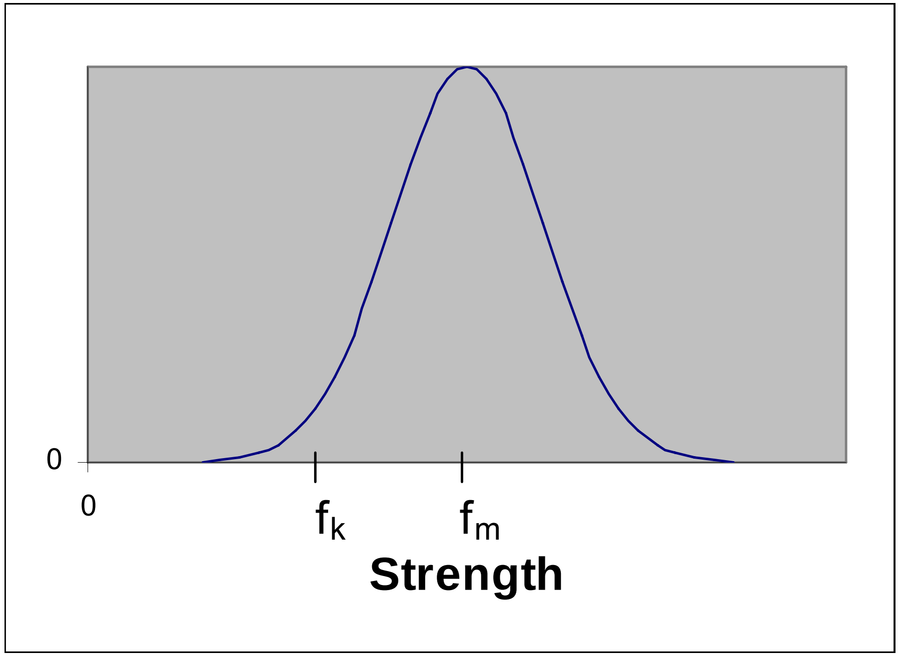
$f_{c,K150} \rightarrow f_{c,C150}$ as in EC2

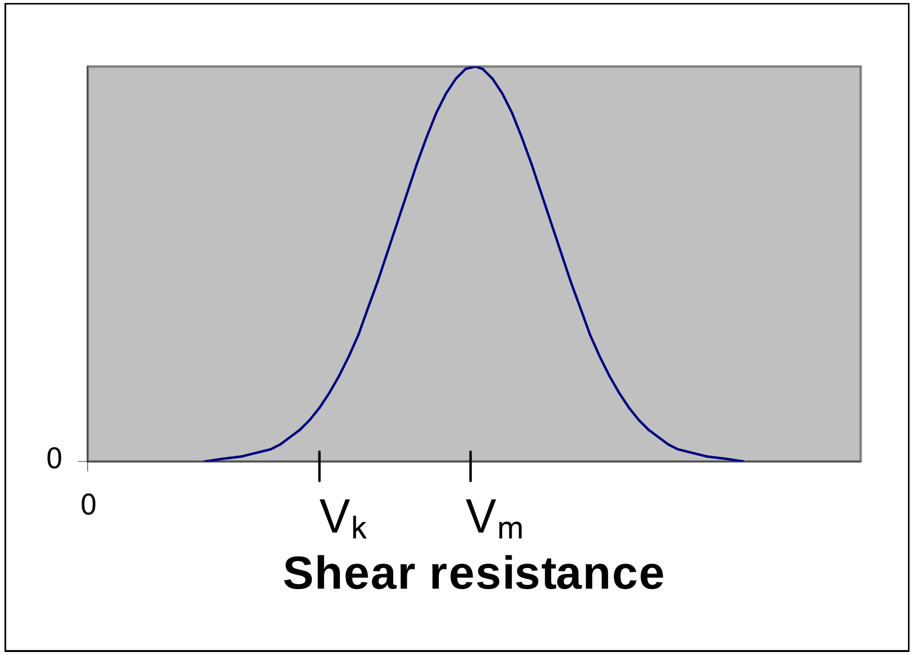
f_{ctm} = calculated as in EC2 from $f_{ck,C150}$ or $f_{cm,C150}$

f_{ctm} used for calculation of theoretical resistance

f_{ctk} used for calculation of characteristic resistance

Which strength should be used in comparison?





Model: $V = V(f_c, \ldots)$

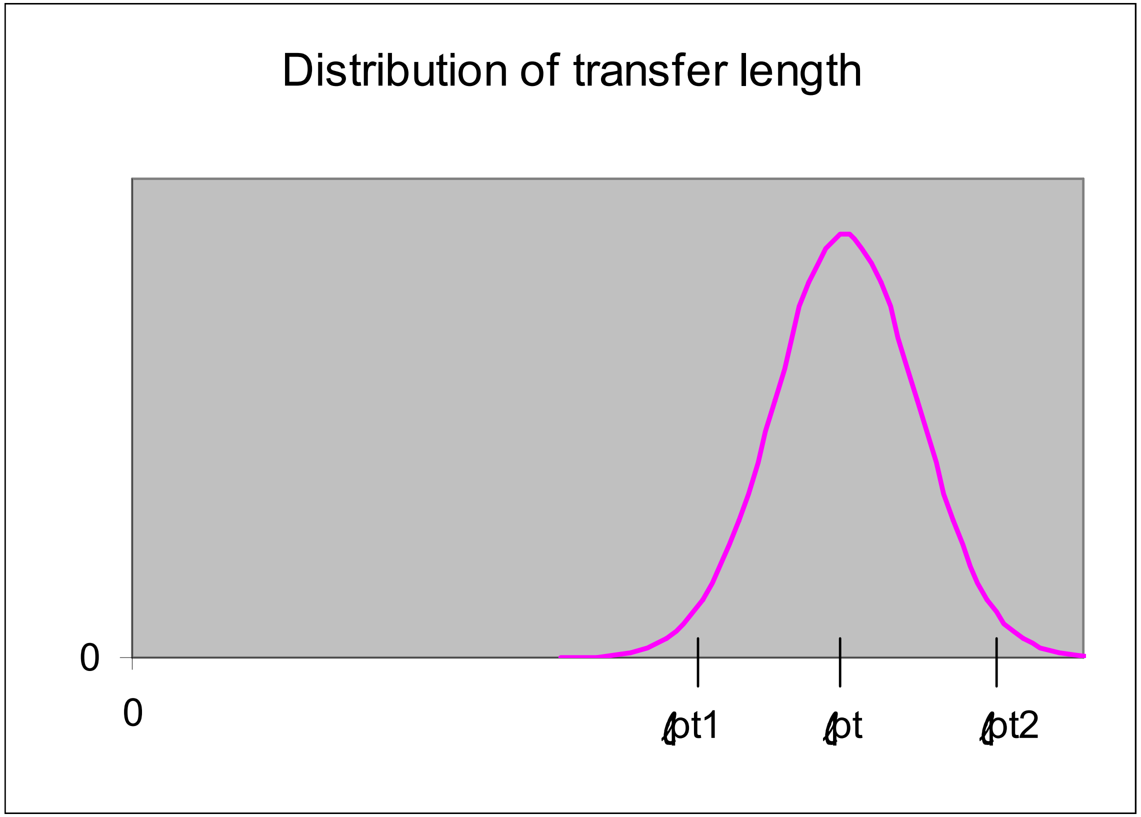
$V_m = V(f_{cm}, \ldots)$

$V_k = V(f_{ck}, \ldots)$

$V_d = V(f_{cd}, \ldots)$

Transfer length in EC2

- l_{pt} = basic value of transfer length
- l_{pt1}=0.8l_{pt} used, when prestressing is unfavourable
- l_{pt2}=1.2l_{pt} used, when prestressing is favourable



$l_{ptm} = l_{pt}$?

l_{pt} used for mean resistance

1.2l_{pt} used for characteristic resistance

On the other hand:

$$l_{pt} = \alpha_1 \alpha_2 \phi \frac{\sigma_{pm0}}{\eta_{p1}\eta_1 f_{ctd(t_{rel})}} = \frac{C}{f_{ctk}(t_{rel})}\gamma_c$$

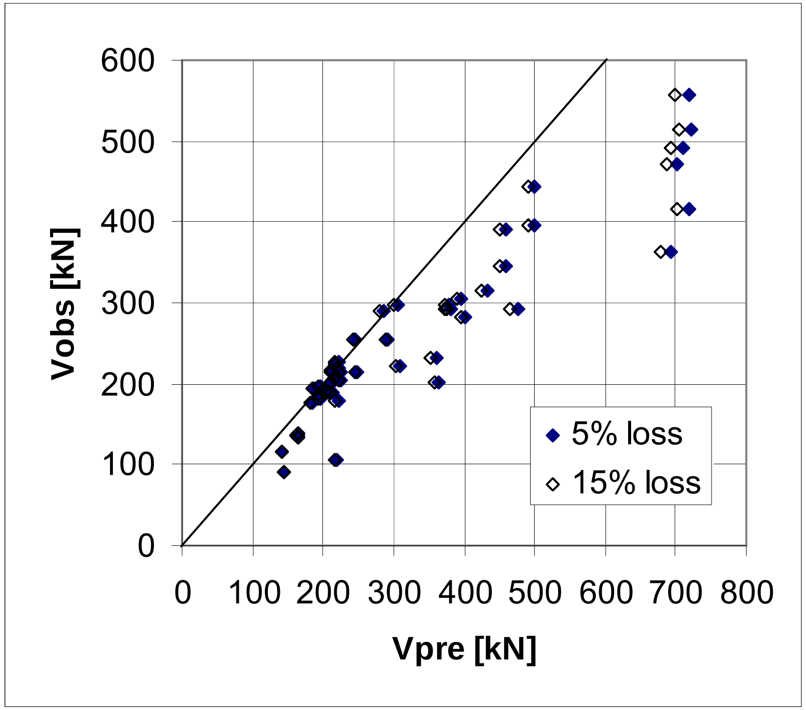
Why not:

$$l_{pt} = \frac{C_1}{f_{ctk}(t_{rel})}$$ **or** $$l_{pt} = \frac{C_2}{f_{ctm}(t_{rel})}$$ **?**

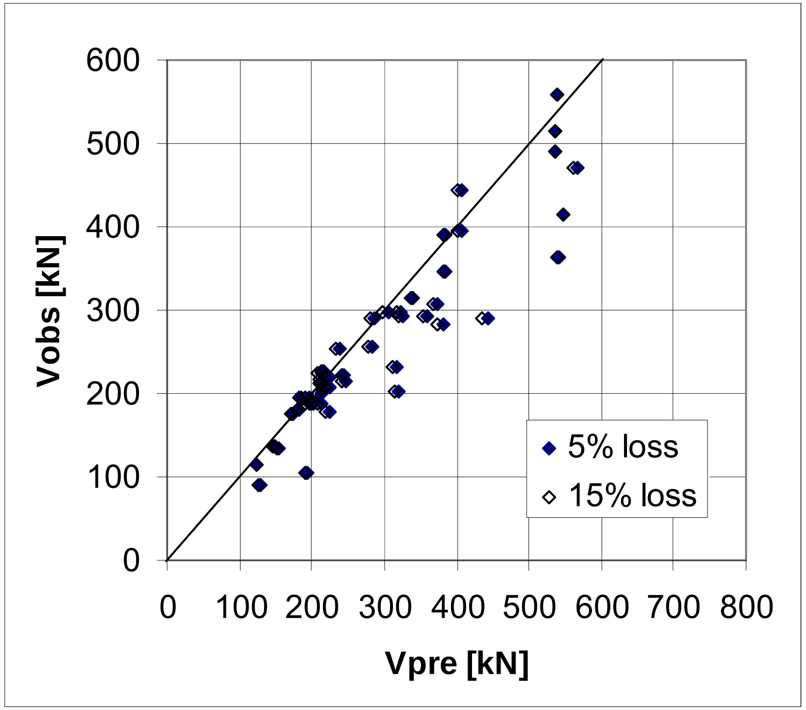
Requirement for mean resistance:

- $V_{observed} / V_{m,predicted} \geq 1.0$ in 50 % of cases

Comparison of mean resistances: $V_{observed} / V_{predicted}$

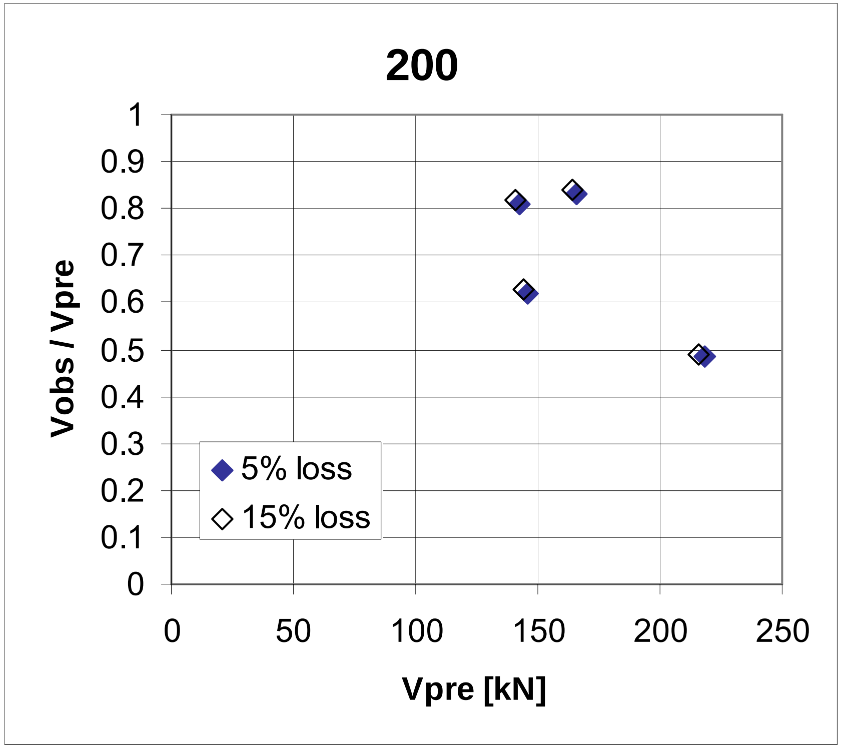


EC2

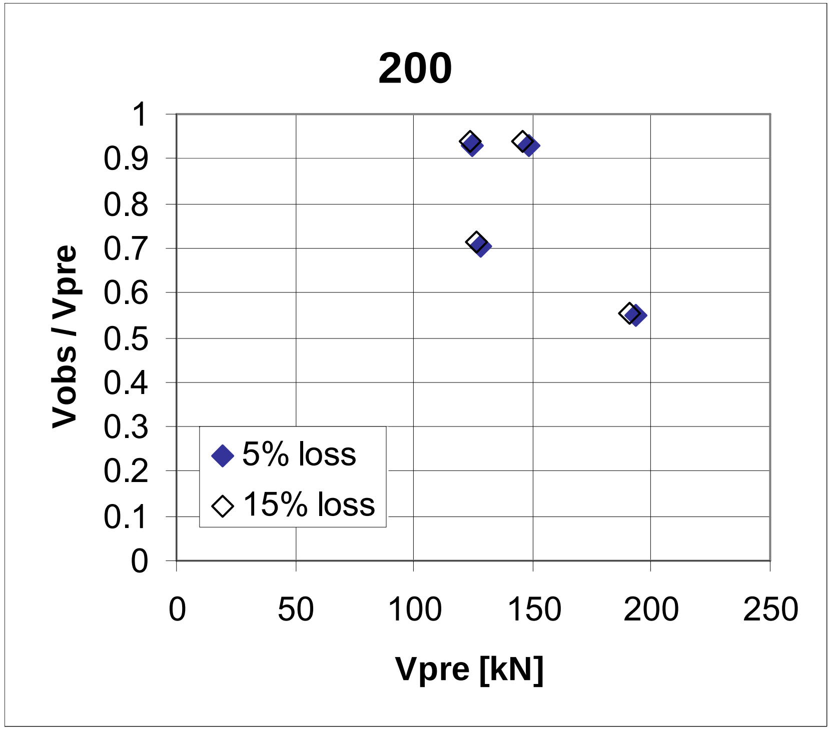


Yang

Fig. 1. All slabs. Relationship of observed (V_{obs}) and predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 and Yang.

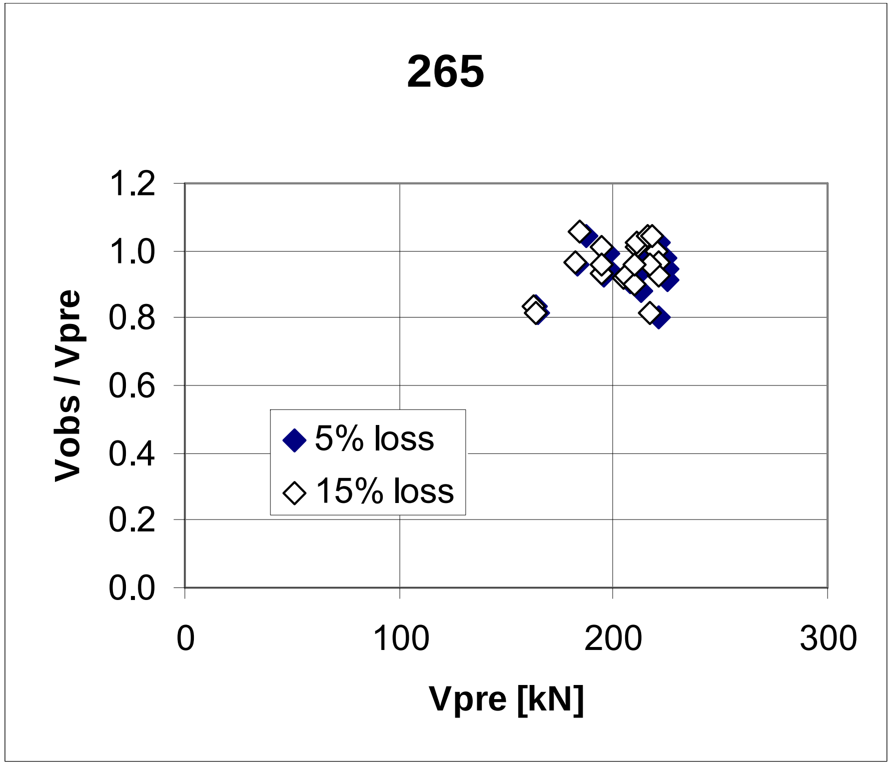


EC2

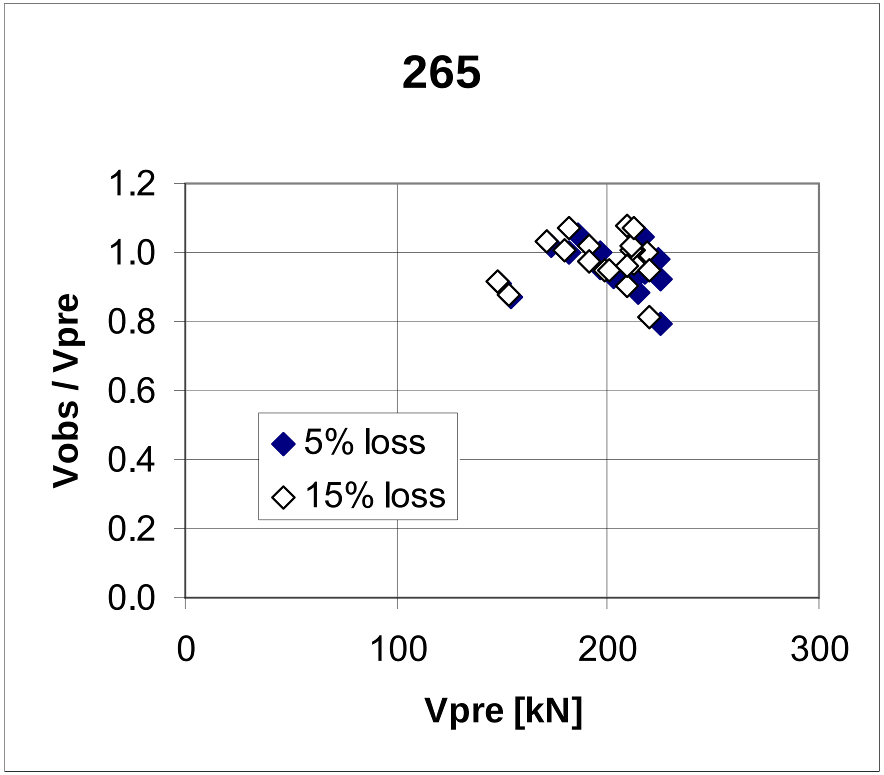


Yang

Fig. 1. 200 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 and Yang.

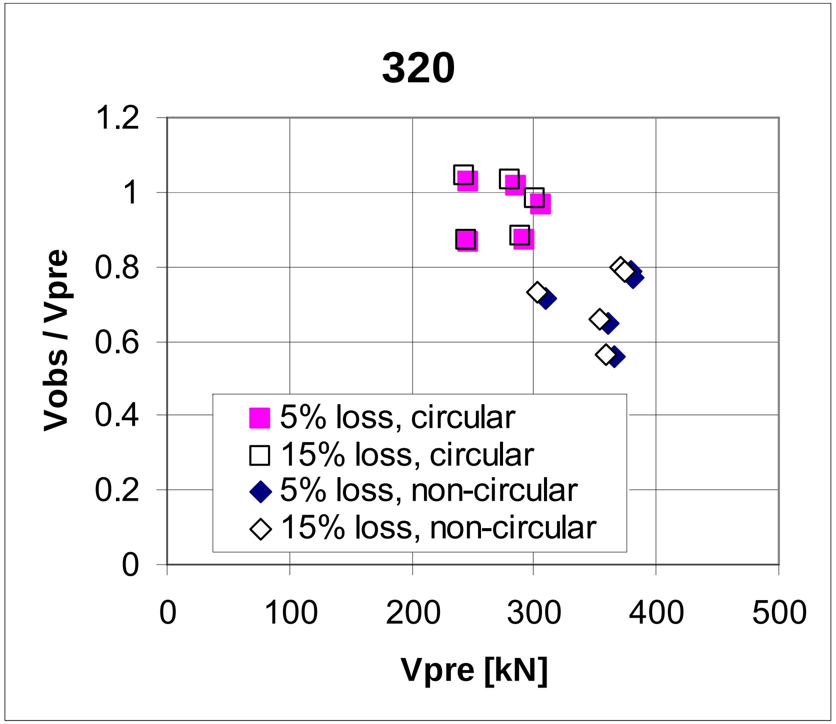


EC2

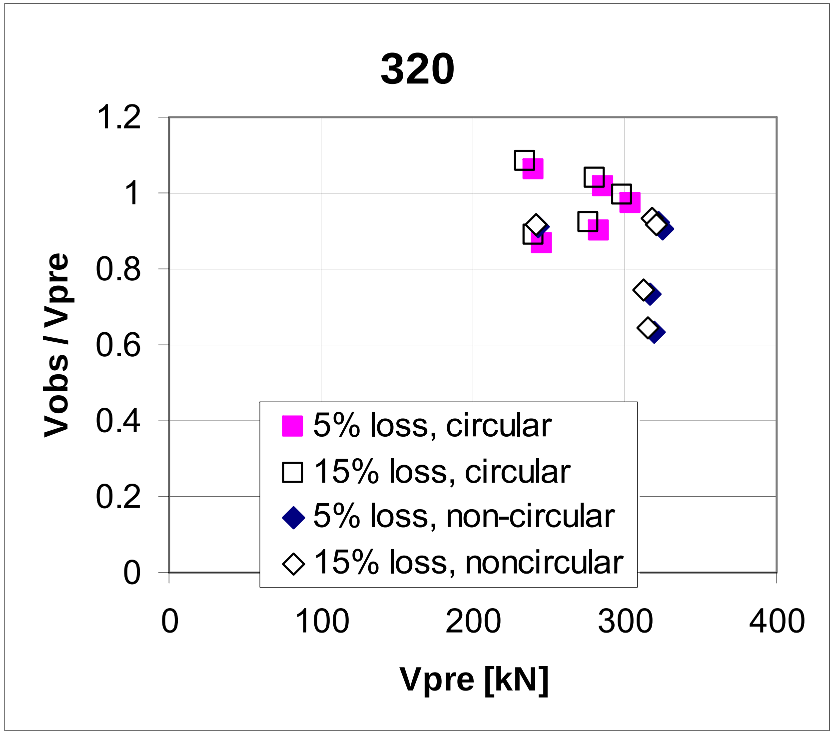


Yang

Fig. 1. 265 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 or Yang.

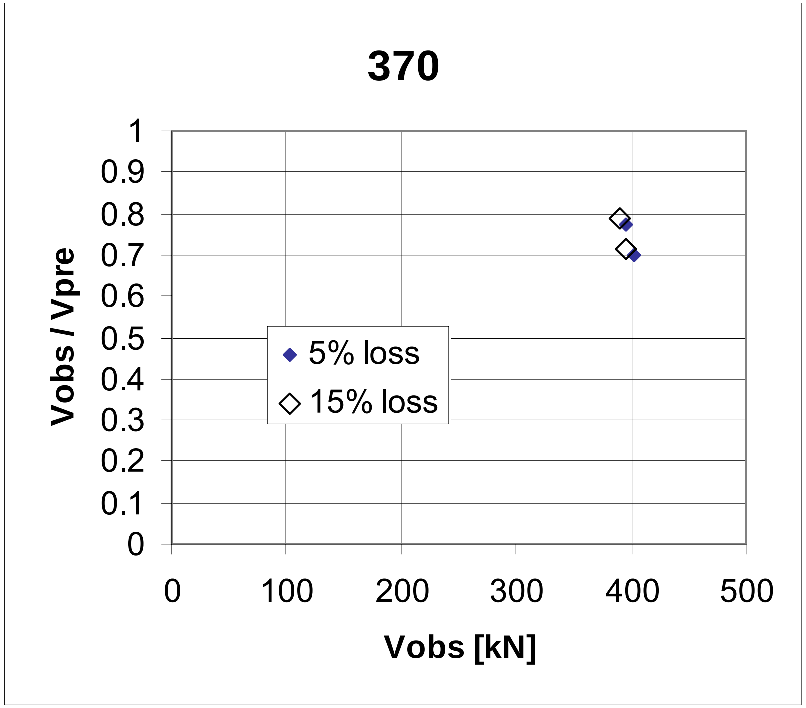


EC2

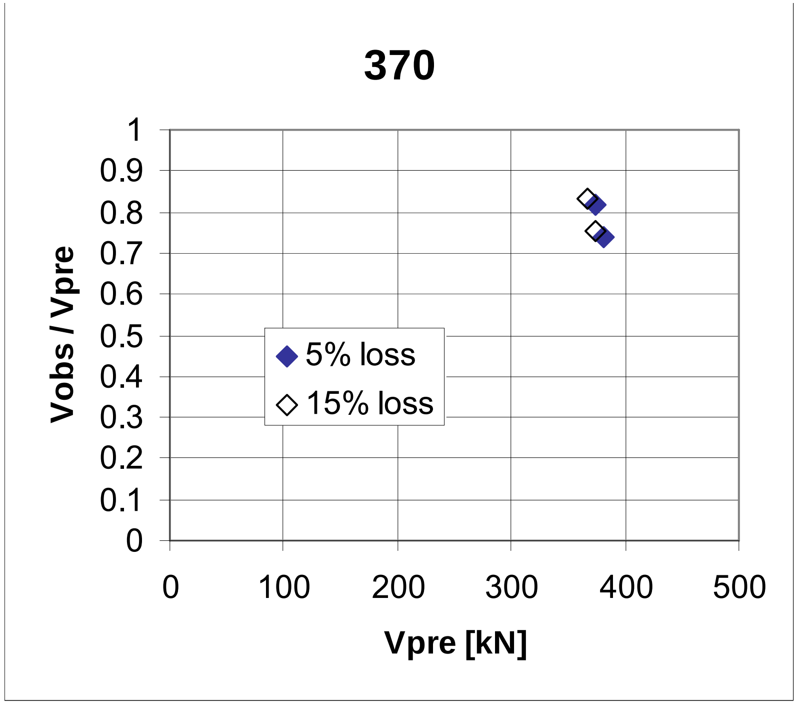


Yang

Fig. 1. 320 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 or Yang.

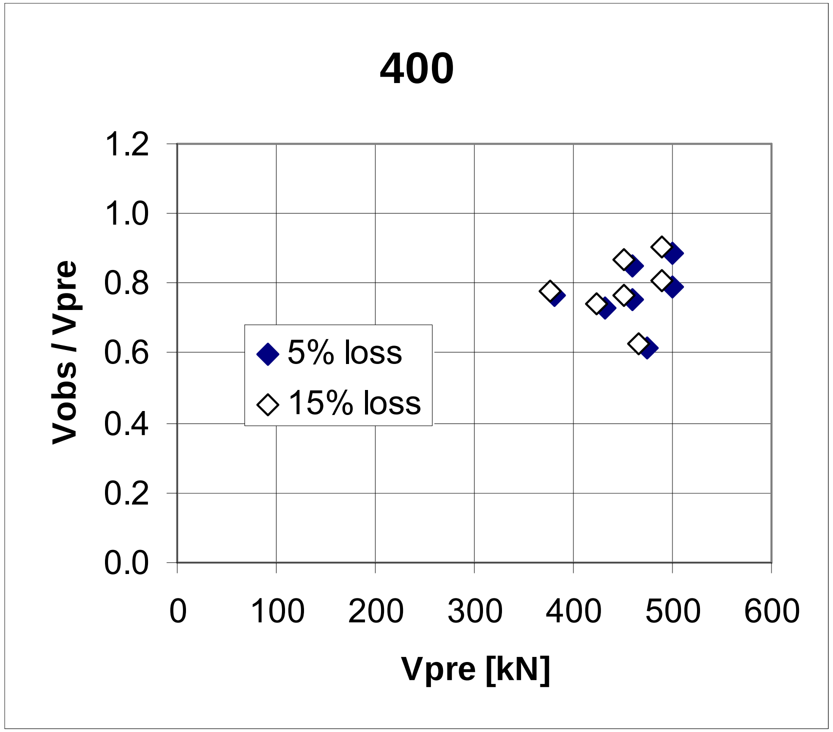


EC2

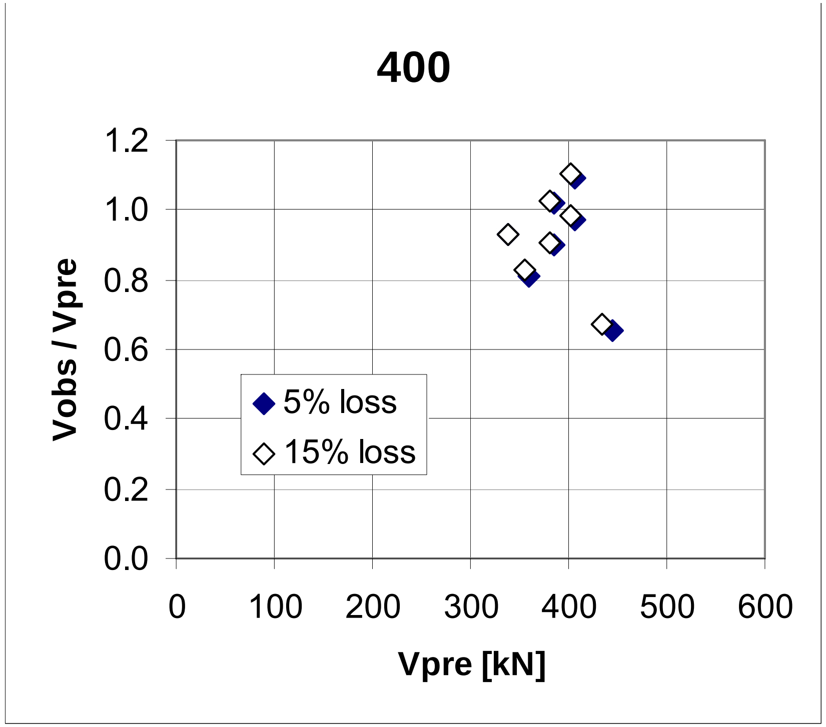


Yang

Fig. 1. 370 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 and Yang.



EC2



Yang

Fig. 1. 400 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 and Yang.

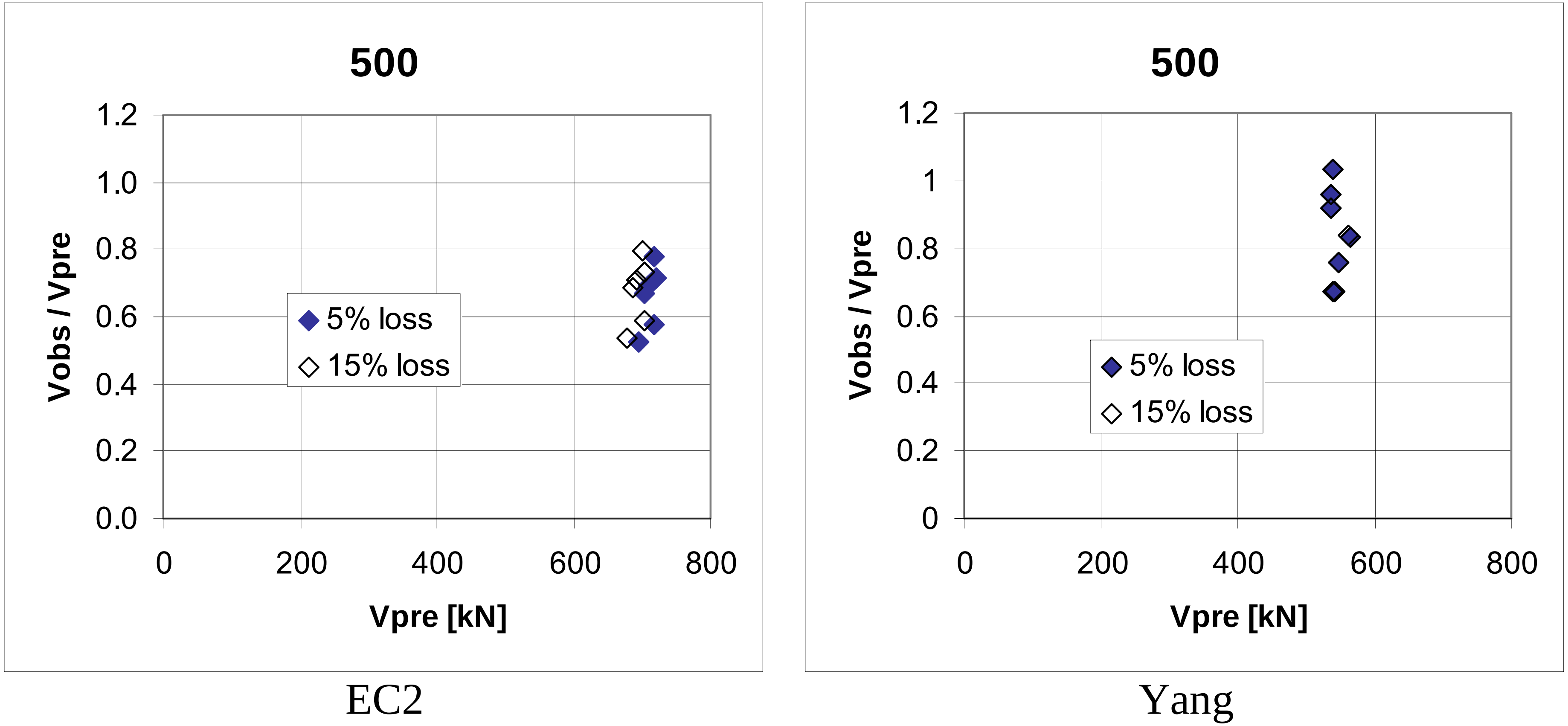


Fig. 1. 500 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using mean tensile strength and the approach of EC2 and Yang.

Comparison of Characteristic resistances: $V_{observed}$ / $V_{predicted,k}$

Requirement for characteristic resistance:

- $V_{observed}$ / $V_{k,predicted}$ $\geq$ 1.0 in 95 % of cases

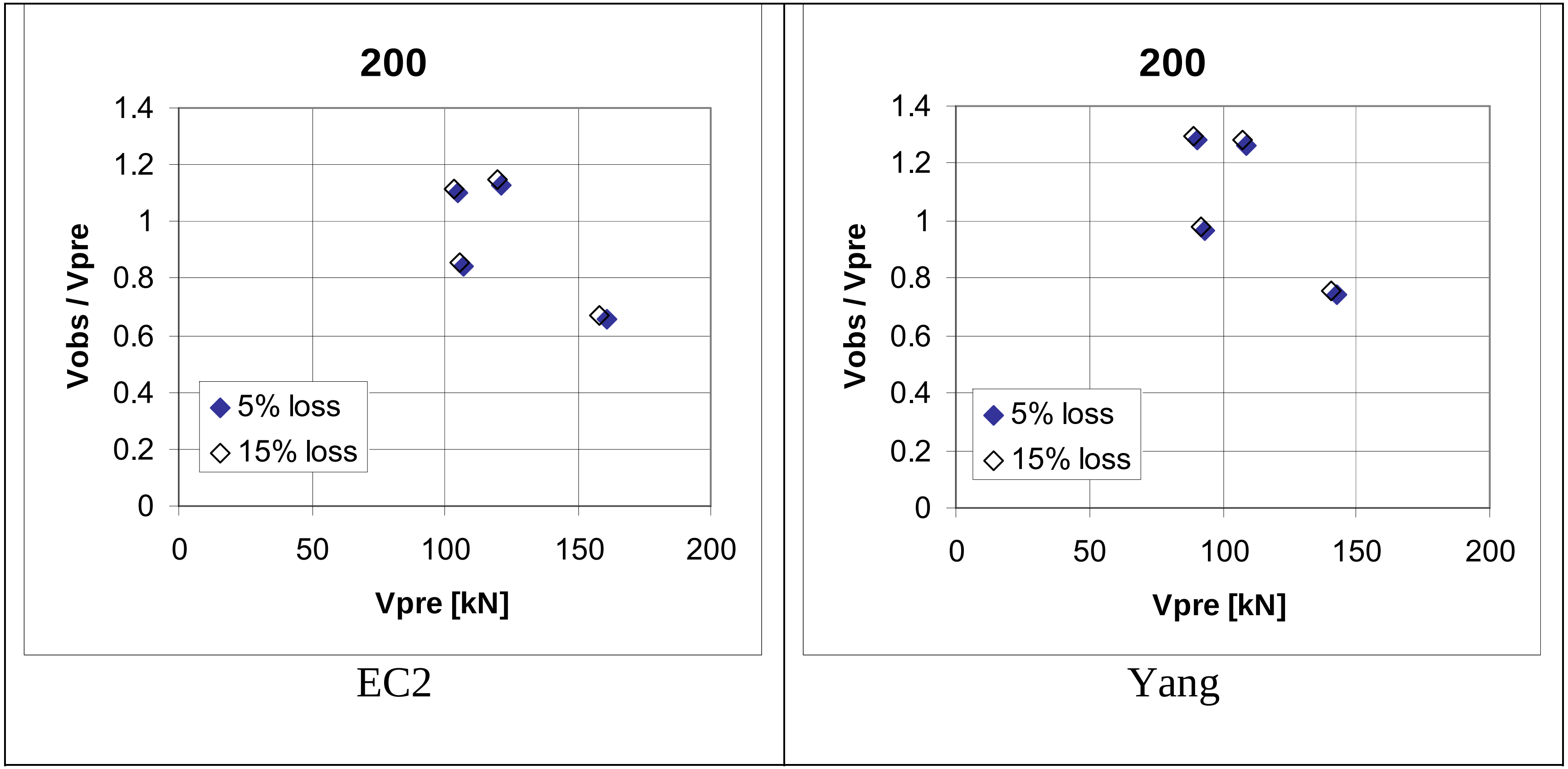


Fig. 1. 200 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

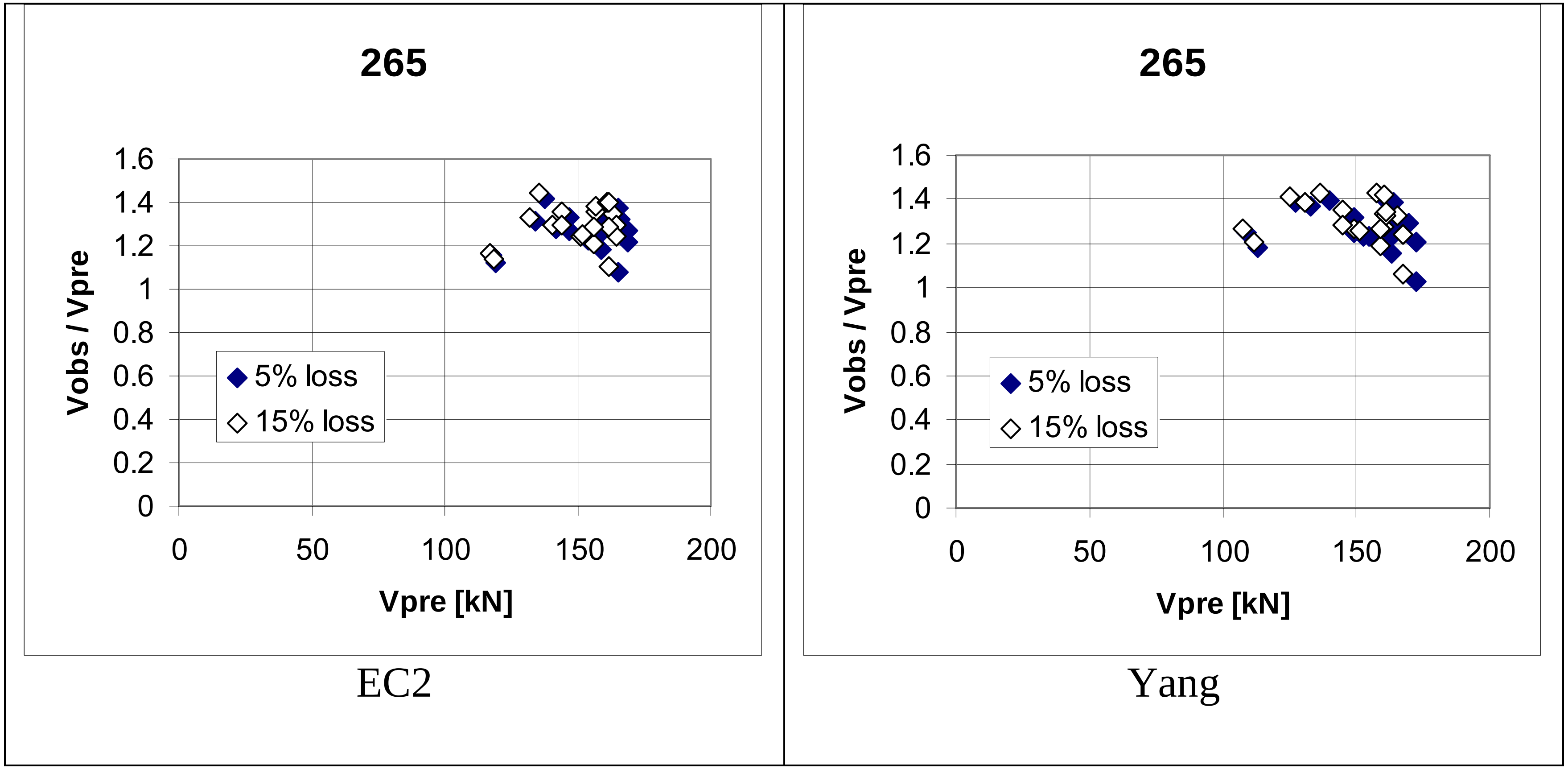


Fig. 1. 265 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

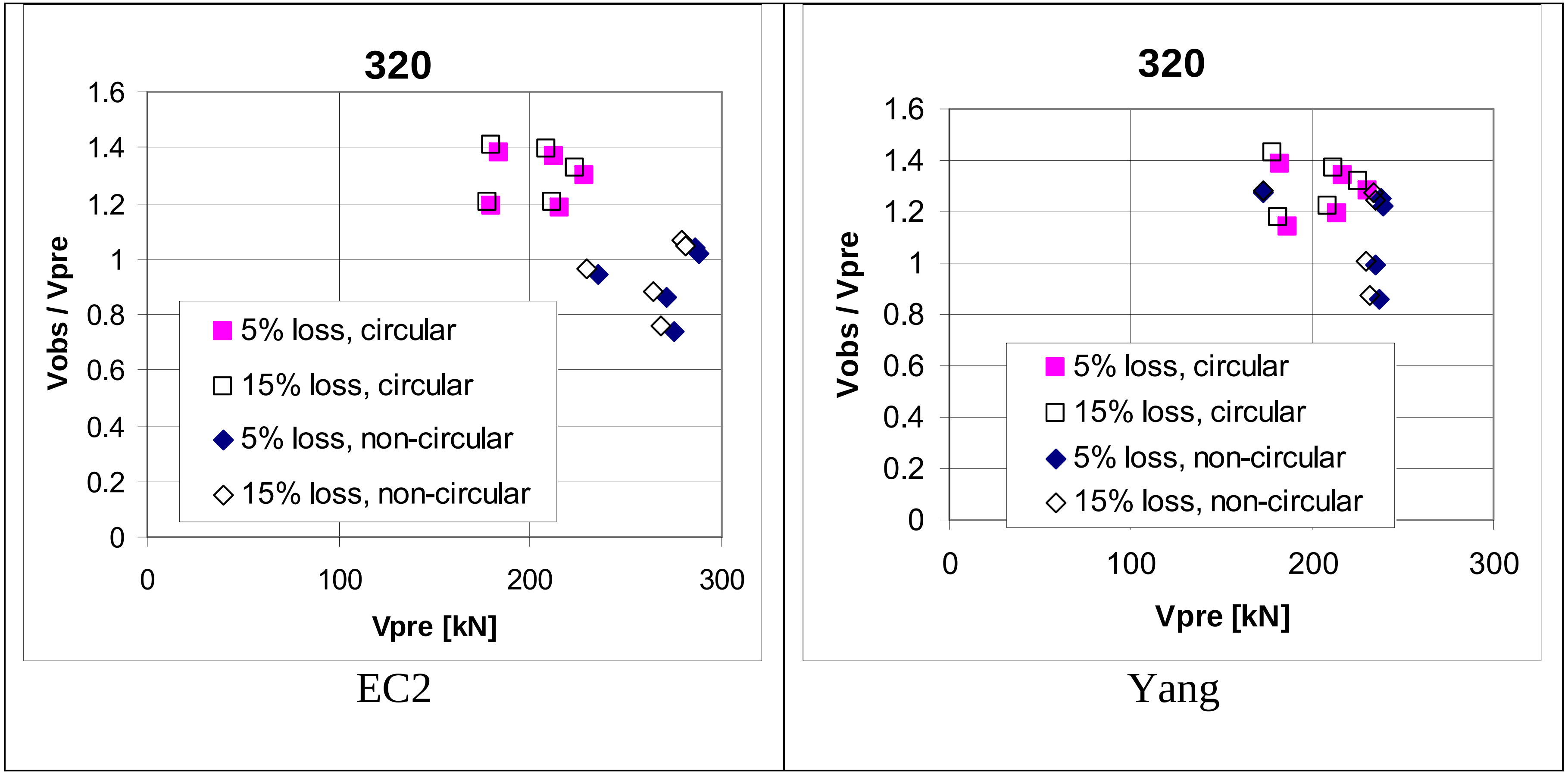


Fig. 1. 320 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

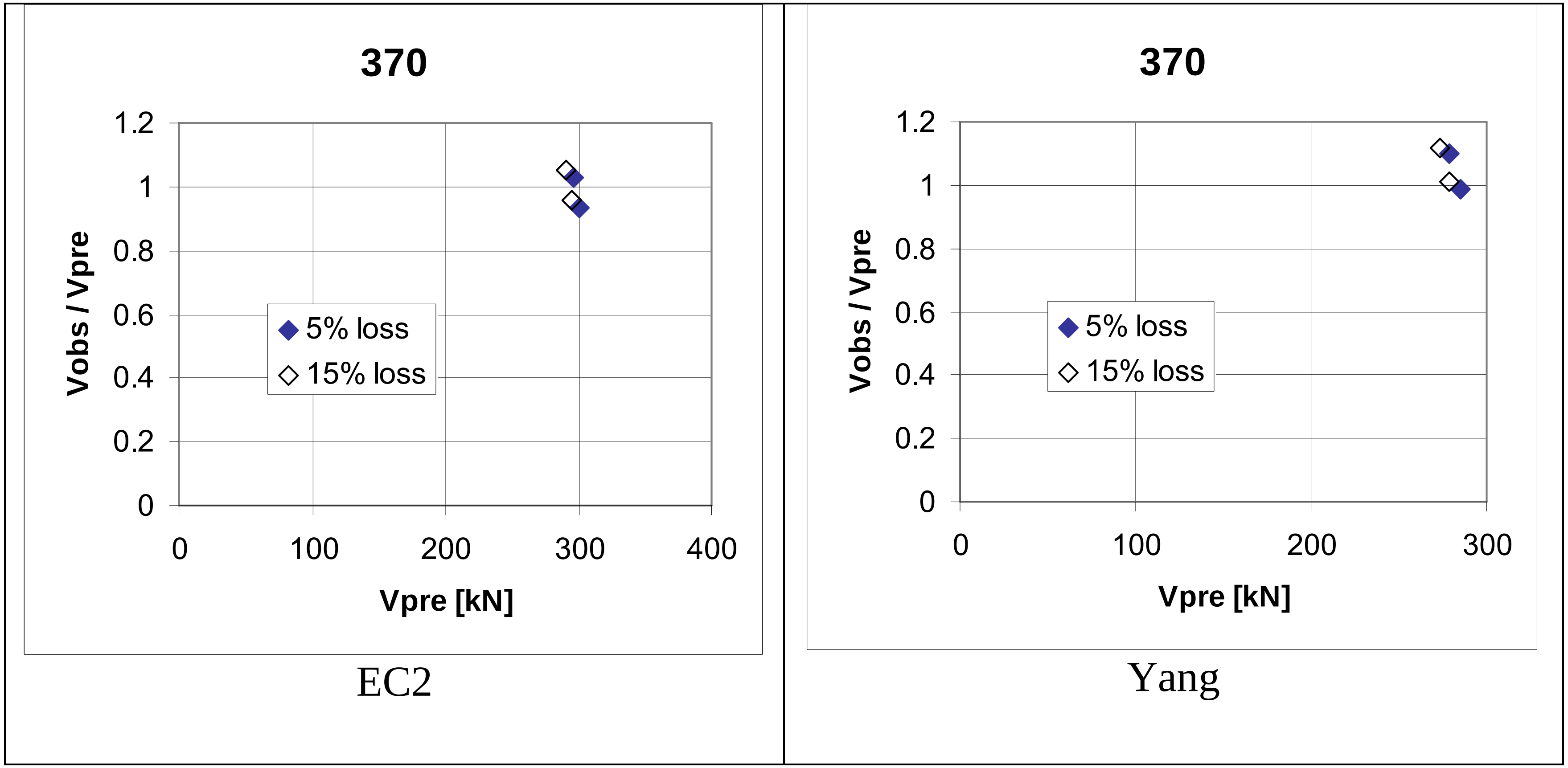


Fig. 1. 370 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

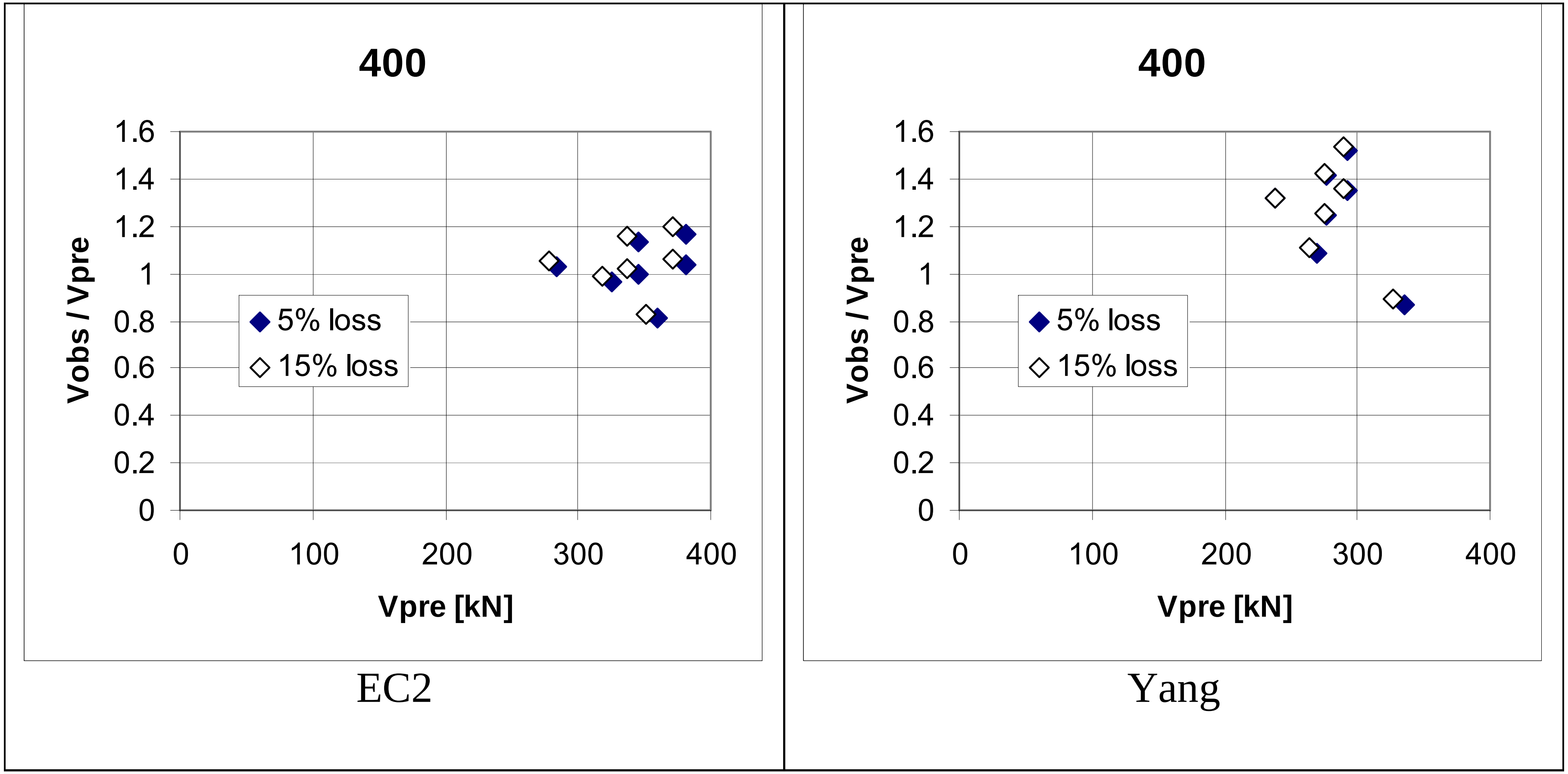


Fig. 1. 400 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

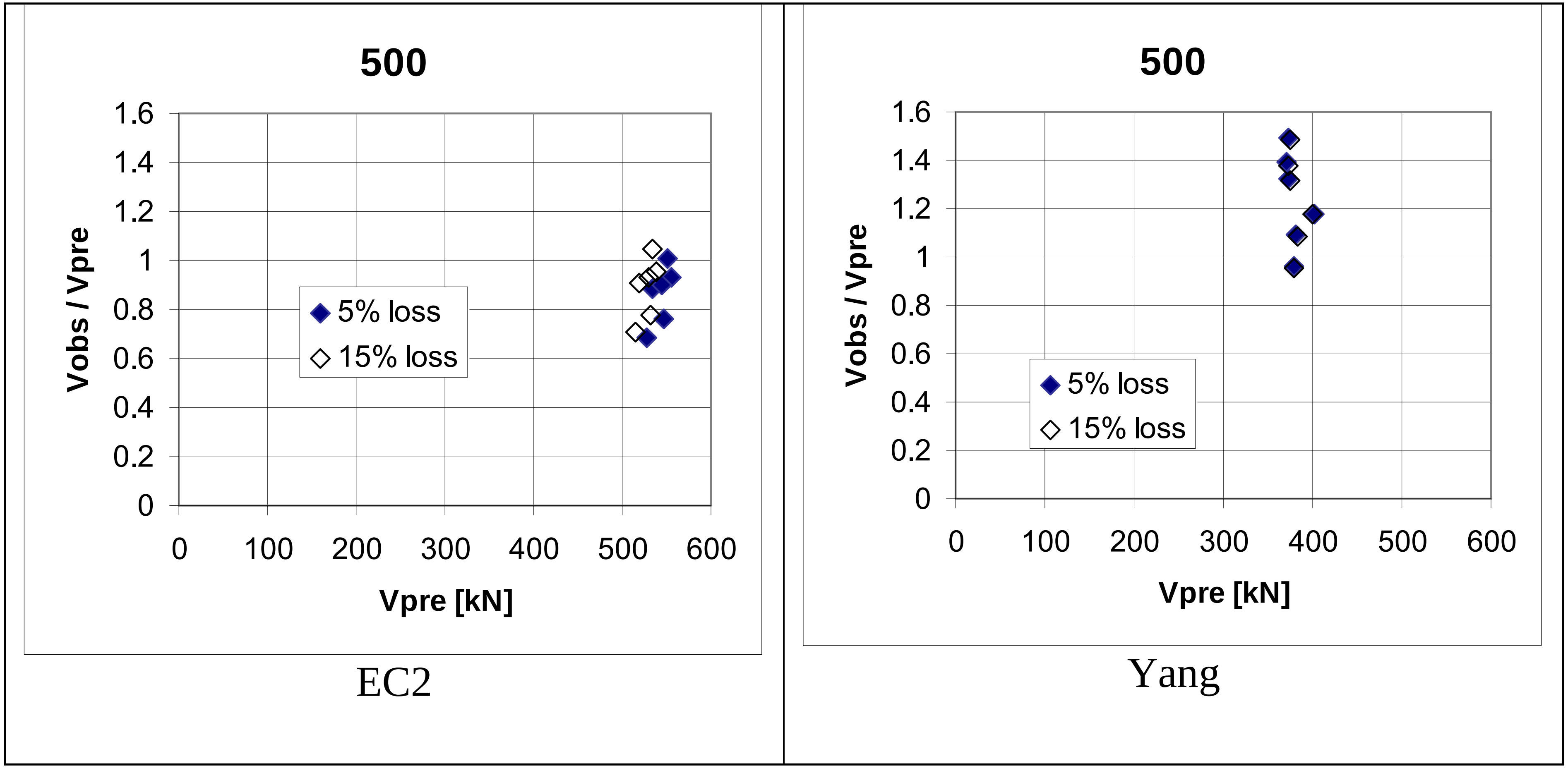
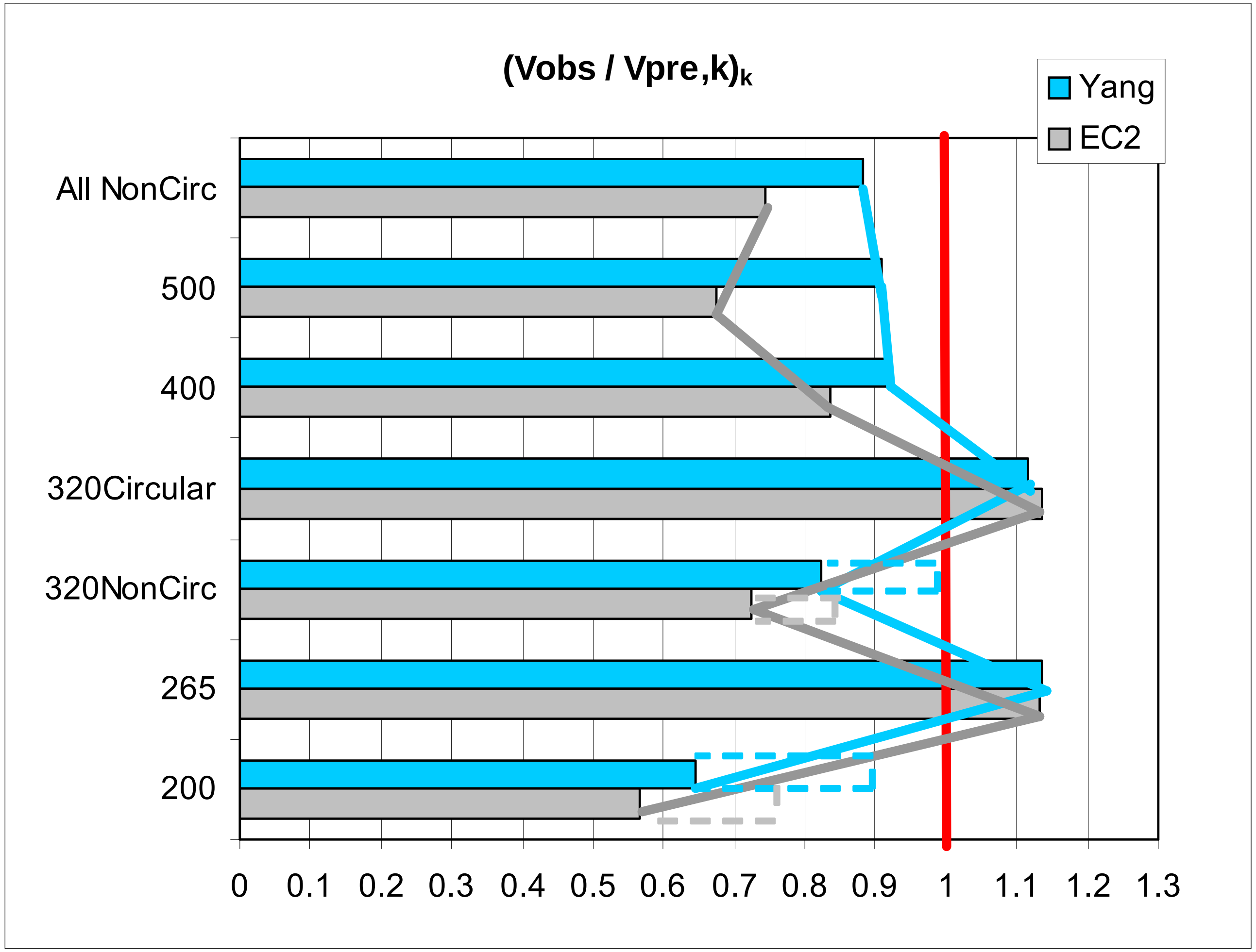
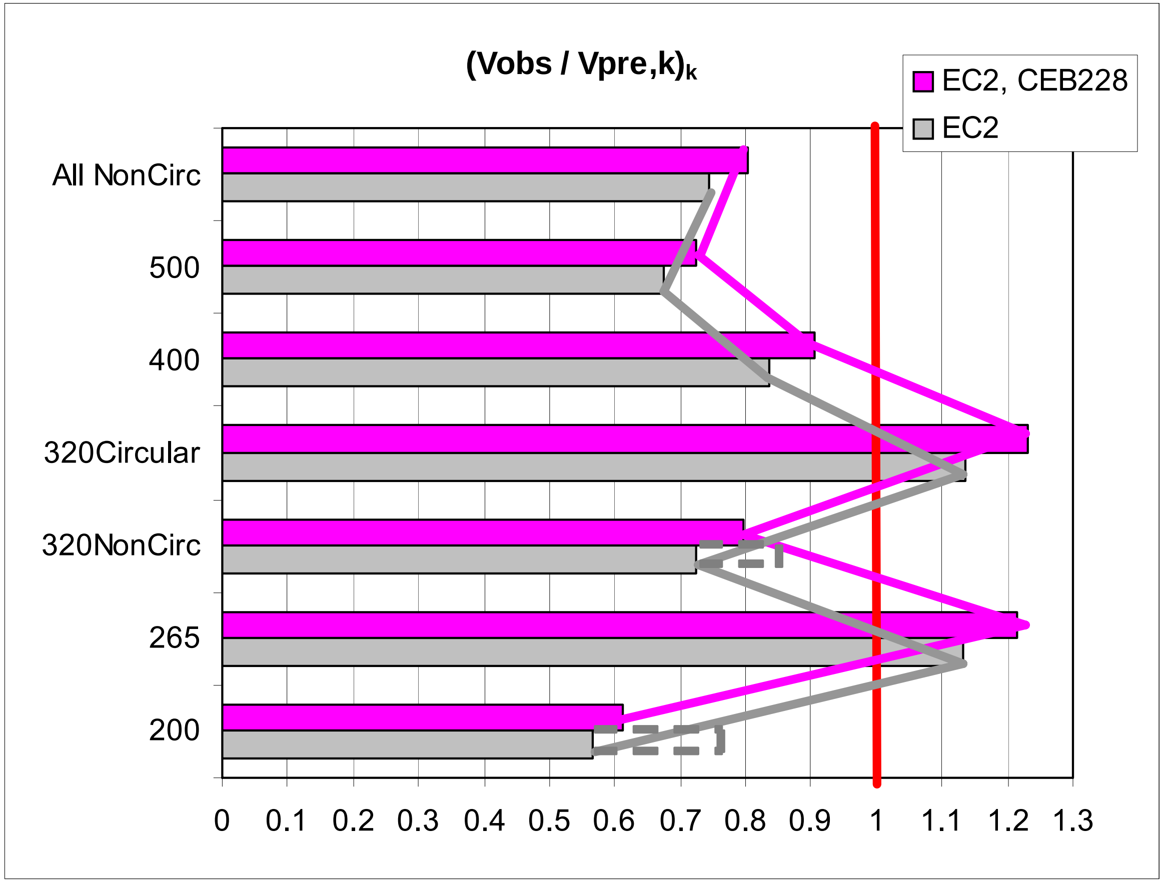


Fig. 1. 500 mm slabs. Ratio of observed (V_{obs}) to predicted resistance (V_{pre}) calculated using characteristic tensile strength and the approach of EC2 and Yang.

Characteristic resistances: test / calculated



Characteristic resistances: test / calculated



Magenta: tensile strength reduced according to CEB Bull. 228

Comparison with other research results

Mean value of ratios of observed shear resistance to predicted shear resistance.

Slab thickness [mm]	All	265	370–400	500
Yang (FIP 1988)	0,92	1,13	0,86	-
Fellinger (EC2)	0,92		0,87 (0,84)*)	-
Present study (EC2)	0,83	0,95	0,77	0,67

**) Excluding four tests would give 0,84 here. In these four tests (PC400-1, PC400-2, ST400-1, ST400-2) the loaded slab ends were provided with transversally reinforced cast-in-situ concrete extending 50 mm into the voids and 50 mm outside the slab ends.*

Conclusions

- The prediction of web shear resistance according to EC2 can be considerably improved by calculating the stresses more accurately e.g. using Yangs' approach
- Based on this, Yang's method or some other similar method should be used instead of EC2 method
- There may still be need for a calibration factor for some slab types
- The differences between slab types when using Yang's approach may be attributable to variations in the tensile strength (compaction etc.)

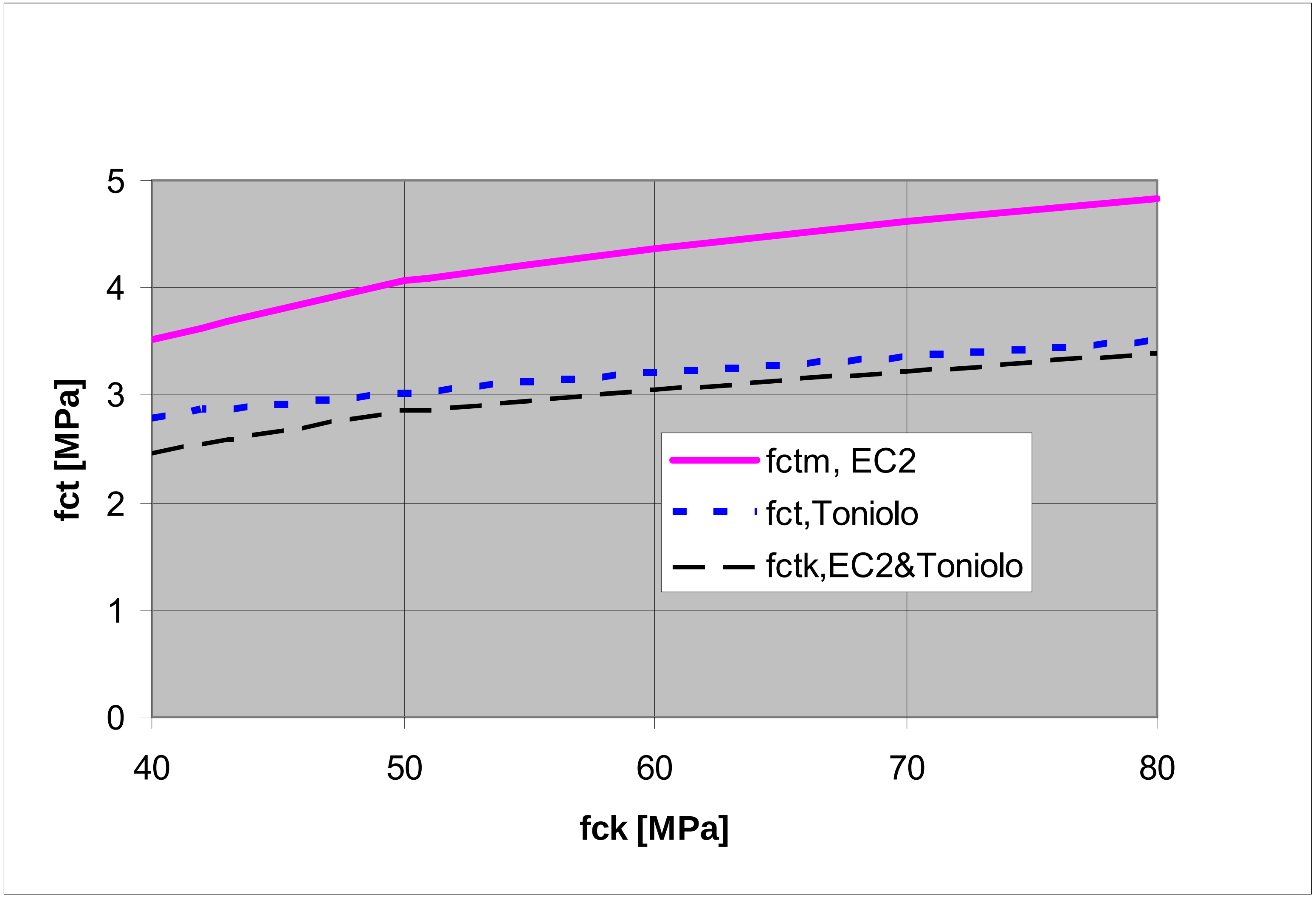
Criticism

- The number of tests / slab type was small
- Many ordinary slab types were missing (slip-formed, elliptical voids etc.)

About Annex J of EN 1168

- The validity of a design formula cannot be verified by three tests only
- According to Annex J, all three tests may end with three different failure modes and still the results are treated as if they belonged to the same population
- Even if the failure modes are the same in all three tests, the characteristic resistance cannot be calculated based on three results only
- Ergo: the resistance must be calculated using the mean strength and compared with the observed mean
- Cores can be drilled, but how to convert the core strengh to cylinder strength? (Proposal: $f_{c,K150} = f_{c,c50}$)

Tensile strength proposed by Toniolo



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VTT Research Notes 2292

2005

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http://www.vtt.fi/inf/pdf/